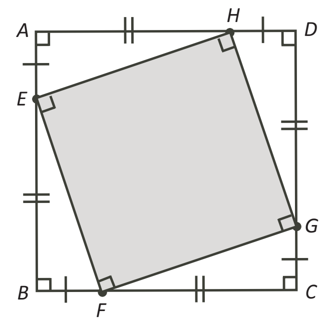
# Problem of the Week
## Problem C and Solution
## The Inner Square




### Problem

$ABCD$ is a square with area 64 $\text{m}^2$. $E, F, G,$ and $H$ are points on sides $AB, BC, CD,$ and $DA$, respectively, such that $AE = BF = CG = DH = 2$ m. $E, F, G,$ and $H$ are connected to form square $EFGH$. Determine the area of $EFGH$.

## Solution

The area of square $ABCD$ is 64 $\text{m}^2$. Therefore the side lengths are 8 m since $8 \times 8 = 64$ and the area is calculated by multiplying the length and the width.

Each of the smaller parts of the sides of square $ABCD$ are 2 m so the longer parts of the sides are $8 - 2 = 6$ m.

Approach #1 to finding the area of square $EFGH$

In right $\triangle HAE$, $AE = 2$ and $AH = 6$. We can use one side as the base and the other as the height in the calculation of the area of the triangle since they are perpendicular to each other. Therefore the area of $\triangle HAE = \frac{AE \times AH}{2} = \frac{2 \times 6}{2} = 6$ $\text{m}^2$. Since each of the triangles has the same base length and height, their areas are equal and the total area of the four triangles is $4 \times 6 = 24$ $\text{m}^2$.

The area of square $EFGH$ can be determined by subtracting the area of the four triangles from the area of square $ABCD$. Therefore the area of square $EFGH = 64 - 24 = 40$ $\text{m}^2$.

Approach #2 to finding the area of square $EFGH$

Some students may be familiar with the Pythagorean Theorem. This theorem states that in a right triangle, the square of the length of the hypotenuse (the longest side) is equal to the sum of the squares of the other two sides. The longest side is located opposite the right angle.

In right $\triangle HAE$, $AE = 2$, $AH = 6$ and $HE$ is the hypotenuse. Therefore,

$$\begin{aligned} HE^2 &= AE^2 + AH^2 \\ &= 2^2 + 6^2 \\ &= 4 + 36 \\ \therefore HE^2 &= 40 \qquad \text{(See the note below.)} \\ \text{Taking the square root,} \qquad HE &= \sqrt{40} \text{ m} \end{aligned}$$

But $EFGH$ is a square so all of its side lengths equal $\sqrt{40}$. The area is calculated by multiplying the length and the width. The area of $EFGH = \sqrt{40} \times \sqrt{40} = 40$ $\text{m}^2$. Therefore, the area of square $EFGH$ is 40 $\text{m}^2$.

**Note:** The area of the square is $HE^2 = 40$ $\text{m}^2$. We found this when we used the Pythagorean Theorem above.