Problem of the Week

Problem C and Solution

Around We Go

Problem

A circle with centre O has a point A on the circumference. Radius OA is rotated 20° clockwise about the centre, resulting in the image OB. Point A is then connected to point B. Radius OB is rotated 20° clockwise about the centre, resulting in the image OC. Point B is then connected to point C.

The process of clockwise rotations continues until some radius rotates back onto OA. Every point on the circumference is connected to the points immediately adjacent to it as a result of the process. A polygon is created.

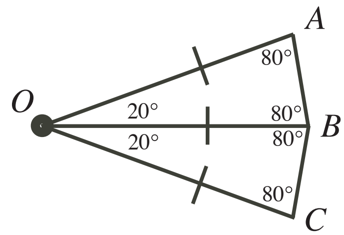
a) Determine the number of sides of the polygon.

b) Determine the sum of the angles in the polygon. That is, determine the sum of the angles at each of the vertices of the polygon.

Solution

Each time the process is repeated, another congruent triangle is created. Each of these triangles has a 20° angle at O, the centre of the circle. But a complete rotation at the centre is 360°. Since each angle in the triangles at the centre of the circle is 20° and the total measure at the centre is 360°, then there are $360 \div 20 = 18$ triangles formed. This means that there are 18 distinct points on the circumference of the circle and the polygon has 18 sides. An 18-sided polygon is called an *octadecagon*, from octa meaning 8 and deca meaning 10.

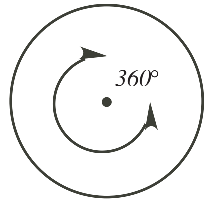
The other two angles in the each of the congruent triangles are equal. (Two sides of the triangle are radii of the circle. The triangles are therefore isosceles.) The angles in a triangle sum to 180° so after the 20° angle is removed, there is 160° remaining for the other two angles. It follows that each of the other two angles in each triangle measures $160^\circ \div 2 = 80^\circ$. The following diagram illustrates this information for the two adjacent triangles AOB and BOC.



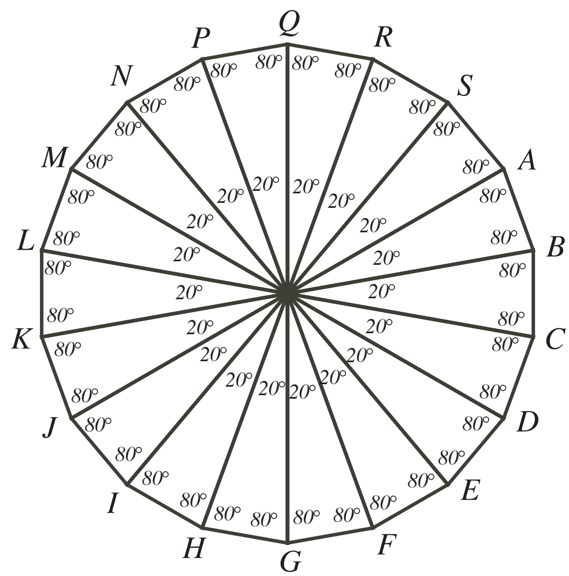
Each angle in the polygon is formed by an 80° angle from one triangle and the adjacent 80° angle from the next triangle. For example, $\angle ABC = \angle ABO + \angle OBC = 80^\circ + 80^\circ = 160^\circ$. There are 18 vertices in the octadecagon and the angle at each vertex is 160°. Therefore the sum of the angles in the octadecagon is $18 \times 160^\circ = 2\,880^\circ$.

Diagrams are provided on the next page to further support the solution.

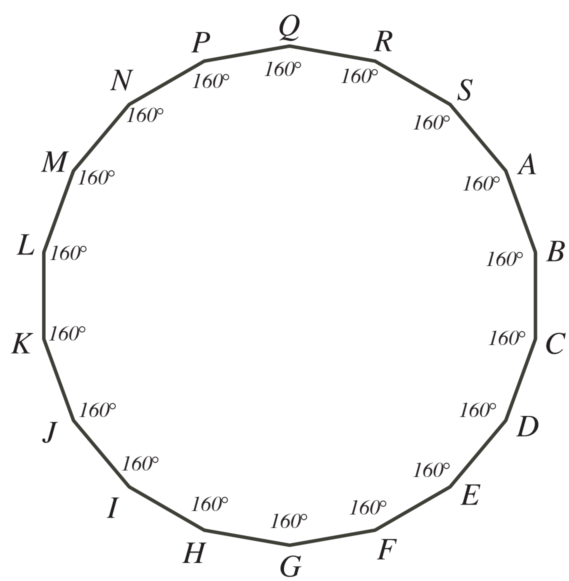
Angle at the Centre of a Circle



Completing the Construction



The Octadecagon - 18 sided Polygon



Notice the vertices of the octadecagon are labelled A to S, but the letter O is missing since it was used in the original construction as the centre of the circle.