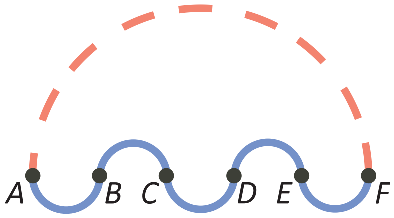

Problem of the Week
Problem C and Solution
Curvy Contest

Problem

Two paths are built from A to F as shown on the diagram above. The distance from A to F in a straight line is 100 m. Points B, C, D, and E lie along AF such that $AB = BC = CD = DE = EF$. The upper path, shown with a dashed line, is a semi-circle with diameter AF. The lower path, shown with a solid line, consists of five semi-circles with diameters AB, BC, CD, DE, and EF. Starting at the same time, Bev and Mike ride their tricycles along these paths from A to F. Bev rides along the upper path from A to F while Mike rides along the lower path from A to F. If they ride at the same speed, who will get to F first?

Solution

The circumference of a circle is found by multiplying its diameter by π. To find the circumference of a semi-circle, divide its circumference by 2.

The length of the upper path is equal to half the circumference of a circle with diameter 100 m. The length of the upper path equals $\pi \times 100 \div 2 = 50\pi$ m. (This is approximately 157.1 m.)

Each of the semi-circles along the lower path have the same diameter. The diameter of each of these semi-circles is $100 \div 5 = 20$ m. The length of the lower path is equal to half the circumference of five circles, each with diameter 20 m. The distance along the lower path equals $5 \times (\pi \times 20 \div 2) = 5 \times (10\pi) = 50\pi$ m.

Since both Bev and Mike ride at the same speed and both travel the same distance, they will arrive at point F at the same time. Neither wins the race since both arrive at the same time. The answer to the problem may surprise you. Most people, at first glance, would think that the upper path is longer.

If you were to extend the problem so that Bev travels the same route but Mike travels along a lower path made up of 100 semi-circles of equal diameter from A to F, they would still both travel exactly the same distance, 50π m. Check it out!