Problem of the Week
Problem C and Solution
What's Your Angle?

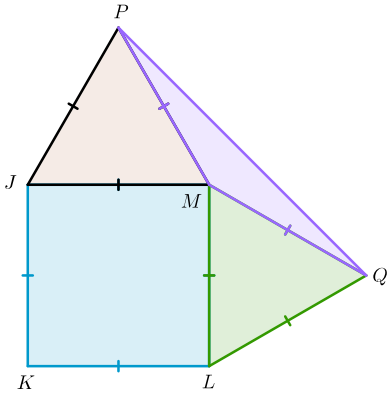
Problem

$JKLM$ is a square. Points P and Q are outside the square such that both $\triangle JMP$ and $\triangle MLQ$ are equilateral. Determine the measure, in degrees, of $\angle MPQ$.

Solution

Since $JKLM$ is a square, $JK = KL = LM = MJ$.
Since $\triangle JMP$ is equilateral, $MJ = JP = MP$.
Since $\triangle MLQ$ is equilateral, $LM = LQ = QM$.

It follows that $JK = KL = LM = MJ = JP = MP = LQ = QM$. These equal side lengths are shown on the diagram.



Each angle in a square is $90°$. Therefore, $\angle JML = 90°$.

Each angle in an equilateral triangle is $60°$. Therefore, $\angle JMP = 60°$ and $\angle LMQ = 60°$.

A complete revolution is $360°$. Since $\angle PMQ$, $\angle JMP$, $\angle JML$ and $\angle LMQ$ form a complete revolution, then

$$\begin{aligned} \angle PMQ &= 360° - \angle JMP - \angle JML - \angle LMQ \\ &= 360° - 60° - 90° - 60° \\ &= 150° \end{aligned}$$

In $\triangle MPQ$, $MP = QM$ and the triangle is isosceles. It follows that $\angle MPQ = \angle MQP$.

In a triangle, the sum of the three angles is $180°$. Since $\angle PMQ = 150°$, then the sum of the two remaining equal angles must be $30°$. Therefore, each of the remaining two angles must equal $15°$ and it follows that $\angle MPQ = 15°$.