



Problem of the Week

Problem C

That's Not Fair Or Is It?

Anna and Elle are twins. Everything they do together must be fair. For a school mathematics project they created a game that uses a specially made pair of six-sided dice. One die has the even numbers 2, 4, 6, 8, 10, and 12 on its faces and the other die has the odd numbers 1, 3, 5, 7, 9, and 11 on its faces.

A turn consists of rolling the dice and using the two numbers that appear on the top faces. Anna and Elle take turns rolling the dice.

Anna performs the following steps after each roll to determine whether or not she gets a point.

1. Anna determines the sum, S , of the numbers on the top faces. On the roll shown below, $S = 9$.
2. Using S , Anna determines, D , the digit sum. If S is a single digit number, then D is the same as S . If S is a two digit number, then D is the sum of the two digits. (If the roll is a 6 and a 3 like below, then the digit sum and the sum are both 9. If the roll is a 5 and 10, then the sum is 15 and the digit sum is $1 + 5 = 6$. If the roll is a 9 and 10, then the sum is 19 and the digit sum is $1 + 9 = 10$.)

Anna gets a point if the digit sum D is a multiple of 4.

Elle gets a point if one of the numbers on the top face is a multiple of the number on the other top face. With the dice roll shown below, Elle would get a point since 6 is a multiple of 3.

Is this game fair? That is, do Anna and Elle have the same probability of getting a point on any roll? Justify your answer.

