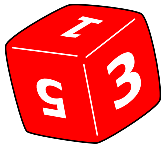
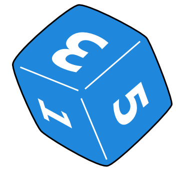


Problem of the Week

Problem C and Solution

Totally Unusual



Problem

The dice shown above are unusual. A usual six-sided die would have the numbers 1, 2, 3, 4, 5 and 6 on the sides. These dice, however, are unusual because the numbers on the six sides are 1, 2, 3, 5, 7 and 9. Two of these unusual dice, one red and one blue, are rolled and the numbers on the upper faces are added together. A winning roll occurs when the sum is either a perfect square or a prime number. Determine the probability that you win on any particular roll.

A *prime number* is an integer greater than 1 that has only two positive divisors, 1 and itself. For example, the number 17 is prime. A *perfect square* is an integer that is the product of some integer and itself. For example, 9 is a perfect square since $3 \times 3 = 9$.

Solution

To solve this problem we will create a chart showing all of the possible rolls and the corresponding sums.

		Upper Face of Blue Die					
		1	2	3	5	7	9
Upper Face of Red Die	1	2	3	4	6	8	10
	2	3	4	5	7	9	11
	3	4	5	6	8	10	12
	5	6	7	8	10	12	14
	7	8	9	10	12	14	16
	9	10	11	12	14	16	18

From the table, we see that there are 36 possible outcomes. We also see that the perfect squares 4, 9 and 16 appear in the table seven times.

The lowest number in the table is 2 and the highest number in the table is 18. The prime numbers appearing in the table in this range of numbers are 2, 3, 5, 7, and 11. These numbers appear in the table a total of nine times.

Since a number cannot be both a prime number and a perfect square, we can be certain that we have not counted a desirable outcome more than once. The total number of prime number sums and perfect square sums is $7 + 9 = 16$.

To determine the probability of a specific outcome, we divide the number of times the specific outcome occurs by the total number of possible outcomes. The probability of winning on a particular roll is $16 \div 36 = \frac{4}{9}$. You have approximately a 44% chance of winning. A game is considered "fair" if you have a 50% chance of winning.