Problem of the Week
Problem B
How Will Dennis Fare?

Problem

To encourage Dennis to work harder at his math problems, his mother promised she would pay him 10 cents for each right answer, but subtract 5 cents for each wrong answer.

a) If he earned 20 cents after doing 32 problems, how many problems did Dennis get correct? How many did he get wrong?

b) Dennis answered another 32 problems. What is the least number of problems he would have to get correct in order to earn more than one dollar?

Think about how to explain this mathematically in an efficient way.

Solution

Dennis earns 10 cents for each right answer, but loses 5 cents for each wrong answer.

a) Since two wrong answers cancel the earnings of one right answer, Dennis would earn nothing if he had twice as many wrong answers as right. If he earned 20 cents on 32 problems, then he must have done better than that. Let's try some possibilities, starting near a two-to-one split of wrong versus right.

Trial 1: 22 wrong and 10 right gives 10×10 cents $- 22 \times 5$ cents $=$ a 10 cent loss.

Trial 2: 21 wrong and 11 right gives 11×10 cents $- 21 \times 5$ cents $= +5$ cents.

Trial 3: 20 wrong and 12 right gives 12×10 cents $- 20 \times 5$ cents $= +20$ cents.

So Dennis got 12 problems right and 20 problems wrong.

b) Using the trials in a), it appears that each increase of 1 in the number of correct answers (and a corresponding decrease of 1 in the number of wrong answers) gains Dennis 15 cents. To earn at least \$1.00, he needs to earn at least 80 cents more than the 20 cents he earned in part a). Since each increase in correct answers gains him 15 cents, he will earn more than \$1.00 if he has 6 more than in a), since he will gain $6 \times 15 = 90$ cents.

Checking: 18 right and 14 wrong gives 18×10 cents $- 14 \times 5$ cents $= +110$ cents, or \$1.10. (However, 17 right and 15 wrong gives 17×10 cents $- 15 \times 5$ cents $= +95$ cents.)

So Dennis needs at least 18 right answers to earn more than \$1.00.