



Problem of the Week

Problem B and Solution

Confusion Rules

Problem

- a) Tian and Mary disagree about the value of the expression $6 \times 4 - 8 \div 2 + 5$.

Tian thinks it should be 13, but Mary says it should be 25. Try to explain how each of them arrived at their answer.

- b) The expression $6 \times 4 - 8 \div 2 + 5$ mixes the operations of addition and subtraction with that of multiplication and division. To avoid confusion in evaluating expressions like these, a special rule is followed: “**Perform all multiplication and division first. Then perform all the addition and subtraction.**”

To help keep things straight, we can put brackets around the terms involving multiplication or division. The above expression could be written $(6 \times 4) - (8 \div 2) + 5$.

If the special rule is followed in the expression $6 \times 4 - 8 \div 2 + 5$, which student, Tian or Mary, has the correct answer?

- c) Evaluate each of the following expressions using the special rule.
- $6 \div 3 + 4 - 2 \times 2 + 5$
 - $2 \times 3 \times 4 - 11 - 6 \times 2 + 20 \div 5$
 - $0.5 \times 24 + 15 \div 5 - 13$

Extension: Find the values of A and B so that each of the following expressions equals zero.

- $22 \div 11 + 3 \times A - 7 \times 2$
- $24 \div B - 2 \times 3 + 7 - 3 \times 3$

Solution

- a) Tian performed his operations moving left to right.

$$6 \times 4 - 8 \div 2 + 5 \rightarrow 24 - 8 \div 2 + 5 \rightarrow 16 \div 2 + 5 \rightarrow 8 + 5 \rightarrow 13$$

Mary performed her operations by doing multiplication and division first followed by subtraction and addition last.

$$6 \times 4 - 8 \div 2 + 5 \rightarrow 24 - 8 \div 2 + 5 \rightarrow 24 - 4 + 5 \rightarrow 20 + 5 \rightarrow 25$$

- b) Using the special rule, Mary has the correct answer.

$$6 \times 4 - 8 \div 2 + 5 = 24 - 8 \div 2 + 5 = 24 - 4 + 5 = 20 + 5 = 25$$





c) Evaluating each of the expressions using the special rule, we obtain:

$$\begin{array}{ll}
 \text{(i)} & (6 \div 3) + 4 - (2 \times 2) + 5 \\
 & = 2 + 4 - (2 \times 2) + 5 \\
 & = 2 + 4 - 4 + 5 \\
 & = 6 - 4 + 5 \\
 & = 2 + 5 \\
 & = 7 \\
 \text{(ii)} & (2 \times 3 \times 4) - 11 - (6 \times 2) + (20 \div 5) \\
 & = (6 \times 4) - 11 - (6 \times 2) + (20 \div 5) \\
 & = 24 - 11 - (6 \times 2) + (20 \div 5) \\
 & = 24 - 11 - 12 + (20 \div 5) \\
 & = 24 - 11 - 12 + 4 \\
 & = 13 - 12 + 4 \\
 & = 1 + 4 \\
 & = 5
 \end{array}$$

$$\begin{array}{l}
 \text{(iii)} \quad (0.5 \times 24) + (15 \div 5) - 13 \\
 = 12 + (15 \div 5) - 13 \\
 = 12 + 3 - 13 \\
 = 15 - 13 \\
 = 2
 \end{array}$$

Extension: This part may have been challenging.

$$\begin{array}{l}
 \text{(i)} \qquad (22 \div 11) + (3 \times A) - (7 \times 2) \\
 = \qquad 2 \quad + (3 \times A) - 14
 \end{array}$$

We want 2 plus something to equal 14 so that when we subtract 14, the result is 0. The something would have to be 12. But 12 is $3 \times A$ so $A = 4$. We can verify that we have the correct value for A by letting $A = 4$ in the expression.

$$\begin{array}{l}
 (22 \div 11) + (3 \times 4) - (7 \times 2) \\
 = 2 \quad + (3 \times 4) - (7 \times 2) \\
 = 2 \quad + 12 - (7 \times 2) \\
 = 2 \quad + 12 - 14 \\
 = 0
 \end{array}$$

(ii) We will work out the multiplication parts and then determine the value of B .

$$\begin{array}{l}
 (24 \div B) - (2 \times 3) + 7 - (3 \times 3) \\
 = (24 \div B) - 6 + 7 - (3 \times 3) \\
 = (24 \div B) - 6 + 7 - 9
 \end{array}$$

Here you can play with possible values of B . If $B = 3$, $24 \div 3 = 8$, $8 - 6 = 2$, $2 + 7 = 9$ and $9 - 9 = 0$. $B = 3$ is the required value.

