



## Problem of the Week

### Problem B

### Speedy or Not, You May Be Caught!

**Problem**

Various animals can travel at the following top speeds for short distances:

- cheetahs race along at 120 km/h (that is, they travel 120 km every hour);
- cows can gallop at 12.8 km/h;
- snails can slither along at 0.0125 m/s;

A human's top speed is 10.4 m/s.

- a) How many seconds would it take each animal to run a 100 m race? How long would it take a human to complete the 100 m?
- b) While hunting, cheetahs can only maintain their amazing speed for about 15 s, after which they generally give up the chase. Gazelles can run 60 km/h when evading predators. Suppose a cheetah spots a gazelle 300 m away. If both animals start at the same time and run in a straight line, will the gazelle escape?

**For further consideration:**

If the top speeds of the animals could be maintained over a long distance (physically impossible), approximately how long would it take for the animals to run a 42.2 km marathon? How long would it take a human to complete the marathon?

**Solution**

- a) Since we want our answer in seconds, it is helpful to convert hours to seconds:

$$1 \text{ h} = 60 \text{ minutes} = 60 \times 60 = 3600 \text{ seconds}$$

Also, remember that 1 km = 1000 m.

For the cheetah:	$\begin{array}{r} 120 \text{ km in } 1 \text{ h} \\ 120\,000 \text{ m in } 3600 \text{ s} \\ \hline \div 1200 \quad 100 \text{ m in } 3600 \div 1200 = 3 \text{ s} \end{array}$
For the cow:	$\begin{array}{r} 12.8 \text{ km in } 1 \text{ h} \\ 12\,800 \text{ m in } 3600 \text{ s} \\ \hline \div 128 \quad 100 \text{ m in } 3600 \div 128 \approx 28.1 \text{ s} \end{array}$
For the snail:	$\begin{array}{r} 0.0125 \text{ m in } 1 \text{ s} \\ \hline \div 0.0125 \quad 1 \text{ m in } 1 \div 0.0125 = 80 \text{ s} \\ \times 100 \quad 100 \text{ m in } 80 \times 100 = 8000 \text{ s} \end{array}$
For the human:	$\begin{array}{r} 10.4 \text{ m in } 1 \text{ s} \\ \hline \div 10.4 \quad 1 \text{ m in } 1 \div 10.4 \text{ s} \\ \times 100 \quad 100 \text{ m in } 1 \div 10.4 \times 100 \approx 9.6 \text{ s} \end{array}$

To run 100 m, the cheetah would take 3 s, the cow would take approximately 28.1 s, the snail would take 8000 s, and the human would take approximately 9.6 s.





- b) Since the cheetah will give up the chase after 15 s, we need only see where both animals are at that time.

The gazelle runs 60 km in 60 minutes. This is the same as 1 km in 1 minute. This is the same as 1000 m in 60 seconds. Dividing by 4, we determine that the gazelle can run 250 m in 15 seconds.

The cheetah runs 120 km in 60 minutes. This is twice as fast as the gazelle. Therefore, the cheetah can run  $2 \times 250 = 500$  m in 15 seconds.

Since the gazelle had a 300 m headstart, it will be 550 m from where the cheetah started. This is 50 m ahead of the cheetah and the gazelle will escape to live another day.

Let's look at this using a chart. In 1 second, the gazelle runs  $\frac{1000}{60} \approx 16.67$  m and the cheetah will run  $\approx 33.33$  m. Therefore, every second the cheetah will get  $\approx 16.67$  m closer to the gazelle. Every 3 seconds the cheetah will gain 50 m on the gazelle. The cheetah starts at 0 m and the gazelle starts at 250 m. The chart illustrates that the cheetah is getting closer but never reaches the gazelle in 15 seconds.

Time (in s)	Location of cheetah (in m)	Location of Gazelle	Distance Apart (in m)
0	0	300	300
1	33.33	316.67	283.33
2	66.66	333.33	266.67
3	100	350	250
4	133.33	366.67	233.33
5	166.66	383.33	216.67
6	200	400	200
7	233.33	416.67	183.33
8	266.66	433.33	166.67
9	300	450	150
10	333.33	466.67	133.33
11	366.66	483.33	116.67
12	400	500	100
13	433.33	516.67	83.33
14	466.66	533.33	66.67
15	500	500	50

### For further consideration:

A marathon is 42.2 km or 42 200 m. We can obtain the times for a marathon by just multiplying our answers in a) by 422.

For the cheetah, it will take  $3 \times 422 = 1266$  s. If we divide this by 3600, we convert our answer to hours. It would take 0.35 h which is approximately 21 minutes.

For the cow, it will take  $28.1 \times 422 = 11\,858.2$  s. If we divide this by 3600, we convert our answer to hours. It would take 3.3 h which is approximately 3 h 18 minutes.

For the snail, it will take  $8000 \times 422 = 3\,376\,000$  s. If we divide this by 3600, we convert our answer to hours. It would take 937.8 h which is approximately 937 h 47 minutes. This is just over 39 days!

For the human, it will take  $9.6 \times 422 = 4051.2$  s. If we divide this by 3600, we convert our answer to hours. It would take 1.125 h which is approximately 1 h 8 minutes.

