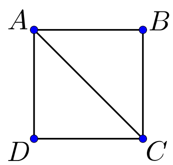
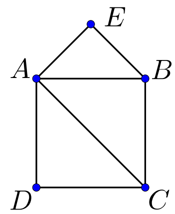
# Problem of the Week
## Problem B
## Look At Rick Shaw Go!

### Problem

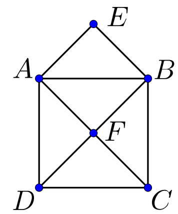
Rick Shaw transports tourists through a park on his human-powered cart. He takes passengers around on several different routes (The Basic, The Classic, The Plus and The Ultimate), depending on how much they are willing to pay. On each route, he travels along each path (line segment) exactly once. He may pass through a node (vertex) more than once to complete a route. He does not have to start and end at the same place.



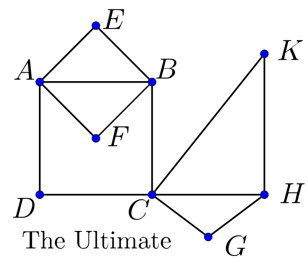
The Basic



The Classic



The Plus



The Ultimate

Above is a picture of Rick and his four possible routes.

a) Determine a way to travel each of the routes. Record the results in the following chart. (There may be more than one successful route.)

b) At each node in any route there are a number of intersecting paths. For each route, record the total number of intersecting paths, the number of intersecting paths at the start point and the number of intersecting paths at the end point.

c) What trends do you notice about the number of intersecting paths at the start and end points of each successful route?

d) What trend do you see in the total number of intersecting paths?

e) Try to draw two routes, one that has a successful path and one that does not. Switch your new routes with a partner. Predict which of your partner's routes will work and which will not. Then confirm your predictions.

### Solution

a) b) The completed table is shown below. There are other possible routes.

| Route | No. of Intersecting Paths | | |
|---|---|---|---|
| | Total | Start | End |
| Basic: $A \to B \to C \to A \to D \to C$ | 10 | 3 | 3 |
| Classic: $C \to D \to A \to E \to B \to C \to A \to B$ | 14 | 3 | 3 |
| Plus: $D \to F \to A \to B \to E \to A \to D \to C \to F \to B \to C$ | 20 | 3 | 3 |
| Ultimate: $H \to K \to C \to H \to G \to C \to B \to F \to A \to B \to E \to A \to D \to C$ | 26 | 3 | 5 |

c) All successful routes start and end at a point with an odd number of intersecting paths.

d) All the total numbers of intersecting paths are even.

See the next page for further comments on parts c), d) and e), and on routes in general.

In each of the given diagrams (graphs) in the question, there were two nodes (vertices) with an odd number of intersecting paths. All other nodes had an even number of intersecting paths. In general you would be able to find a successful route if exactly two of the nodes in your diagram had an odd number of intersecting paths and all of the other nodes had an even number of intersecting paths.

An *Euler*, pronounced "oiler", path is a path that uses every edge of the graph exactly once, but starts and ends at a different node. An *Euler* circuit is a path that uses every edge of the graph exactly once, but starts and ends at the same node.

In our problem, each route was an Euler path.

If a graph has no nodes with an odd number of intersecting paths, you can find an Euler circuit. If a graph has exactly two nodes with an odd number of intersecting paths, you can find an Euler path by starting at either of those nodes.

If a graph has three or more nodes with an odd number of intersecting paths, you cannot complete an Euler path or circuit.

The following graphs are fairly similar. Exactly one of the graphs contains an Euler path, exactly one contains an Euler circuit and exactly one contains neither an Euler path or an Euler circuit. Can you correctly identify which type of path is contained in each graph.