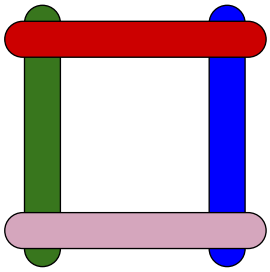
Problem of the Week
Problem A and Solution
Crafty Construction

Problem

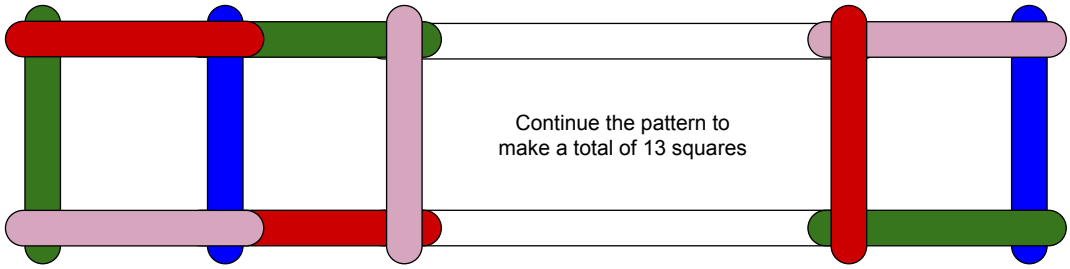
Sam is making square picture frames for his friends and family. He has one box of popsicle sticks and there are 50 sticks in a box. He needs to make 13 frames.

A) If Sam builds individual frames like this:



will he have enough material to build 13 individual frames? Explain your answer.

B) Instead of individual frames, he decides to connect the frames in a line so that any two frames share at most one popsicle stick.



If he built the frames this way, would he have enough sticks in a box of 50 to make connected frames for 13 pictures?

C) Can you draw another layout of the 13 frames where any two frames share at most one popsicle stick?

D) Can you find a way of making 13 connected frames that uses less than 35 popsicle sticks?

Solution

A) Since each frame requires 4 sticks, we can calculate the total number of sticks required this way: $13 \times 4 = 52$

We could also make a table showing the number of frames and the number of sticks required and see the following pattern:

# of Frames	# of Sticks
1	4
2	8
3	12
4	16
5	20
6	24
7	28
8	32
9	36
10	40
11	44
12	48
13	52

Either way, since it will take 52 popsicle sticks to build 13 individual frames, Sam does not have enough popsicle sticks in the box.

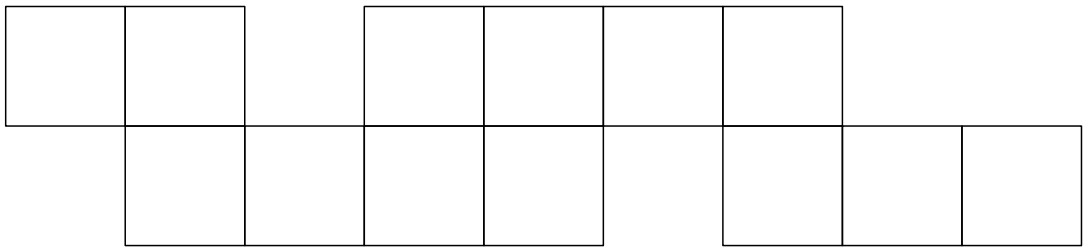
B) Since the first frame takes 4 sticks and the other 12 frames take 3 sticks each, we can calculate the total number of sticks required by adding 4 to 12×3. This gives a total of $4 + 36 = 40$.

Another way to look at this is to make a table to see the following pattern:

# of Frames	# of Sticks
1	4
2	7
3	10
4	13
5	16
6	19
7	22
8	25
9	28
10	31
11	34
12	37
13	40

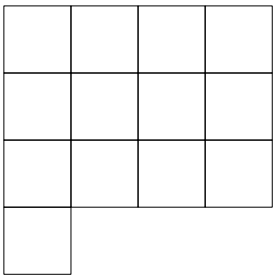
Either way, since it will take 40 popsicle sticks to build 13 connected frames, Sam does have enough popsicle sticks in the box.

C) There are many designs you could make to hold 13 pictures. Here is one:



This one takes only 39 popsicle sticks.

D) The most efficient use of popsicle sticks is to share as many sides as possible with other frames. If you connect the frames to form a shape as close as possible to a square, you can make 13 frames using only 34 popsicle sticks. For example:



Teacher's Notes

Optimization problems appear everywhere in real life, and mathematicians have many techniques to help maximize or minimize some process. Businesses try to minimize costs, farmers try to maximize yield, people try to minimize the time they spend doing things they do not like and maximize the time they spend doing things they do like. Areas of advanced mathematical study such as calculus and linear programming can be used to solve some optimization problems. The key to being able to use these techniques is to translate the real world problem into a mathematical form. This process is called abstraction, and is an essential element of mathematics.

At the end of this particular problem students are asked to minimize the number of popsicle sticks necessary to frame 13 pictures. Another similar optimization problem is to find the maximum area of a shape given a fixed perimeter. For example, what would the maximum area of a quadrilateral be if its perimeter is 100 cm? It can be proven (although not easily) that the solution in this case will be a square. You could try drawing different rectangles that have a perimeter of 100 cm to see that none of them have an area greater than the square that has sides with length 25 cm each. In the general case, to maximize the area of a shape that has n sides and a fixed perimeter you must form a *regular polygon*. A shape where the lengths of all of the sides are equal is known as a regular polygon.