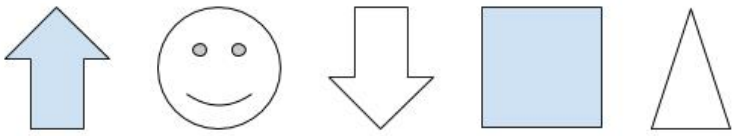
Problem of the Week
Problem A and Solution
Passive Patterns

Problem

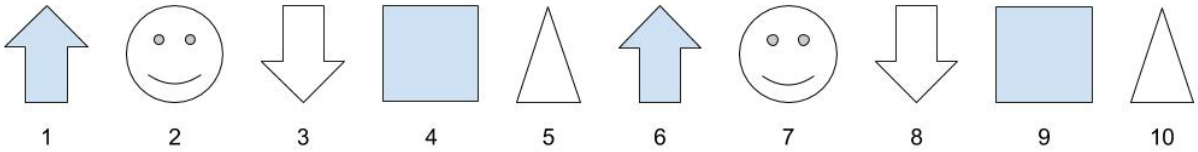
The core of a pattern is the shortest string of objects that repeat in a given order. Below we have the core of a pattern that contains five objects.



What would the 15^{th} object be in this pattern? the 28^{th}?, the 101^{st}?

Solution

You may draw out the shapes and begin to realize the repetition with the pattern as follows:



If you continue numbering, then every shape whose position is a multiple of 5 will be a triangle. So the 15^{th} shape will be a triangle.

Shapes 1, 6, 11, 16, 21, etc. will all be up arrows. In other words, any numbers that end with a 1 or a 6 in the pattern sequence will be an up arrow.
Shapes 2, 7, 12, 17, 22, etc. will all be smiley faces. In other words, any numbers that end with a 2 or a 7 in the pattern sequence will be a smiley face.
Shapes 3, 8, 13, 18, 23, etc. will all be a down arrow. In other words, any numbers that end with a 3 or an 8 in the pattern sequence will be a down arrow.
Shapes 4, 9, 14, 19, 24, etc. will all be a square. In other words, any numbers that end with a 4 or a 9 in the pattern sequence will be a square.

So the 18^{th} shape will be a down arrow and the 101^{st} shape will be an up arrow.

Teacher's Notes

Determining which shape appears at the various positions can be accomplished using the *mod* operation. For positive integers, calculating *mod* is the same as finding the remainder in division. For example, $17 \bmod 5 = 2$, since when we divide 17 by 5 we get a remainder of 2. This is helpful for this problem, since the positions that have the same value *mod* 5 will have the same shape. In other words, since from the core of the pattern:

$1 \bmod 5 = 1$, and the shape at position 1 is an up arrow,
$2 \bmod 5 = 2$, and the shape at position 2 is a smiley face,
$3 \bmod 5 = 3$, and the shape at position 3 is a down arrow,
$4 \bmod 5 = 4$, and the shape at position 4 is a square, and
$5 \bmod 5 = 0$, and the shape at position 5 is a triangle,

to determine the shape at some random position k, we calculate $k \bmod 5$.

Whatever answer we get must be a number between 0 and 4, since the remainder of a division must be smaller than the divisor. We match the result of this calculation with the core results. Whatever number we get determines the shape at that position.

Since $15 \bmod 5 = 0$, the shape at the 15^{th} position is a triangle.
Since $18 \bmod 5 = 3$, the shape at the 18^{th} position is a down arrow.
Since $101 \bmod 5 = 1$, the shape at the 101^{st} position is an up arrow.
If we wanted to know what shape is at position 32836749, we calculate $32836749 \bmod 5 = 4$, which tells us there is a square at that position.

If the length of the core changed, that would change the number that follows *mod* in our calculation. For example, if we had a core that had 8 different shapes repeated, then our calculations would be $k \bmod 8$. The *mod* operation is used in many applications in mathematics and computer science.