Problem of the Week
Problem A and Solution
Passive Patterns

Problem
The core of a pattern is the shortest string of objects that repeat in a given order. Below we have the core of a pattern that contains five objects.

What would the 15th object be in this pattern? the 28th?, the 101st?

Solution
You may draw out the shapes and begin to realize the repetition with the pattern as follows:

If you continue numbering, then every shape whose position is a multiple of 5 will be a triangle. So the 15th shape will be a triangle.

Shapes 1, 6, 11, 16, 21, etc. will all be up arrows. In other words, any numbers that end with a 1 or a 6 in the pattern sequence will be an up arrow.

Shapes 2, 7, 12, 17, 22, etc. will all be smiley faces. In other words, any numbers that end with a 2 or a 7 in the pattern sequence will be a smiley face.

Shapes 3, 8, 13, 18, 23, etc. will all be a down arrow. In other words, any numbers that end with a 3 or an 8 in the pattern sequence will be a down arrow.

Shapes 4, 9, 14, 19, 24, etc. will all be a square. In other words, any numbers that end with a 4 or a 9 in the pattern sequence will be a square.

So the 18th shape will be a down arrow and the 101st shape will be an up arrow.
Teacher’s Notes

Determining which shape appears at the various positions can be accomplished using the mod operation. For positive integers, calculating mod is the same as finding the remainder in division. For example, \(17 \mod 5 = 2\), since when we divide 17 by 5 we get a remainder of 2. This is helpful for this problem, since the positions that have the same value \(mod\ 5\) will have the same shape. In other words, since from the core of the pattern:

1 mod 5 = 1, and the shape at position 1 is an up arrow,
2 mod 5 = 2, and the shape at position 2 is a smiley face,
3 mod 5 = 3, and the shape at position 3 is a down arrow,
4 mod 5 = 4, and the shape at position 4 is a square, and
5 mod 5 = 0, and the shape at position 5 is a triangle,

to determine the shape at some random position \(k\), we calculate \(k \mod 5\).

Whatever answer we get must be a number between 0 and 4, since the remainder of a division must be smaller than the divisor. We match the result of this calculation with the core results. Whatever number we get determines the shape at that position.

Since 15 mod 5 = 0, the shape at the 15\(^{th}\) position is a triangle.
Since 18 mod 5 = 3, the shape at the 18\(^{th}\) position is a down arrow.
Since 101 mod 5 = 1, the shape at the 101\(^{st}\) position is an up arrow.
If we wanted to know what shape is at position 32836749, we calculate 32836749 mod 5 = 4, which tells us there is a square at that position.

If the length of the core changed, that would change the number that follows mod in our calculation. For example, if we had a core that had 8 different shapes repeated, then our calculations would be \(k \mod 8\). The mod operation is used in many applications in mathematics and computer science.