



Problem of the Week

Problem A and Solution

Granny's Quilt

Problem

Granny is making a quilt for her new grandchild. She has 5 different rolls of fabric: green, blue, red, orange, yellow. Each square of her quilt must have 3 different colours.

Granny wants each square to be a unique combination of colours.

- A) What is the maximum number of squares she can have in her quilt?
- B) How many times does red appear in a square?
- C) Would the answer above change for a different colour? Why?

Solution

A) There are 10 different combinations of three colours, chosen from five colours, assuming that the order of the colours does not matter.

- blue, green, red
- blue, green, orange
- blue, green, yellow
- blue, orange, red
- blue, orange, yellow
- blue, red, yellow
- green, orange, red
- green, orange, yellow
- green, red, yellow
- orange, red, yellow

B) Red appears in patterns six times each.

C) Since each colour has the same chance of appearing the square, it does not matter which colour you pick. Each colour appears six times in the patterns.





Teacher's Notes

It is relatively easy to calculate how many different combinations you can make by selecting three colours out of five choices. If you have to select three different items then you have all 5 options for the first selection, 4 choices left for the second selection, and 3 choices left for the last selection. This means you have $5 \times 4 \times 3 = 60$ possible selections.

However, if the order of the colours you select does not matter, within those 60 selections there are many duplicates. For example, selecting blue, then red, then orange, results in the same three colours as selecting orange, then red, then blue. So you need to eliminate these duplicates. When arranging three different items into specific positions, you have 3 choices for the position of the first item, then you have 2 choices for the position of the second item, and only 1 position left for the last item. This means you have $3 \times 2 \times 1 = 6$ different arrangements of three items. So for each set of three colours in your selection, there is a total of 6 different arrangements of those colours. Eliminating duplicates, there are $60 \div 6 = 10$ combinations of three different colours out of five choices.

Some people may find actually listing all of the selections in a way that ensures that you do not miss or duplicate one of the combinations more difficult. Here is one way. Make a table with five columns, where the header of the columns are the colour choices. In each row, you can use the digit 1 to indicate you are selecting that colour and the digit 0 to indicate you are not selecting that colour. Each row needs to have three 1s and two 0s that form a five digit number representing a three colour selection. Place the five digit numbers in the table in ascending order like this:

B	G	O	R	Y
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0

These numbers represent the 10 different combinations of colours for the quilt.

