Problem of the Week
Problem A and Solution
Colin's Coloured Blocks

Problem

Colin arranges blocks in a 12×12 grid. He starts in position A1 then follows a snake pattern to fill the grid. When he reaches the end of the first row, he continues filling up the second row from right to left. When he reaches the third row, he fills from left to right and so on. All the odd numbered rows are filled from left to right and the even numbered rows are filled from right to left.

Colin uses yellow, blue, red and green blocks to fill the grid. He starts with 1 yellow block, then 3 blue blocks, then 5 red blocks, then 7 green blocks. He continues the pattern starting with yellow blocks again. Each change of colour has 2 more blocks than the previous colour.

A) What is the colour of the block at position F8 on the grid?

B) What is the colour of the block at position J9 on the grid?

C) What is the colour of the block at position B11 on the grid?

D) If Colin starts with 50 blocks of each colour, how many yellow, blue, red, and green blocks does he have left over after filling the grid?

Solution

After filling up the grid it looks like this:

	A	B	C	D	E	F	G	H	I	J	K	L
1	Y	B	B	B	R	R	R	R	R	G	G	G
2	Y	Y	Y	Y	Y	Y	Y	Y	G	G	G	G
3	Y	B	B	B	B	B	B	B	B	B	B	B
4	R	R	R	R	R	R	R	R	R	R	R	R
5	R	G	G	G	G	G	G	G	G	G	G	G
6	Y	Y	Y	Y	Y	Y	Y	Y	G	G	G	G
7	Y	Y	Y	Y	Y	Y	Y	Y	Y	B	B	B
8	B	B	B	B	B	B	B	B	B	B	B	B
9	B	B	B	B	R	R	R	R	R	R	R	R
10	R	R	R	R	R	R	R	R	R	R	R	R
11	R	G	G	G	G	G	G	G	G	G	G	G
12	G	G	G	G	G	G	G	G	G	G	G	G

A) F8 is blue.

B) J9 is red.

C) B11 is green.

D) The total number of yellow blocks used is $1 + 9 + 17 = 27$.
The total number of blue blocks used is $3 + 11 + 19 = 33$.
The total number of red blocks used is $5 + 13 + 21 = 39$.
The total number of green blocks used is $7 + 15 + 23 = 45$.

If Colin starts with 50 blocks of each colour, he will have:

$50 - 27 = 23$ yellow blocks left over,
$50 - 33 = 17$ blue blocks left over,
$50 - 39 = 11$ red blocks left over, and
$50 - 45 = 5$ green blocks left over.

Teacher's Notes

Calculating the number of blocks for each colour is an example of determining the value of an *arithmetic series*. An arithmetic series is a sum, where the terms in the sum have a common difference. For example, in this arithmetic series $1 + 9 + 17 = 27$, the difference between the first and second terms is 8, and the difference between the second and third terms is 8. In fact, all of the series used to determine the total number of blocks of each colour have a common difference of 8. This common difference can be explained by the fact that we repeat the colours in a regular pattern. In between repeating the colours, we increase the number of blocks in the pattern by 2. We switch colours 4 times before returning to the same colour in the pattern. Since $2 \times 4 = 8$, the difference in the number of blocks we will use the next time with the same colour will be 8.

There is a formula for calculating the result of an arithmetic sequence.

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

where n is the number of terms in the sequence, a_1 is the value of the first term, and d is the common difference. Using this formula we could calculate the total number of red blocks for example. In this case, we would have to figure out that Colin uses 5 red blocks the first time (a_1), and that red blocks are used 3 times (n). We know that the common difference is 8 (d). Then we calculate the sum as:

$$\begin{aligned} S_n &= \frac{(3)}{2}[2(5) + ((3) - 1)(8)] \\ &= \frac{3}{2}[10 + (2)(8)] \\ &= \frac{3}{2}[26] \\ &= (3)(13) \\ &= 39 \end{aligned}$$

Adding up three terms is probably easier than using the formula to calculate the number of blocks in this problem. However, if we wanted to know how many block are used if the pattern continues for another 50 cycles, the formula would be useful.