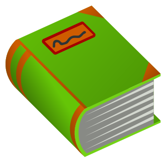

Problem of the Week
Problem A and Solution
Money Management

Problem

Adam only saves quarters and dimes. He has already saved 13 quarters and 5 dimes. In Canada, one quarter equals 25 cents, one dime equals 10 cents, and 100 cents equals \$1 (one dollar).

He wants to purchase a book that costs \$7.75.

Not including the money he already has, what combinations of quarters and dimes could Adam save so he has exactly enough money to buy the book?

Show your thinking.

Solution

Adam already has:

$$\begin{array}{llcr} & \text{13 quarters} & = & \$3.25 \\ + & \text{5 dimes} & = & \$0.50 \\ \hline & & & \$3.75 \end{array}$$

So Adam needs $\$7.75 - \$3.75 = \$4.00$ to buy the book. Since the total number of cents required (400) is an even number, and Adam only has dimes and quarters, if he uses quarters to pay for the book he must use an even number of quarters. Note that 5 dimes and 2 quarters both equal 50¢. The first combination calculates the total using all quarters. As we move down the table, we can remove two quarters each time and replace them with five dimes. This keeps the total the same, and is a good way to make sure all possible combinations have been considered.

Combination	10¢ (dime)	25¢ (quarter)
1	0	16
2	5	14
3	10	12
4	15	10
5	20	8
6	25	6
7	30	4
8	35	2
9	40	0

Therefore, there are 9 different combinations of quarters and dimes that Adam could collect to equal four dollars.

Teacher's Notes

The solution to this problem looks very much like a table of values for an equation that uses two variables. We can describe relationships between two or more values using equations. For example:

$$y = 5x + 1$$

describes a relationship between the variables x and y. Essentially, for any value we choose for x, the value of y will be one more than five times the value chosen. From this equation we can generate a table that contains a sample of the values for x and y that satisfy the relationship. You can create the table by picking a random value for one of the variables (known as the *independent* variable) and that will determine the value of the other variable (known as the *dependent* variable). If we pick the values 1, 2, 3, 4, and 5 for x, we get the following table:

x	y
1	6
2	11
3	16
4	21
5	26

We can write the relationship between the number of dimes and number of quarters needed to have enough money to purchase the book as an equation as well. If we use a variable d to represent the number of dimes Adam may use, and a variable q to represent the number of quarters he may use, we can write the following equation:

$$10d + 25q = 400$$

since each dime is worth 10¢, each quarter is worth 25¢, and Adam needs 400¢ more to buy the book.

Looking at each combination row in the table, we have values for d and q that make this equation true. For example, using combination 4, we can substitute 15 for d and 10 for q to get:

$$10(15) + 25(10) = 150 + 250 = 400$$

The equation describing this problem has a couple of restrictions that are not necessarily part of all equations. Since the values of the variables are representing physical coins, then the values can not be negative numbers, nor can they be fractions. The possible values for d are between 0 and 40. The possible values for q are between 0 and 16. The possible values for variables can be described as the *domain* and *range*. Since all values for these variables must be whole numbers, then this equation is describing a *discrete function*.

All of these concepts: independent variable, dependent variable, domain, range, and discrete functions are investigated in mathematics courses students may study in the future.