



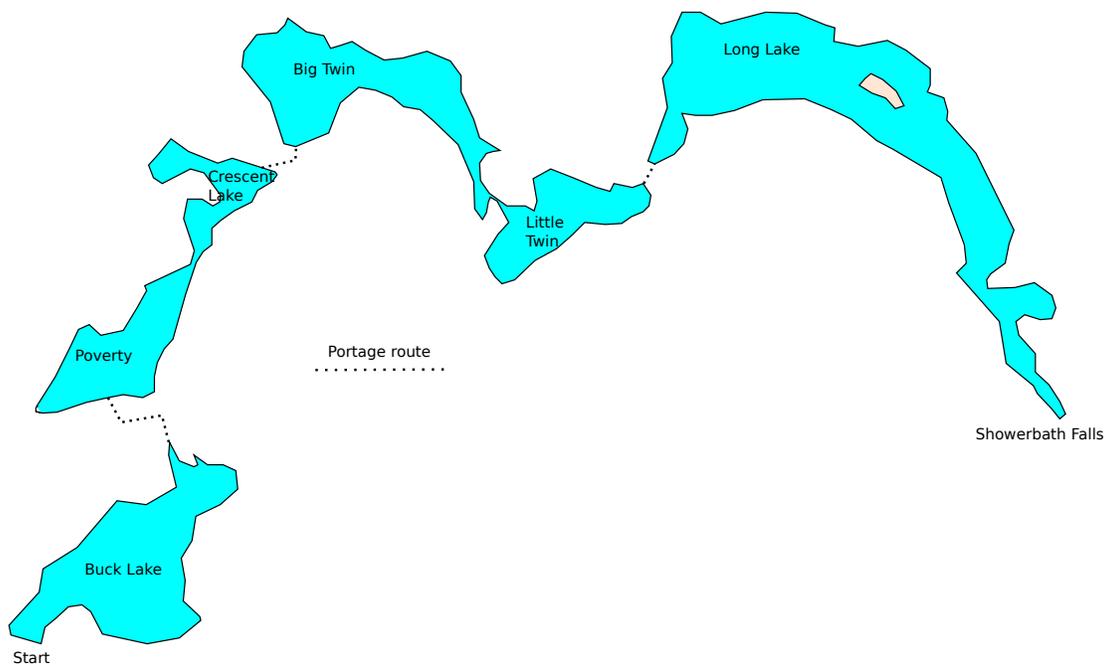
Problem of the Week

Problem A and Solution

Canoeing to Showerbath Falls

Problem

James and Katie decide to paddle their canoe to Showerbath Falls. There are six lakes to paddle through to get from the starting point of the trip to the end. The lakes are Buck, Poverty, Crescent, Big Twin, Little Twin, and Long Lake. Some of the lakes are connected by narrows, where you can paddle straight through, and some lakes are connected by portages, which means James and Katie have to get out of the canoe and carry it across a path to the next lake. Poverty and Crescent are connected by narrows, and Big Twin and Little Twin are connected by narrows. There are portages between Buck and Poverty, Crescent and Big Twin, and Little Twin and Long Lake.



They start in Buck Lake and it takes them 30 minutes to paddle across it. The portage between Buck and Poverty takes 10 minutes to cross. It takes 20 minutes to paddle across Poverty and Crescent. The portage between Crescent and Big Twin takes 15 minutes. From Big Twin to the final portage is a 25 minute paddle. This last portage is only 5 minutes long. Long Lake is well named and it takes 50 minutes to get to the end. Showerbath Falls is at the end of Long Lake.

- A) How much more time do they spend paddling rather than portaging the canoe during the trip from Buck Lake to Showerbath Falls?
- B) Katie and James spent an hour at Showerbath Falls eating lunch and then returned home. It took them the same amount of time to travel back. If they left at 9:00 in the morning, what time did they get home?





Solution

A) The total time paddling is: $30 + 20 + 25 + 50 = 125$ minutes.

The total time portaging is: $10 + 15 + 5 = 30$ minutes.

Katie and James spent $125 - 30 = 95$ minutes more paddling than portaging. Since there are 60 minutes in 1 hour, we can do repeated subtraction to convert minutes into a combination of minutes and hours. In this case, $95 - 60 = 35$. Since $35 < 60$, we cannot subtract 60 any more. So 95 minutes is equal to 1 hour and 35 minutes.

B) The total time for the trip one way is: $125 + 30 = 155$ minutes. The total time for the round trip including lunch is: $155 + 60 + 155 = 370$ minutes. Using repeated subtraction we calculate

$$370 - 60 = 310$$

$$310 - 60 = 250$$

$$250 - 60 = 190$$

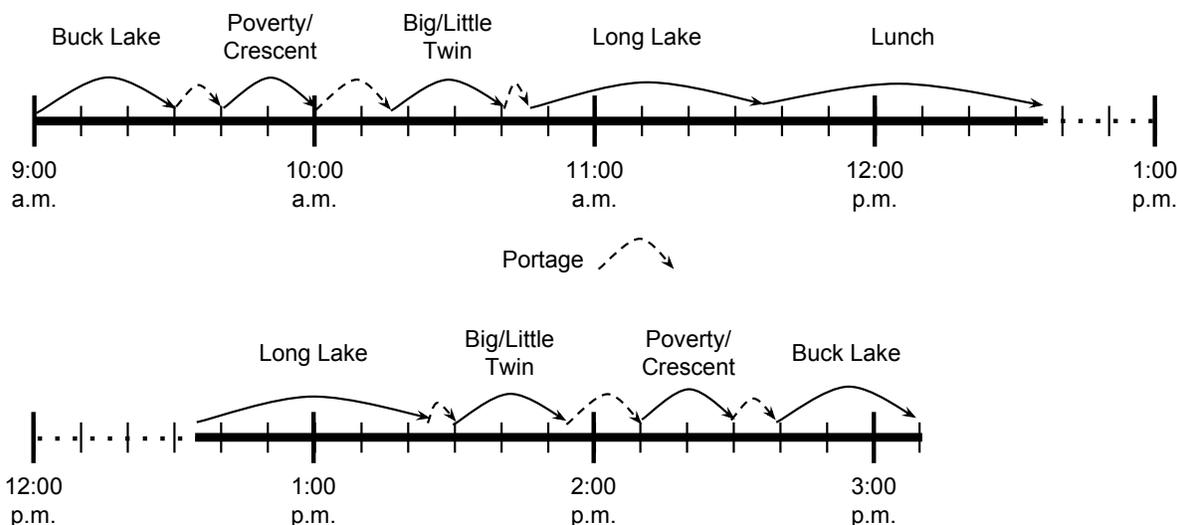
$$190 - 60 = 130$$

$$130 - 60 = 70$$

$$70 - 60 = 10$$

Since 60 is repeatedly subtracted 6 times before the difference is less than 60, and last difference is 10, then 370 minutes is equal to 6 hours and 10 minutes. If James and Katie left at 9:00 a.m. then they would get back at 3:10 p.m.

You can also find the time it takes for James and Katie to complete the trip using a timeline.





Teacher's Notes

Converting a total number of minutes for the trip to the format using hours and minutes is another good example of why it is important to be able to calculate the quotient and the remainder when dividing. The *modulo* or *mod* operation that is used in mathematics and in many programming languages computes the same value as the remainder for positive integers. This operation is also part of an area of mathematics known as *modular arithmetic*.

Modular arithmetic describes relationships between integers where the numbers repeat themselves after reaching a certain value. It is often referred to “clock arithmetic” since our use of a 12-hour clock is a very good example of this idea. When we describe time using a 12-hour clock, the maximum number of minutes is 59 and the maximum number of hours is 12. If it is 3:14 right now, then after 60 minutes, it will be 4:14. The number of minutes in both those times is the same, even though 60 minutes has elapsed. If it is 3:14 right now, then in exactly five days, it will also be 3:14. In this example the elapsed time is 120 hours or 7200 minutes, and yet we refer to the time using exactly the same number of minutes and hours. That happens because when we deal with time this way, every 60 minutes we repeat the number of minutes, and every 12 hours we repeat the number of hours.

The CEMC has an activity known as Math Circles which is presented at the University of Waterloo. The materials from past years of this activity are available online at:

http://www.cemc.uwaterloo.ca/events/mathcircle_presentations.html

The materials from Fall 2016 on November 8-9 for the Junior Grade 6 students introduce the idea of modular, or clock, arithmetic.

