Problem of the Week
Problem A and Solution
What Would You Buy?

Problem

School Council donates $75.00 to your school to purchase soccer balls, basketballs, skipping ropes, and frisbees. In order to be fair, you must purchase at least 2 of each type of equipment, but no more than 8 of each type.
The prices are:

- Soccer balls: $4.90
- Basketballs: $5.85
- Skipping ropes: $3.50
- Frisbees: $2.00

A) What would you buy to use as much of the $75.00 donation as possible?

B) How much money would be left over?

Solution

Since you need to buy at least two of each piece of equipment, start by calculating the cost of two soccer balls, two basketballs, two skipping ropes, and two frisbees. There are several ways of calculating this total. Here is one way:

- Two soccer balls cost: $2 \times 4.90 = \$9.80$
- Two basketballs cost: $2 \times 5.85 = \$11.70$
- Two skipping ropes cost: $2 \times 3.50 = \$7.00$
- Two frisbees cost: $2 \times 2.00 = \$4.00$

So the total spent on two of each item is: $9.80 + 11.70 + 7.00 + 4.00 = \$32.50$

Another way to calculate this total is to figure out how much it costs for one of each item: $4.90 + 5.85 + 3.50 + 2.00 = \$16.25$
If we want to buy two of each, then the cost will double: $2 \times 16.25 = \$32.50$

Either way, the total cost of two of each type of equipment is $32.50.
So, there is $75.00 - 32.50 = \$42.50$ left to spend.

One way to spend exactly \$42.50 is to buy two more soccer balls, two more basketballs, and six more skipping ropes, since:

- Two soccer balls cost: $2 \times 4.90 = \$9.80$
- Two basketballs cost: $2 \times 5.85 = \$11.70$
- Six skipping ropes cost: $6 \times 3.50 = \$21.00$

So the total spent on these extra items is: $9.80 + 11.70 + 21.00 = \$42.50$
So, if we bought four soccer balls, four basketballs, eight skipping ropes, and two frisbees, we would spend exactly \$75.00. No money would be left over.

Another way to spend exactly \$42.50 is to purchase five more soccer balls, four more skipping ropes, and two more frisbees, since:

- Five soccer balls cost: $5 \times 4.90 = \$24.50$
- Four skipping ropes cost: $4 \times 3.50 = \$14.00$
- Two frisbees cost: $2 \times 2.00 = \$4.00$

So the total spent on these extra items is: $24.50 + 14.00 + 4.00 = \$42.50$
In this case we would buy seven soccer balls, two basketballs, six skipping ropes, and four frisbees, and spend exactly \$75.00.

Teacher's Notes

There is no simple way find the solutions to this problem. One possibility would be to try all combinations showing how many of each item you might buy, and then find the values that would have a total purchase cost that is as close to \$75.00 as possible without going over that amount. However, since you can buy between 2 and 8 of each item, there are $7^4 = 2401$ different combinations to check. This is a very typical type of problem in the real world, where you have many variables and some restrictions on those variables, and you need to optimize your resources. Advanced mathematical techniques such as linear programming can be used to solve these kinds of problems.

Another way solve this problem is to guess and check. If you make educated guesses, you can often reduce the number of combinations that need to be checked. When we do not have a clear, deterministic way of solving a problem, we can use *heuristic* methods. A heuristic is any problem solving technique that is not necessarily precise, but will produce a solution that is acceptable in a reasonable amount of time.

Let's try to reduce the number of guesses we have to make. We can hope that there is a solution where we spend exactly \$75.00. This would mean that after buying the first two of each item, we need to spend exactly \$42.50. Now we can check only the combinations that would give us a total cost that ends with 0.50. Let's investigate the multiples of each of our prices.

- Any multiple of 2.00 will be a whole number.
- The even multiples of 3.50 will be whole numbers; the odd multiples will be values that end with 0.50. If we buy an odd number of skipping ropes, then we can look for a total cost of the rest of the items we purchase that is a whole number of dollars.
- The multiples of 4.90 to consider are: 4.90, 9.80, 14.70, 19.60, 24.50, and 29.40
- The multiples of 5.85 to consider are: 5.85, 11.70, 17.55, 23.40, 29.25, and 35.10

The maximum amount we can spend on skipping ropes and frisbees is:

$$(6 \times 3.50) + (6 \times 2.00) = 21.00 + 12.00 = 33.00$$

since the most we can buy is six more of each item. This is much less than we have to spend, so we need to buy at least one soccer ball or basketball. Now we can focus on combinations of multiples of these two items that total a whole number of dollars or have a total that ends with 0.50. This is a much smaller number of combinations to check. We see that:

- $5 \times 4.90 = 24.50$, which comes from 5 soccer balls.
- $9.80 + 11.70 = 21.50$, which comes from 2 soccer balls and 2 basketballs.
- $4.90 + 35.10 = 40.00$ which comes from 1 soccer ball and 6 basketballs.
- $19.60 + 23.40 = 43.00$, which is more than we have to spend.
- $29.40 + 35.10 = 64.50$, which is more than we have to spend.

So now we know that any combination that gets us to exactly \$42.50 will involve either 5 soccer balls, a combination of 2 soccer balls and 2 basketballs, or a combination of 1 soccer ball and 6 basketballs. We can eliminate the last combination by observing that after making that purchase, we only have \$2.50 left to spend. This means we could only buy a skipping rope which cost \$2.00. This total would be \$74.50.

When we look for a solution that involves one of the remaining combinations of soccer balls and basketballs we have a new problem to solve.
In the first case, we still have to spend $42.50 - 24.50 = \$18.00$
In the second case, we still have to spend $42.50 - 21.50 = \$21.00$
So in both cases we are looking at differences that are a whole number of dollars, and we only have skipping ropes and frisbees left to buy. A whole number total can only come from purchasing an even number of skipping ropes and some number frisbees. This is a relatively small number of combinations to check.

Using a combination of educated guesses, and a process that leads to smaller problems that need to be solved, we can find our two solutions.