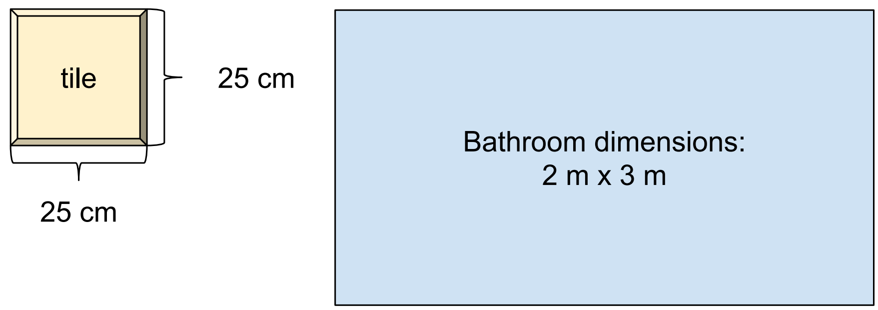
Problem of the Week

Problem A and Solution

Mrs. Thomas's Tidy Tiles

Problem

Mrs. Thomas wants to tile her 2m × 3m bathroom floor. Each tile is 25cm × 25cm.



A) How many tiles will she need to cover her bathroom floor?

B) If each tile costs \$1.50, how much will it cost Mrs. Thomas to tile her bathroom?

Solution

A) Each tile is 25 cm × 25 cm. This means that if we put 4 tiles in a line they would form a rectangle where the length of the long side is 1 m. If we put 16 tiles together (4 × 4) to form a square, the dimensions of that square would be $1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^2$. The area needed to be covered by tiles is $2 \text{ m} \times 3 \text{ m} = 6 \text{ m}^2$. So, Mrs. Thomas would need $16 \times 6 = 96$ tiles to complete her bathroom.

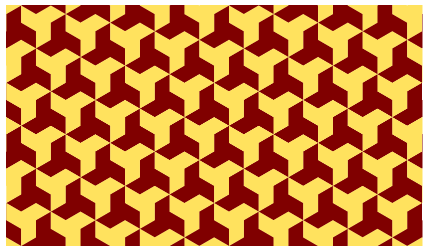
Alternatively, we could look at the bathroom dimensions and convert them into centimetres. The length of the bathroom is $3 \times 100 = 300$ cm. The width of the bathroom is $2 \times 100 = 200$ cm. If we divide the dimension of the bathroom by the width of the tile, then we know how many tiles are required along each edge. For the length, it is $300 \div 25 = 12$ tiles. For the width, it is $200 \div 25 = 8$ tiles. Therefore we need $12 \times 8 = 96$ tiles to cover the area of the bathroom.

B) If she needs 96 tiles and they each cost \$1.50, she would spend $96 \times 1.50 = (96 \times 1) + (96 \times 0.50) = 96 + (96 \div 2) = 96 + 48 = \$144.00$. She would spend \$144 for her tiles.

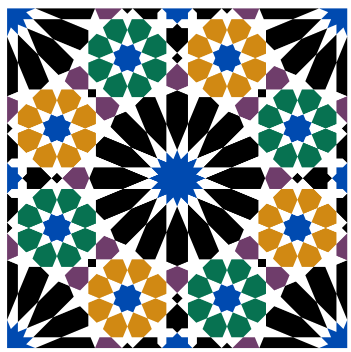
Teacher's Notes

This is a literal example of a tiling problem. In mathematics, tiling describes the process of covering a flat surface (or *plane*) with geometric shapes known as tiles. We imagine that the plane extends infinitely in all directions. Two examples are illustrated below.

The first example illustrates tiling with a single geometric shape.



The second example illustrates tiling with multiple geometric shapes.



You can imagine that these patterns could be continued indefinitely in all four directions. A real world tiling has boundaries. The problem of trying to fit geometric shapes into a bounded area is called a packing problem. Packing problems are studied by mathematicians and computer scientists. This type of problem asks the question, what is the maximum number of objects I can fit into a fixed space. That space can be two or three dimensional. Unlike with tilings, the packing problem allows gaps between the objects. Filling the entire space without any gaps would be optimal. However, there are some situations where you unable to avoid gaps. For example, consider trying to maximize the number of circles you can fit into a square. It is not possible to arrange the circles inside the square without gaps. Another example would be maximizing the number of squares inside a triangle. Again, you are guaranteed to have gaps. Solving packing problems has real life applications such as determining a company's shipping and storage requirements.