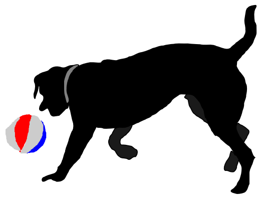


Problem of the Week
Problem A and Solution
Playing Fetch

Problem

Robbie loves playing fetch with his dog Spencer. Spencer always starts by sitting beside Robbie before Robbie throws the ball. When Robbie throws the ball, Spencer runs to it and brings the ball back to the same spot. Robbie throws the ball three times.

- The first time he throws it 8 metres.
- The second time he throws it twice as far as the first time.
- The third time he throws it 5 metres less than the second time.

How far does Spencer run in total?

Solution

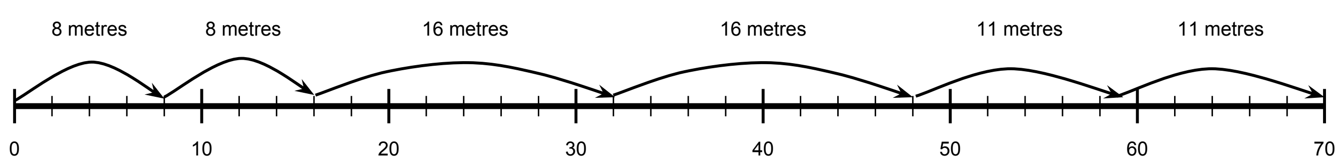
On the first throw, Spencer runs $2 \times 8 = 16$ metres.

The distance of the second throw is $8 \times 2 = 16$ metres.
On the second throw, Spencer runs $2 \times 16 = 32$ metres.

The distance of the third throw is $16 - 5 = 11$ metres.
On the third throw, Spencer runs $2 \times 11 = 22$ metres.

The total distance Spencer runs is: $16 + 32 + 22 = 70$ metres.

We can also show the distance Spencer runs on a number line.



Teacher's Notes

This problem can be described algebraically. We can use a variable d to represent the distance that Robbie throws the ball the first time. Then, the rest of the distances can be described in terms of d.

The distance he throws the ball the second time is $2d$.
The distance he throws the ball the third time is $2d - 5$.

Since Spencer runs to get the ball and returns to the original spot, every time Robbie throws the ball, Spencer will run that distance twice. We can use t to represent the total distance Spencer runs. Here is one way to calculate t:

$$t = d + d + 2d + 2d + 2d - 5 + 2d - 5$$
$$t = 10d - 10$$

We can also think of the total distance that Spencer runs as two times the total distance that Robbie throws the ball. So here is another way to calculate t:

$$t = 2 \times (d + 2d + 2d - 5)$$
$$t = 2 \times (5d - 5)$$
$$t = 10d - 10$$

Using the second approach, we had to use the *distributive law* to calculate that

$$2 \times (5d - 5) \text{ is equal to } 10d - 10$$

In other words, we multiplied 2 by $5d$ and by -5.

There are other ways we could derive the total distance, however when we simplify the equation, the result will always be:

$$t = 10d - 10$$

Now we can calculate the value of t by substituting the value we have for d. So

$$t = 10(8) - 10$$
$$t = 80 - 10$$
$$t = 70$$

This seems like a lot of work to get the answer we could have just counted using a number line. If we are only interested calculating the distance for one set of throws, then creating the equation is not particularly helpful. However, if we saw the same pattern for throwing with different starting distances, then the equation can be helpful. Suppose Robbie always throws in the same pattern, but this time his first throw is 5 metres. The total distance Spencer runs in this case would be:

$$t = 10(5) - 10$$
$$t = 50 - 10$$
$$t = 40$$

Algebraic equations describe a general case, and they can be very helpful when there are multiple specific cases to consider.