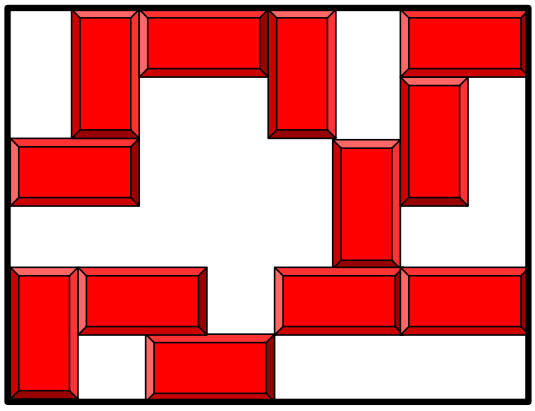
Problem of the Week
Problem A and Solution
Building Blocks

Problem

Johnna Lee loves to build with interlocking building blocks. She starts building on a large, white, flat piece for the base, but the rest of the blocks she uses are red. Looking from overhead, this is what Johnna Lee built.



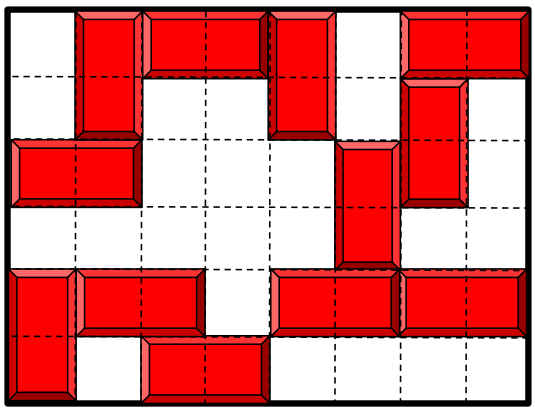
Each of the red blocks has a length of 2 cm and a width of 1 cm.

A) What is the area of the white base?

B) What fraction of the area of the base is covered by red blocks?

Solution

You can draw grid lines that align with the edges of the blocks. The squares formed by the grid lines are 1 cm $\times$ 1 cm.



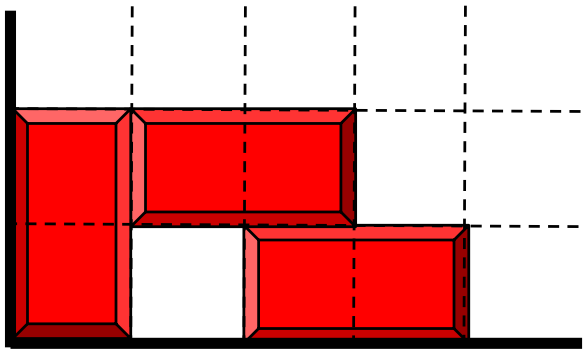
A) If you count the total number of squares formed by the grid lines, you see that there are 48. Alternatively, you can see that the base is 8 cm wide and 6 cm high. The area of the base is $8 \text{ cm} \times 6 \text{ cm} = 48 \text{ cm}^2$.

B) Count the number of squares formed by the grid lines that overlap the blocks. The total is 24. The fraction of the area of covered by blocks over the area of the base is $\frac{24}{48}$. Since $48 = 24 \times 2$ we can also write the fraction as $\frac{1}{2}$.

Here is another way to calculate this fraction. The area of one of the blocks is $1 \text{ cm} \times 2 \text{ cm} = 2 \text{ cm}^2$. There are 12 blocks showing on the base. The total area they cover is $12 \times 2 \text{ cm}^2 = 24 \text{ cm}^2$. Since the area of the base is 48 cm^2, the blocks cover $\frac{24}{48}$ or $\frac{1}{2}$ the area of the base.

Teacher's Notes

The second part of this problem could be solved without knowing any specific information about the dimensions of the blocks. Everything you need to determine what fraction of the base is being covered by the blocks is available in the picture. The key is knowing that the ratio of the length to the width of the building block is $2 : 1$. This can be determined by examining the bottom left corner of the diagram.



From this part of the picture, we see that the twice the shorter side of the block is equal to the length of the longer side of the block. This relationship, along with the way the blocks are aligned in the rest of the diagram, allows us to add the grid lines to the diagram. These grid lines form unit squares. The actual size of those squares is irrelevant; their measurements could be $1 \text{ cm} \times 1 \text{ cm} = 1 \text{ cm}^2$, or $1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^2$, or $1 \text{ Wiffle} \times 1 \text{ Wiffle} = 1 \text{ Wiffle}^2$. What is important for us, is that they are squares. Since we are computing a fraction, the units will disappear in the calculation.

Ultimately the solution for this problem can be computed based on the number of unit squares that form the base of the structure and the number of unit squares the blocks cover. In this case we have a fraction of:

$$\frac{24 \ \cancel{\text{units}^2}}{48 \ \cancel{\text{units}^2}} = \frac{1}{2}$$