Problem of the Week
Problem A and Solution
Getting to School

Problem
Twenty grade 3 students and 25 grade 4 students attend the School of Math. Twelve grade 3 students take the bus, and the rest walk to school. Out of all the students in grades 3 and 4, there are 23 who walk to school. At an assembly for grade 3 and 4 students, a name is drawn for a prize.

A) What is the probability that the prize winner is in grade 3 and walks to school?

B) What is the probability that the prize winner is in grade 4 and takes the bus to school?

Solution
The solution for both parts is based on determining the total number of students at the assembly. There are a total of $20 + 25 = 45$ students at the assembly.

A) Since there are 20 students in grade 3, and 12 of them take the bus,
then $20 - 12 = 8$ grade 3 students walk to school.

The probability that the prize winner is in grade 3 and walks to school is $8$ out of $45$ or $\frac{8}{45}$.

B) Since 23 students walk to school, and 8 of them are grade 3 students,
then $23 - 8 = 15$ grade 4 students walk to school.

Since there are 25 students in grade 4 and 15 of them walk to school,
then $25 - 15 = 10$ grade 4 students take the bus to school.

The probability that the prize winner is in grade 4 and takes the bus to school is $10$ out of $45$ or $\frac{10}{45}$ or $\frac{2}{9}$. 
Teacher’s Notes

The probability of some outcome can be described in many different ways. The solution uses the words “out of” essentially as a replacement for the line between the numerator and the denominator of a fraction. The solution also uses fractions to describe the probability.

We expect the numerator and denominator of a fraction describing probability to have specific characteristics. The numerator must be a non-negative number (i.e. greater than or equal to 0), and the denominator must be a positive number that is greater than or equal to the numerator. Since there are infinitely many equivalent fractions, there are many fractions we could use to describe the same probability. In this problem, the probability of the prize winner being in grade 4 was described as \( \frac{10}{45} \) or \( \frac{2}{9} \). But we could also describe the probability as \( \frac{100}{450} \) or \( \frac{12}{54} \).

If we compare all of these fractions by doing division with a calculator, the result is the same:

\[
\frac{10}{45} = \frac{2}{9} = \frac{100}{450} = \frac{12}{54} = 0.22222
\]

As shown here, all probabilities can also be described as a number between 0 and 1. So in this problem, we could describe the probability as approximately 0.22222.

(Note that since dividing the numerator and denominator of any of the fractions produces a repeating decimal, we have to describe it as approximately the same.)