The problems in this booklet are organized into strands. A problem often appears in multiple strands. The problems are suitable for most students in Grade 5 or higher.
Data Management
&
Probability
Problem of the Week
Problem B
Batter Up!

Use the following clues to fill in the table below, and determine who won this classic 2016 World Series game between the Cleveland and Chicago baseball teams. HINT: For an easy start, put in any known scores.

1. Chicago scored one run in the first inning, but did not score again until the 4th inning.

2. Cleveland tied it up in the third inning.

3. Chicago scored the same number of runs in the 4th, 5th, and 10th innings.

4. Neither team scored in the 7th inning or 9th inning.

5. Both teams scored 2 runs in the 5th inning.

6. Chicago scored one run fewer in the 6th inning than they did in the 5th inning, and did not score again until the 10th inning.

7. Cleveland scored in only 4 innings, and tied it up again in the 8th inning.

8. Cleveland batted in the 10th inning and scored one run.

<table>
<thead>
<tr>
<th>Inning</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chicago</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Cleveland</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

**Strand**
Data Management and Probability
Batter Up!

Problem
Use the following clues to fill in the table provided in the problem. Use the information to determine who won this classic 2016 World Series game between the Cleveland and Chicago baseball teams. HINT: For an easy start, put in any known scores.

1. Chicago scored one run in the first inning, but did not score again until the 4th inning.
2. Cleveland tied it up in the third inning.
3. Chicago scored the same number of runs in the 4th, 5th, and 10th innings.
4. Neither team scored in the 7th inning or 9th inning.
5. Both teams scored 2 runs in the 5th inning.
6. Chicago scored one run fewer in the 6th inning than they did in the 5th inning, and did not score again until the 10th inning.
7. Cleveland scored in only 4 innings, and tied it up again in the 8th inning.
8. Cleveland batted in the 10th inning and scored one run.

Solution
Using the first two clues, we can fill the chart to the following point.

<table>
<thead>
<tr>
<th>Inning</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chicago</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cleveland</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Using the third and fourth clues, we can fill more information in the chart. We will use a ■ to show that Chicago scored the same number of runs in each of innings 4, 5, and 10.

<table>
<thead>
<tr>
<th>Inning</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chicago</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>■</td>
<td>■</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>■</td>
<td></td>
</tr>
<tr>
<td>Cleveland</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Using the fifth clue, we can put a 2 in for both teams in inning number 5. But Chicago scored the same number of runs in innings 4, 5 and 10 so we can replace □ with a 2 in the three spots.

<table>
<thead>
<tr>
<th>Inning</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chicago</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cleveland</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Using the sixth clue, we can put a 1 in for Chicago in inning number 6. We can fill in zeros for Chicago in inning 8 and since we have all the information for Chicago, we can fill in Chicago’s final score with an 8.

<table>
<thead>
<tr>
<th>Inning</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chicago</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>Cleveland</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Using the eighth clue, we know that Cleveland scored 1 run in the tenth inning.

<table>
<thead>
<tr>
<th>Inning</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chicago</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>Cleveland</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>7</td>
</tr>
</tbody>
</table>

Finally, using the seventh clue, we know that Cleveland tied the game in the eighth inning and only scored in four innings altogether. So Cleveland must have scored 3 runs in their half of the eighth inning and no runs in the fourth and sixth innings. Cleveland’s final score can now be filled in with a 7.

<table>
<thead>
<tr>
<th>Inning</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chicago</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>Cleveland</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>7</td>
</tr>
</tbody>
</table>

Based on the clues provided, we have determined that the Chicago team won the game by the score 8 to 7 over the Cleveland team in this 2016 World Series game.
Problem of the Week
Problem B
How Dense is This?

In honour of Canada 150, let’s explore how much space it has. In terms of its land area, 9 156 521 km$^2$, Canada is the fourth largest country in the world. Russia, China, and the U.S.A. have greater land area, but Canada’s population is significantly less.

a) Find the area and population* of the following countries. Add three of your own choices to the table.

<table>
<thead>
<tr>
<th>Country</th>
<th>Land Area</th>
<th>Population</th>
<th>Persons per km$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>9 156 521 km$^2$</td>
<td>36.63 million</td>
<td>$\approx$ 4.0 persons/km$^2$</td>
</tr>
<tr>
<td>Russia</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ukraine</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>India</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finland</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brazil</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sudan</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Which countries have the least, and greatest, population density? In each of these two countries, how many square km of land area are there for each person?

c) The city of Mumbai in India has an area of 603.4 km$^2$, and a population of 21.69 million. How many square metres of land area are there for each person in Mumbai?

d) How many square metres of classroom space are there for each person in your classroom?

* For populations and areas, try the website

http://www.worldometers.info/world-population/population-by-country/

NOTE: Land area data varies, depending on your chosen source.

**STRAND**  DATA MANAGEMENT AND PROBABILITY
Problem of the Week
Problem B and Solution
How Dense is This?

Problem
In honour of Canada 150, let’s explore how much space it has. In terms of its land area, \(9,156,521\) km\(^2\), Canada is the fourth largest country in the world. Russia, China, and the U.S.A. have greater land area, but Canada’s population is significantly less.

a) Find the area and population\(^*\) of the following countries. Add three of your own choices to the table.

<table>
<thead>
<tr>
<th>Country</th>
<th>Land Area</th>
<th>Population</th>
<th>Persons per km(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>9,156,521 km(^2)</td>
<td>36.63 million</td>
<td>(\approx 4.0) persons/km(^2)</td>
</tr>
<tr>
<td>Russia</td>
<td>16,292,614 km(^2)</td>
<td>143.38 million</td>
<td>(\approx 8.8) persons/km(^2)</td>
</tr>
<tr>
<td>Japan</td>
<td>364,503 km(^2)</td>
<td>126.05 million</td>
<td>(\approx 345.8) persons/km(^2)</td>
</tr>
<tr>
<td>Ukraine</td>
<td>578,945 km(^2)</td>
<td>44.41 million</td>
<td>(\approx 76.7) persons/km(^2)</td>
</tr>
<tr>
<td>India</td>
<td>2,973,450 km(^2)</td>
<td>1,342.51 million</td>
<td>(\approx 451.5) persons/km(^2)</td>
</tr>
<tr>
<td>Finland</td>
<td>304,466 km(^2)</td>
<td>5.54 million</td>
<td>(\approx 18.2) persons/km(^2)</td>
</tr>
<tr>
<td>Brazil</td>
<td>8,349,534 km(^2)</td>
<td>211.24 million</td>
<td>(\approx 25.3) persons/km(^2)</td>
</tr>
<tr>
<td>Sudan</td>
<td>1,764,281 km(^2)</td>
<td>42.17 million</td>
<td>(\approx 23.9) persons/km(^2)</td>
</tr>
<tr>
<td>Germany</td>
<td>348,621 km(^2)</td>
<td>80.64 million</td>
<td>(\approx 231.3) persons/km(^2)</td>
</tr>
<tr>
<td>China</td>
<td>9,386,293 km(^2)</td>
<td>1,388.23 million</td>
<td>(\approx 147.9) persons/km(^2)</td>
</tr>
<tr>
<td>U. S. A.</td>
<td>9,144,930 km(^2)</td>
<td>326.47 million</td>
<td>(\approx 35.7) persons/km(^2)</td>
</tr>
</tbody>
</table>

b) Which countries have the least, and greatest, population density? In each of these two countries, how many square km of land area are there for each person?

c) The city of Mumbai in India has an area of 603.4 km\(^2\), and a population of 21.69 million. How many square metres of land area are there for each person in Mumbai?

d) How many square metres of classroom space are there for each person in your classroom?

Solution
a) The completed table plus optional countries is shown above, along with the calculated population densities. (*Note that answers will vary, depending on the source of data on land area and population.)

b) Canada has the least population density, 4.0 persons per km\(^2\); India has the greatest, 408.4 persons per km\(^2\). There are \(9,156,521 \div 36,630,000 \approx 0.249973\) km\(^2\) of land area per person in Canada. There are \(2,973,450 \div 1,342,510,000 \approx 0.00221484\) km\(^2\) of land area per person in India, less than 1% of Canada’s space per person.

c) There are \(603,400,000 \div 21,690,000 \approx 27.8\) m\(^2\) (or 0.000278 km\(^2\)) of land area per person in Mumbai. Thus Mumbai is much more densely populated than India as a whole.

d) Taking an average classroom as 10 m by 7 m, or 70 m\(^2\), with 25 persons, the space per person is \(70 \div 25 = 2.8\) m\(^2\). (Your classroom may differ in both size and/or numbers.)
Problem of the Week
Problem B
Olympic Proportions

Olympic Games are held every two years, alternating between summer and winter Olympics. For example, Rio de Janeiro held the Summer Olympics in 2016, while Pyeongchang, South Korea will host the Winter Olympics this month.

a) Using the website https://www.olympic.org/olympic-games, make a list of all the countries which have hosted Olympic games from 1994 to 2016.

b) Look up the number of countries on each continent. Then complete the table below to demonstrate the fraction of countries on each continent which have hosted Olympic games from 1994 to 2016.
(http://www.enchantedlearning.com/geography/continents/Extremes.shtml)

<table>
<thead>
<tr>
<th>Continent</th>
<th>No. of Countries</th>
<th>No. of Host Countries</th>
<th>Fraction (%) Hosts/Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asia</td>
<td>47</td>
<td>3</td>
<td>$\frac{3}{47}$ (6.4%)</td>
</tr>
<tr>
<td>Europe</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. America</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S. America</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Africa</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c) If you were trying to distribute hosting countries evenly, which continent should be next to host either games? Justify your choice.

**Strands**  Number Sense and Numeration, Data Management and Probability
Problem

Olympic Games are held every two years, alternating between summer and winter Olympics. For example, Rio de Janeiro held the Summer Olympics in 2016, while Pyeonchang, South Korea will host the Winter Olympics this month.

a) Using the website https://www.olympic.org/olympic-games, make a list of all the countries which have hosted Olympic games from 1994 to 2016.

b) Look up the number of countries on each continent. Then complete the table below to demonstrate the fraction of countries on each continent which have hosted Olympic games.

<table>
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<tr>
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<th>No. of Host Countries</th>
<th>Fraction (% Hosts/Countries)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asia</td>
<td>47</td>
<td>3</td>
<td>$\frac{3}{47}$ (6.4%)</td>
</tr>
<tr>
<td>Europe</td>
<td>43</td>
<td>4</td>
<td>$\frac{4}{43}$ (9.3%)</td>
</tr>
<tr>
<td>N. America</td>
<td>23</td>
<td>2 (3)</td>
<td>$\frac{3}{23}$ (13.0%)</td>
</tr>
<tr>
<td>S. America</td>
<td>12</td>
<td>1</td>
<td>$\frac{1}{12}$ (8.3%)</td>
</tr>
<tr>
<td>Australia</td>
<td>14</td>
<td>1</td>
<td>$\frac{1}{14}$ (7.1%)</td>
</tr>
<tr>
<td>Africa</td>
<td>54</td>
<td>0</td>
<td>$\frac{0}{54}$ (0%)</td>
</tr>
</tbody>
</table>

c) If you were trying to distribute hosting countries evenly, which continent should be next to host either games? Justify your choice.

Solution

a) Host countries are as follows:

1994 Lillehammer, Norway 2002 Salt Lake City, USA 2010 Vancouver, Canada
1996 Atlanta, USA 2004 Athens, Greece 2012 London, UK
1998 Nagano, Japan 2006 Turin, France 2014 Sochi, Russia
2000 Sydney, Australia 2008 Beijing, China 2016 Rio de Janeiro, Brazil

b) See the completed table above. Source http://www.enchantedlearning.com/geography/continents/Extremes.shtml

c) Ideally, Africa should host; however, both the hot climate and a lack financial resources work against this.

Given the percentages of host countries on each continent, Asia should be the next host (which is to happen in Pyeonchang, Korea).
Problem of the Week
Problem B
On the Podium

Gregory, Adam and Paul are athletes who competed in the downhill skiing event in the Winter Olympics.

Gregory, Adam and Paul each finished in first, second or third. There were no ties.

Each athlete is also from a different country. One is from Canada, one is from France and one is from Japan.

Using the following clues, determine who placed first, second and third, and for which country each athlete was competing.

1. Gregory was faster than Adam.
2. Gregory is not Canadian, and he did not finish in second place.
3. The Japanese athlete was faster than the French athlete.
4. Adam is not Japanese and he did not finish in third place.
5. The Canadian athlete was faster than the French athlete.

You may find the following table a helpful way to organize your solution to this problem.

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>France</th>
<th>Japan</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gregory</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adam</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paul</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1st
2nd
3rd

**Strand** Data Management and Probability
Problem
Gregory, Adam and Paul are athletes who competed in the downhill skiing event in the Winter Olympics. Gregory, Adam and Paul each finished in first, second or third. There were no ties. Each athlete is also from a different country. One is from Canada, one is from France and one is from Japan. Using the following clues, determine who placed first, second and third, and for which country each athlete was competing.

1. Gregory was faster than Adam.
2. Gregory is not Canadian, and he did not finish in second place.
3. The Japanese athlete was faster than the French athlete.
4. Adam is not Japanese and he did not finish in third place.
5. The Canadian athlete was faster than the French athlete.

Solution
In our solution, we will go through each clue and update the table based on the information in the clue. We will put an $\times$ in a cell if the combination indicated by the row and column for that cell is not possible, or a $\checkmark$ if it must be true.

From clue (1), since Gregory was faster than Adam, we know that Adam could not have finished first and that Gregory did not finish third. We can therefore put an $\times$ in the cells corresponding to Adam in first and Gregory in third.

From clue (2), we can put an $\times$ in the cells corresponding to Gregory being Canadian and also to Gregory in second. The table is updated as follows:

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>France</th>
<th>Japan</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gregory</td>
<td>$\times$</td>
<td></td>
<td></td>
<td>$\times$</td>
<td>$\times$</td>
<td></td>
</tr>
<tr>
<td>Adam</td>
<td></td>
<td></td>
<td></td>
<td>$\times$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paul</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We see that Gregory must have finished first. We can add a $\checkmark$ to the corresponding cell in the table. Since Gregory finished first and there were no ties, we know that Paul did not finish first. We can add an $\times$ to the corresponding cell in the table. The table is updated and shown on the top of the next page.
<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>France</th>
<th>Japan</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gregory</td>
<td>X</td>
<td>X</td>
<td>✓</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Adam</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paul</td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>1st</td>
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<td>X</td>
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<td>2nd</td>
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</tr>
<tr>
<td>3rd</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>

From clue (3), we know the French athlete did not finish first and that the Japanese athlete did not finish third. We can therefore put an X in the cells corresponding to the French athlete in first and the Japanese athlete in third. Since we now know that Gregory came in first, this clue also tells us that Gregory is not French. We can put an X in the corresponding cell. The table is updated below.

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>France</th>
<th>Japan</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gregory</td>
<td>X</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Adam</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paul</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st</td>
<td></td>
<td>X</td>
<td>✓</td>
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<tr>
<td>2nd</td>
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<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

We see that Gregory must compete for Japan. Since Gregory finished first, this also tells us that the Japanese athlete finished first. We can add a ✓ to the corresponding cells in the table.

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>France</th>
<th>Japan</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gregory</td>
<td>X</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Adam</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paul</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st</td>
<td></td>
<td>X</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Since the athletes are from different countries, we now know that Adam and Paul are not from Japan, and so we can add X’s to the corresponding cells in the table. Since we know the Japanese athlete finished first, he could not have finished second too, and the Canadian did not finish first, so we can add X’s to the corresponding cells in the table. The table is updated below.

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>France</th>
<th>Japan</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gregory</td>
<td>X</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Adam</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paul</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st</td>
<td></td>
<td>X</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2nd</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>
From clue (4), we can add an X in the cells corresponding to Adam finishing in third. The table is updated as follows.

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>France</th>
<th>Japan</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gregory</td>
<td>X</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Adam</td>
<td></td>
<td>X</td>
<td>X</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paul</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We can now see that Adam must have finished in second and Paul must have finished in third. We can add the corresponding ✓’s and X’s. The table is updated below.

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>France</th>
<th>Japan</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gregory</td>
<td>X</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Adam</td>
<td>✓</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>✓</td>
<td>X</td>
</tr>
<tr>
<td>Paul</td>
<td>X</td>
<td>✓</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From clue (5), we know the Canadian athlete was faster than the French athlete. Since the Japanese athlete finished first, this means the Canadian must have finished second and the French athlete must have finished third. Since Adam finished second and the Canadian athlete finished second, then Adam is the Canadian athlete and Paul is the French athlete. We can add the corresponding ✓’s and X’s. The table is updated as follows.

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>France</th>
<th>Japan</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gregory</td>
<td>X</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Adam</td>
<td>✓</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>✓</td>
<td>X</td>
</tr>
<tr>
<td>Paul</td>
<td>X</td>
<td>✓</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In conclusion:

Gregory competed for Japan and finished in first place.

Adam competed for Canada and finished in second place.

Paul competed for France and finished in third place.
Consider the winter Olympic sports of downhill skiing and 1500 metre speed skating, which have recorded numeric results. In the table below are the gold medalist times for men and women in these events in the ten Winter Olympics since 1980 (from the website https://www.olympic.org/sports).

<table>
<thead>
<tr>
<th>Year</th>
<th>Downhill Skiing</th>
<th>Speed Skating</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Men</td>
<td>Women</td>
</tr>
<tr>
<td>1980</td>
<td>1:45.50</td>
<td>1:37.52</td>
</tr>
<tr>
<td>1984</td>
<td>1:45.59</td>
<td>1:13.36</td>
</tr>
<tr>
<td>1988</td>
<td>1:59.63</td>
<td>1:25.86</td>
</tr>
<tr>
<td>1992</td>
<td>1:50.37</td>
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</tr>
<tr>
<td>1994</td>
<td>1:45.75</td>
<td>1:35.93</td>
</tr>
<tr>
<td>1998</td>
<td>1:50.11</td>
<td>1:28.89</td>
</tr>
<tr>
<td>2002</td>
<td>1:39.13</td>
<td>1:39.56</td>
</tr>
<tr>
<td>2006</td>
<td>1:48.80</td>
<td>1:56.49</td>
</tr>
<tr>
<td>2010</td>
<td>1:54.31</td>
<td>1:44.19</td>
</tr>
<tr>
<td>2014</td>
<td>2:06.23</td>
<td>1:41.57</td>
</tr>
</tbody>
</table>

a) How many times has the winning time improved in each sport from one Olympics to the next, and what was the greatest improvement?

b) How much faster is the most recent record-breaking time than the 1980 winning time for each sport? Who achieved this fastest time?

c) Create two graphs (one for each sport) of the winning times over these ten Winter Olympics. Use whichever type of graph you think will best display the data.

**STRANDS**  Data Management and Probability, Number Sense and Numeration
Problem of the Week
Problem B and Solution
Citius, Altius, Fortius

Problem
Consider the winter Olympic sports of downhill skiing and 1500 metre speed skating, which have recorded numeric results. In the table below are the gold medalist times for men and women in these events in the ten Winter Olympics since 1980 (from the website https://www.olympic.org/sports).

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<tr>
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<td>1980</td>
<td>1:45.50</td>
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</tr>
<tr>
<td>1992</td>
<td>1:50.37</td>
<td>1:52.55</td>
</tr>
<tr>
<td>1994</td>
<td>1:45.75</td>
<td>1:35.93</td>
</tr>
<tr>
<td>1998</td>
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<td>2014</td>
<td>2:06.23</td>
<td>1:41.57</td>
</tr>
</tbody>
</table>

a) How many times has the winning time improved in each sport from one Olympics to the next, and what was the greatest improvement?
b) How much faster is the most recent record-breaking time than the 1980 winning time for each sport? Who achieved this fastest time?
c) Create two graphs (one for each sport) of the winning times over these ten Winter Olympics. Use whichever type of graph you think will best display the data.

Solution
a) In downhill skiing, the men’s times improved three times, the greatest being 10.98 seconds from 1998 to 2002; the women’s times improved five times, the greatest being 24.16 seconds from 1980 to 1984.
In speed skating, the men’s times improved six times, the greatest being 6.30 seconds from 1984 to 1988; the women’s times improved six times, the greatest being 7.53 seconds from 1980 to 1984.

b) In downhill skiing, the men’s most recent record-breaking time, achieved by Fritz Strobl of Austria in 2002, was 1:39.13, 6.37 seconds faster than 1980; the women’s most recent record-breaking time, achieved by Michela Figini of Switzerland in 1984, was 1:13.36, 24.16 seconds faster than 1980.
In speed skating, the men’s most recent record-breaking time, achieved by Derek Parra of the USA in 2002, was 1:43.95, 11.49 seconds faster than 1980; the women’s most recent record-breaking time, achieved by Jorien Ter Mors of the Netherlands in 2014, was 1:53.51, 17.44 seconds faster than 1980.
c) Here are the required graphs

**Olympic Downhill Skiing Results**

- **Men's Results**
- **Women's Results**

**Olympic 1500m Speed Skating Results**

- **Men's Results**
- **Women's Results**

**Extension:**
Do some research and write a paragraph about why these changes in winning times have occurred.
Problem of the Week
Problem B
I Say My Name

An onomatopoeic chickadee flies from one of three different trees to one of two feeders, and then flies back to one of the trees.

a) Make a ‘tree’ diagram to show all possible routes the chickadee might take. How many routes are there in total?

b) Look at the number of possible choices of trees the chickadee might leave, the number of possible choices of feeders on which it could land, and then the number of possible choices of trees to which it could return. With what arithmetic operation (e.g., addition, multiplication, etc.) could you combine these three numbers in order to obtain the total number of routes found in part a)?

c) After dining at one of the feeders, the chickadee decides to stop at either a birdbath, or on your hand for some seeds before returning to the trees. Use your result from b) to determine the new number of total possible routes.
Problem of the Week
Problem B and Solution
I Say My Name

Problem
An onomatopoeic chickadee flies from one of three different trees to one of two feeders, and then flies back to one of the trees.

a) Make a ‘tree’ diagram to show all possible routes the chickadee might take. How many routes are there in total?

b) Look at the number of possible choices of trees the chickadee might leave, the number of possible choices of feeders on which it could land, and then the number of possible choices of trees to which it could return. With what arithmetic operation (e.g., addition, multiplication, etc.) could you combine these three numbers in order to obtain the total number of routes found in part a)?

c) After dining at one of the feeders, the chickadee decides to stop at either a birdbath, or on your hand for some seeds before returning to the trees. Use your result from b) to determine the new number of total possible routes.

Solution

a) In the tree diagram at the right, the feeders are labelled 1, 2, and the trees are labelled 1, 2, 3. The tree diagram reveals that there are 18 different routes the chickadee could take.

b) For each of the three trees the chickadee could leave there are two possible feeders on which it could land. So there are $3 \times 2 = 6$ ways it could do this part of its journey. Then for each of these, there are 3 different trees to which it could fly back. So in total there are $3 \times 2 \times 3 = 18$ different routes it could take, obtained by multiplying the number of possibilities at each stage.

c) If it could also stop at a bird bath or to eat some seeds from your hand, then two further possibilities would be inserted after the feeders but before the return to the trees. Thus there would now be a total of $3 \times 2 \times 2 \times 3 = 36$ routes.
StatsCan is doing a census. Jayne has been hired to enumerate 330 households.

a) If Jayne can deliver, on average, one form every 8 minutes, how many hours will she need to deliver all 330 forms?

b) On average, she has to travel 0.14 km between households to deliver the forms, and she is paid 92 cents per kilometre. If she is also paid $14.42 per hour for enumerating, how much money will she earn?

c) An average household has 1.7 adults and 1.2 children. Pick an appropriate type of graph to show how many adults and how many children we could expect to see in this region.
Problem of the Week  
Problem B and Solution  
Number Census and Enumeration

Problem  
StatsCan is doing a census. Jayne has been hired to enumerate 330 households.

a) If Jayne can deliver, on average, one form every 8 minutes, how many hours will she need to deliver all 330 forms?

b) On average, she has to travel 0.14 km between households to deliver the forms, and she is paid 92 cents per kilometre. If she is also paid $14.42 per hour for enumerating, how much money will she earn?

c) An average household has 1.7 adults and 1.2 children. Pick an appropriate type of graph to show how many adults and how many children we could expect to see in this region.

Solution

a) Jayne will need \(8 \times 330 = 2640\) minutes, or \(2640 \div 60 = 44\) hours to deliver all the forms.

b) There will be 329 spaces of 0.14 km between the 330 households, so Jayne will travel a total distance of \(0.14 \times 329 = 46.06\) km. Thus she will be paid \(46.06 \times 0.92 \approx $42.38\) for travel.

In addition, she will receive \(44 \times 14.42 = $634.48\) for enumerating. Thus she will earn \(634.48 + 42.38 = $676.86\) in total.

c) This region will have, on average, \(330 \times 1.7 = 561\) adults, and \(330 \times 1.2 = 396\) children. Below are two types of graphs which illustrate this.
Problem of the Week

Problem B
A Whale of a Shark!

Whale-sharks are relatively friendly fish on whom divers have been known to hitch rides. They are ‘filter-feeders’ with some 300 rows of teeth, and travel long distances at a leisurely 5 km per hour, or 120 km per day.


When they are tagged, their lengths are measured and they are fitted with an electronic tracking device. It transmits their locations each time they surface, and records the distance from where they last surfaced.

In the table are the names of eleven of the whale-sharks being tracked (these may change over time, so replace any that are no longer available). The entries for Susi have been completed for you.

<table>
<thead>
<tr>
<th>Name</th>
<th>Length</th>
<th>Distance</th>
<th>Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Susi</td>
<td>6.2 m</td>
<td>19.1 km</td>
<td></td>
</tr>
<tr>
<td>Hula</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nexus</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mr. Casper</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Barack</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yoda</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cheggers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dipsy</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Kaimana</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sebastian</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Giti</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Go to the website http://www.conservation.org/projects/Pages/Track-Whale-Sharks.aspx and fill in the lengths and most recent distances for the whale-sharks in the table.

a) Find the mean, median, mode, and range of the lengths of the 11 fish.

b) Find the mean, median, mode, and range of the distances travelled by the 11 whale-sharks since their previous transmissions.

c) Why do you think the distances travelled vary so much, whereas the lengths of the whale-sharks do not?

d) Determine the number of days of travel for each whale-shark which travelled more than 100 km since its previous transmission. (Recall the average speed of 120 km/day.)

**STRAND** Data Management and Probability
Problem of the Week
Problem B and Solution
A Whale of a Shark!

Problem
Whale-sharks are relatively friendly fish on whom divers have been known to hitch rides. They are ‘filter-feeders’ with some 300 rows of teeth, and travel long distances at a leisurely 5 km per hour, or 120 km per day. The website http://www.conservation.org/projects/Pages/Track-Whale-Sharks.aspx contains much information about tagged whale-sharks.

When they are tagged, their lengths are measured and they are fitted with an electronic tracking device. It transmits their locations each time they surface, and records the distance from where they last surfaced.

In the table are the names of eleven of the whale-sharks being tracked (these may change over time, so replace any that are no longer available). The entries for Susi have been completed for you.

<table>
<thead>
<tr>
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<th>Distance</th>
<th>Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Susi</td>
<td>6.2 m</td>
<td>85.1 km</td>
<td></td>
</tr>
<tr>
<td>Hula</td>
<td>7.2 m</td>
<td>5 km</td>
<td></td>
</tr>
<tr>
<td>Nexus</td>
<td>3.93 m</td>
<td>69.5 km</td>
<td></td>
</tr>
<tr>
<td>Mr. Casper</td>
<td>4.65 m</td>
<td>16.1 km</td>
<td></td>
</tr>
<tr>
<td>Barack</td>
<td>7.51 m</td>
<td>5.1 km</td>
<td></td>
</tr>
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<td>Yoda</td>
<td>4.83 m</td>
<td>1 km</td>
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</tr>
<tr>
<td>Cheggers</td>
<td>5.64 m</td>
<td>3.6 km</td>
<td></td>
</tr>
<tr>
<td>Dipsy</td>
<td>4.57 m</td>
<td>4 km</td>
<td></td>
</tr>
<tr>
<td>Kaimana</td>
<td>4.46 m</td>
<td>329.6 km</td>
<td>2.747</td>
</tr>
<tr>
<td>Sebastian</td>
<td>4.37 m</td>
<td>3.6 km</td>
<td></td>
</tr>
<tr>
<td>Giti</td>
<td>4.75 m</td>
<td>107.7 km</td>
<td>0.8975</td>
</tr>
</tbody>
</table>

Go to the website and fill in the lengths and most recent distances for the whale-sharks in the table.

a) Find the mean, median, mode, and range of the lengths of the 11 fish.
b) Find the mean, median, mode, and range of the distances travelled by the 11 whale-sharks since their previous transmissions.
c) Why do you think the distances travelled vary so much, whereas the lengths of the whale-sharks do not?
d) Determine the number of days of travel for each whale-shark which travelled more than 100 km since its previous transmission. (Recall the average speed of 120 km/day.)

Solution
The table has been completed with data given as of April 30, 2017, including the calculated days of travel for those whale-sharks with distances greater than 100 km in part d).

a) The mean of the lengths is approximately 5.28 m; the median is 4.75 m; there is no mode; the range of lengths from 3.93 m to 7.51 m is 3.58 m.
b) The mean of the distances travelled is 57.3 km; the median is 5.1 km; the mode is 3.6 km; the range of distances from 1 km to 329.6 km is 328.6 km.
c) The distances measured depend on how often the fish surfaces, so they can vary a lot, whereas the length of a mature whale-shark will naturally be much more consistent from fish to fish.
d) Kaimana’s transmissions were 2.747 days apart. This is approximately 2 days and 18 hours. Giti’s transmissions were 0.8975 days apart. This is approximately 21.5 hours.
Geometry & Spatial Sense

TAKE ME TO THE COVER
Red and Justin are planning a countryside ride on their ATV. They will follow this pattern:
- drive five kilometres and turn right;
- drive five kilometres and turn right;
- drive five kilometres and turn left;
- keep repeating these three steps until they return to their starting point.

a) On the grid below, the side of each square is one kilometre. Map out the boys’ route, starting from the point $S$ in the direction shown. Then determine how far they will have travelled when they get back to $S$.

b) What is the name of the shape enclosed by their route?

c) What is the area of this shape?
Problem of the Week
Problem B and Solution
Crossing Paths

Problem
Red and Justin are planning a countryside ride on their ATV. They will follow this pattern: drive five kilometres and turn right; drive five kilometres and turn right; drive five kilometres and turn left. Keep repeating these three steps until they return to their starting point.

a) On the grid below, the side of each square is one kilometre. Map out the boys’ route, starting from the point $S$ in the direction shown. Then determine how far they will have travelled when they get back to $S$.
b) What is the name of the shape enclosed by their route?
c) What is the area of this shape?

Solution

a) The boys’ route is shown on the above grid as a solid line, with arrows indicating the directions travelled. Since each line segment has length 5 km, the total distance they travelled is $12 \times 5 = 60$ km.

b) The 12-sided geometric shape enclosed by their route is called an irregular dodecagon.

c) The dashed lines on the grid show that this shape consists of 5 squares, each with side length 5 km. Thus the total area enclosed is $5 \times (5 \times 5) = 125$ km$^2$. 
Problem of the Week
Problem B
Code Freddy Home

Starting at the top left square of the maze, and facing left (symbolized by ←), Freddy the Robot must follow your program commands to get to his Home Charging Station at H. Freddy can only turn right (clockwise on the diagram), and will only turn the number of degrees which you tell him. As well, he will go exactly the number of squares you tell him.

For example, if he starts facing ←, and your command said, “Turn 180 and go 3 spaces”, he would turn 180° right (clockwise) and then move 3 spaces forward (to the right).

Write a set of 15 commands which will guide Freddy from the start to Home. Each command should be of the form: Turn ____ and go ____ spaces.

EXTENSION: Draw your own maze and create a program to help Freddy go Home.

STRAND    GEOMETRY AND SPATIAL SENSE
Problem of the Week
Problem B and Solution
Code Freddy Home

Problem
Starting at the top left square of the maze, and facing left (symbolized by ←), Freddy the Robot must follow your program commands to get to his Home Charging Station at H. Freddy can only turn right (clockwise on the diagram), and will only turn the number of degrees which you tell him. As well, he will go exactly the number of squares you tell him.
For example, if he starts facing ←, and your command said, “Turn 180° and go 3 spaces”, he would turn 180° right (clockwise) and then move 3 spaces forward (to the right).

Write a set of 15 commands which will guide Freddy from the start to Home. Each command should be of the form: Turn ____ and go ____ spaces.

Extension: Draw your own maze and create a program to help Freddy go Home.

Solution
Here are the required 15 commands: (Note there two options for turns 12 to 14. The second option is in parentheses.)

1. Turn 180° and go 3 spaces.
2. Turn 90° and go 2 spaces.
3. Turn 90° and go 3 spaces.
4. Turn 270° and go 3 spaces.
5. Turn 270° and go 3 spaces.
6. Turn 90° and go 3 spaces.
7. Turn 270° and go 2 spaces.
8. Turn 270° and go 8 spaces.
9. Turn 90° and go 2 spaces.
10. Turn 90° and go 1 space.
11. Turn 270° and go 2 spaces.
12. Turn 90° and go 3 (or 2) spaces.
13. Turn 90° and go 2 spaces.
14. Turn 270° and go 4 (or 5) spaces.
15. Turn 270° and go 2 spaces.

Now Freddy is home!
Problem of the Week
Problem B
Take a Turn

Let’s explore some ‘turns’ in different contexts.

a) Shredder does snowboard helicopter jumps with ease. If he rotates all the way around in a horizontal circle, through how many degrees does he rotate? What if he only rotates halfway around?

b) Through how many degrees does the minute hand on an analog clock move during
   i) $\frac{1}{4}$ hour (say from 12:45 to 1:00 o’clock)?
   ii) 1 minute (say from 12:59 to 1:00 o’clock)?

c) Through how many degrees does the hour hand move in one hour (say from 1:00 to 2:00 o’clock)?

d) If a clock is keeping time correctly, the minute hand rotates at a rate of $360^\circ$ per hour. One day, Sam notices that the time on the clock and on her digital watch is the same at noon, but when her watch says 1:00 o’clock, the clock says 1:05. Assuming that Sam’s digital watch keeps accurate time, at what rate is the minute hand of the clock rotating now?
Problem of the Week
Problem B and Solution
Take a Turn

Problem
Let’s explore some ‘turns’ in different contexts.

a) Shredder does snowboard helicopter jumps with ease. If he rotates all the way around in a horizontal circle, through how many degrees does he rotate? What if he only rotates halfway around?
b) Through how many degrees does the minute hand on an analog clock move during
   i) \( \frac{1}{4} \) hour (say from 12:45 to 1:00 o’clock)?
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c) Through how many degrees does the hour hand move in one hour (say from 1:00 to 2:00 o’clock)?
d) If a clock is keeping time correctly, the minute hand rotates at a rate of 360° per hour. One day, Sam notices that the time on the clock and on her digital watch is the same at noon, but when her watch says 1:00 o’clock, the clock says 1:05. Assuming that Sam’s digital watch keeps accurate time, at what rate is the minute hand of the clock rotating now?

Solution

a) In doing a full turn, Shredder rotates horizontally through 360°; in a half turn, he rotates 180°
b) i) Since the minute hand rotates through 360° in 1 hour, in \( \frac{1}{4} \) hour, it rotates through \( 360° \div 4 = 90° \).
ii) One minute is \( \frac{1}{60} \) th of an hour. Since the minute hand rotates through 360° in 1 hour, in \( \frac{1}{60} \) hour, it rotates through \( 360° \div 60 = 6° \).
c) In one hour, the hour hand moves \( \frac{1}{12} \) of a full circle, or \( 360° \div 12 = 30° \)
d) Assuming that Sam’s digital watch keeps accurate time, the minute hand of the clock has moved an extra \( \frac{1}{12} \) of a full circle in one hour of real time. Thus it is moving at a rate of \( 360° + 30° = 390° \) per hour.
Problem of the Week
Problem B
No Business Like Snow Business

The snow is deep and it’s perfect snow fort weather. You’ve decided to build a wall 1.5 m high, 9 m long and 30 cm deep. You find that, by packing snow into your trusty plastic bin, you can make cubic ‘snow bricks’ which have dimensions 30 cm by 30 cm by 30 cm.

a) How many bricks will you need to build the wall?

b) Suppose you had a bin in the shape of a rectangular prism that was 60 cm long by 30 cm wide by 15 cm high. How many rectangular prism bricks would you need to build a wall with the same dimensions?

c) In the end, you decide to use a 2 : 1 ratio of rectangular bricks to cubic bricks. How many cubic bricks will you need?
Problem of the Week
Problem B and Solution
No Business Like Snow Business

Problem

The snow is deep and it’s perfect snow fort weather. You’ve decided to build a wall 1.5 m high, 9 m long and 30 cm deep. You find that, by packing snow into your trusty plastic bin, you can make cubic ‘snow bricks’ which have dimensions 30 cm × 30 cm × 30 cm.

a) How many bricks will you need to build the wall?
b) Suppose you had a rectangular prism bin 60 cm long by 30 cm wide by 15 cm high. How many rectangular prism bricks would you need to build a wall with the same dimensions?
c) In the end, you decide to use a 2:1 ratio of rectangular bricks to cubic bricks. How many cubic bricks will you need?

Solution

a) Since each brick is a 30 cm cube, it has height and length equal to 0.3 m. Thus a wall 1.5 m high and 9 m long will need to be $1.5 \div 0.3 = 5$ bricks high, and $9 \div 0.3 = 30$ bricks long. So the total number of bricks required will be $5 \times 30 = 150$ bricks.

b) The number of blocks needed is 150. This can be done in many ways. Here is one solution. Since the wall is 30 cm deep (or thick), you must use that dimension of the rectangular prism bricks as the depth. Using the 15 cm (or 0.15 m) dimension for the height, a 1.5 m high wall would need to be $1.5 \div 0.15 = 10$ bricks high. Using the 60 cm (or 0.6 m) dimension for the length, the 9 m length would require $9 \div 0.6 = 15$ bricks. Thus the total number of rectangular prism bricks required would be $10 \times 15 = 150$ bricks.

c) For a 2:1 ratio of rectangular bricks to cubic bricks, you want half as many cubic bricks as rectangular (i.e., $\frac{2}{3}$ rectangular and $\frac{1}{3}$ cubic). We see from a) and b) that 150 of either type of snow brick is needed to completely build the wall. So we can use 100 rectangular bricks (10 high and 10 wide) to cover 6 m of the length, and 50 cubic bricks (5 high by 10 wide) to cover the remaining 3 m of length.
Problem of the Week
Problem B
This Strikes a Chord

A line segment that has its endpoints on a circle is called a chord. The diameter of a circle is a special chord which passes through the centre of the circle. Two chords can divide a circle in two different ways:

1. non-intersecting chords divide the circle into three pieces, or
2. the two chords can also intersect, giving four pieces.

NOTE: An intersection only refers to chords crossing inside the circle, and for this problem, no more than two chords can intersect at one point.

a) Three chords offer more possibilities. What is the maximum number of pieces into which a circle can be divided by three chords?

b) Sketch all the ways a circle can be subdivided by three chords. How is the number of intersections related to the number of pieces?

c) Find all the ways a circle can be subdivided by four chords. Is the number of intersections related to the number of pieces in the same way as in part b)?

d) If you used six chords, what would you predict to be the maximum number of pieces? Explain your reasoning.

Strands  Geometry and Spatial Sense, Patterning and Algebra
Problem
A line segment that has its endpoints on a circle is called a chord.

a) What is the maximum number of pieces into which a circle can be divided by three chords?
b) Sketch all the ways a circle can be subdivided by three chords. How is the number of intersections related to the number of pieces?
c) Find all the ways a circle can be subdivided by four chords. Is the number of intersections related to the number of pieces in the same way as in part b)?
d) If you used six chords, what would you predict to be the maximum number of pieces? Explain your reasoning.

Solution
In the following, the letter \( i \) refers to the number of intersections, and the letter \( p \) refers to the corresponding number of pieces into which the circle is divided.

a),b) The ways a circle can be divided by three chords is shown below. The maximum number of pieces is 7. Note that in each case, the number of pieces is 4 more than the number of intersections.

\[
\begin{align*}
&\text{i}_0, \text{p}_4 \\
&\text{i}_1, \text{p}_5 \\
&\text{i}_2, \text{p}_6 \\
&\text{i}_3, \text{p}_7
\end{align*}
\]

c) For four chords, the possibilities are shown below. In this case the number of pieces exceeds the number of intersections by 5 in each case. Thus we see that for 2, 3, and 4 chords, if \( n \) is the number of chords, then \( p = i + n + 1 \).

\[
\begin{align*}
&\text{i}_0, \text{p}_5 \\
&\text{i}_1, \text{p}_6 \\
&\text{i}_2, \text{p}_7 \\
&\text{i}_3, \text{p}_8 \\
&\text{i}_4, \text{p}_9 \\
&\text{i}_5, \text{p}_{10} \\
&\text{i}_6, \text{p}_{11}
\end{align*}
\]

d) Think of placing the chords one after the other. Since each chord can intersect all the previous chords, the maximum number of intersections for six chords will be \( 1 + 2 + 3 + 4 + 5 = 15 \). Observing the results of parts a), b), c), we predict that the number of pieces here will be \( p = i + 6 + 1 \). Thus, for six chords, the maximum number of pieces will be \( 15 + 6 + 1 = 22 \), as shown below.
Measurement

TAKE ME TO THE COVER
Problem of the Week
Problem B
Crossing Paths

Red and Justin are planning a countryside ride on their ATV. They will follow this pattern:
• drive five kilometres and turn right;
• drive five kilometres and turn right;
• drive five kilometres and turn left;
• keep repeating these three steps until they return to their starting point.

a) On the grid below, the side of each square is one kilometre. Map out the boys’ route, starting from the point S in the direction shown. Then determine how far they will have travelled when they get back to S.

b) What is the name of the shape enclosed by their route?

c) What is the area of this shape?

**STRANDS** Measurement, Geometry and Spatial Sense
Problem of the Week
Problem B and Solution
Crossing Paths

Problem
Red and Justin are planning a countryside ride on their ATV. They will follow this pattern: drive five kilometres and turn right; drive five kilometres and turn right; drive five kilometres and turn left. Keep repeating these three steps until they return to their starting point.
a) On the grid below, the side of each square is one kilometre. Map out the boys’ route, starting from the point $S$ in the direction shown. Then determine how far they will have travelled when they get back to $S$.
b) What is the name of the shape enclosed by their route?
c) What is the area of this shape?

Solution

a) The boys’ route is shown on the above grid as a solid line, with arrows indicating the directions travelled. Since each line segment has length 5 km, the total distance they travelled is $12 \times 5 = 60$ km.

b) The 12-sided geometric shape enclosed by their route is called an irregular dodecagon.

c) The dashed lines on the grid show that this shape consists of 5 squares, each with side length 5 km. Thus the total area enclosed is $5 \times (5 \times 5) = 125$ km$^2$. 
Problem of the Week
Problem B
Train Watching

Henry loves to watch the trains enter, then leave the 2 km tunnel through the mountain across the valley from his home.

One day, he sees the engine of a 1 km long train enter the tunnel at 12:10 p.m. If the train is travelling at 60 km per hour, at what time will Henry see the caboose leave the other end of the tunnel?
Problem of the Week
Problem B and Solution
Train Watching

Problem
Henry loves to watch the trains enter, then leave the 2 km tunnel through the mountain across the valley from his home. One day, he sees the engine of a 1 km long train enter the tunnel at 12:10 p.m. If the train is travelling at 60 km per hour, at what time will Henry see the caboose leave the other end of the tunnel?

Solution
As the front of the engine enters the tunnel, the back of the caboose is 1 km from the entrance of the tunnel and 1 + 2 = 3 km from the other end of the tunnel. The caboose must travel a total of 3 km.

To travel 3 km at 60 km per hour, it will take the caboose $3 \div 60 = \frac{1}{20}$ of an hour, or 3 minutes. Alternatively, since the train travels at 60 km per hour, it travels 60 km in 60 minutes or 1 km each minute. So to travel 3 km the caboose will take 3 minutes.

Henry will see the caboose leave the other end of the tunnel 3 minutes after 12:10 p.m., that is, at 12:13 p.m.
Problem of the Week

Problem B

Take a Turn

Let’s explore some ‘turns’ in different contexts.

a) Shredder does snowboard helicopter jumps with ease. If he rotates all the way around in a horizontal circle, through how many degrees does he rotate? What if he only rotates halfway around?

b) Through how many degrees does the minute hand on an analog clock move during
   i) \( \frac{1}{4} \) hour (say from 12:45 to 1:00 o’clock)?
   ii) 1 minute (say from 12:59 to 1:00 o’clock)?

c) Through how many degrees does the hour hand move in one hour (say from 1:00 to 2:00 o’clock)?

d) If a clock is keeping time correctly, the minute hand rotates at a rate of 360° per hour. One day, Sam notices that the time on the clock and on her digital watch is the same at noon, but when her watch says 1:00 o’clock, the clock says 1:05. Assuming that Sam’s digital watch keeps accurate time, at what rate is the minute hand of the clock rotating now?

Strands

Geometry and Spatial Sense, Measurement
Problem of the Week
Problem B and Solution
Take a Turn

Problem
Let’s explore some ‘turns’ in different contexts.

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   i) $\frac{1}{4}$ hour (say from 12:45 to 1:00 o’clock)?
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c) Through how many degrees does the hour hand move in one hour (say from 1:00 to 2:00 o’clock)?

d) If a clock is keeping time correctly, the minute hand rotates at a rate of $360^\circ$ per hour. One day, Sam notices that the time on the clock and on her digital watch is the same at noon, but when her watch says 1:00 o’clock, the clock says 1:05. Assuming that Sam’s digital watch keeps accurate time, at what rate is the minute hand of the clock rotating now?

Solution

a) In doing a full turn, Shredder rotates horizontally through $360^\circ$; in a half turn, he rotates $180^\circ$

b) i) Since the minute hand rotates through $360^\circ$ in 1 hour, in $\frac{1}{4}$ hour, it rotates through $360^\circ \div 4 = 90^\circ$.
   ii) One minute is $\frac{1}{60}$ th of an hour. Since the minute hand rotates through $360^\circ$ in 1 hour, in $\frac{1}{60}$ hour, it rotates through $360^\circ \div 60 = 6^\circ$.

c) In one hour, the hour hand moves $\frac{1}{12}$ of a full circle, or $360^\circ \div 12 = 30^\circ$

d) Assuming that Sam’s digital watch keeps accurate time, the minute hand of the clock has moved an extra $\frac{1}{12}$ of a full circle in one hour of real time. Thus it is moving at a rate of $360^\circ + 30^\circ = 390^\circ$ per hour.
Problem of the Week
Problem B
Sh-Rinking

Over the summer, Randy and Sarah have built the base for a rectangular outdoor skating rink which is 25 m by 10 m, surrounded by boards. Now it’s winter, and time to make the rink!

a) How many litres of water will they need to fill the rink to a depth of 10 cm, assuming the ground is level?

b) To resurface the rink, their Dad gets out his 4-wheeler and blade and scrapes off 3 mm of ice. What is the volume of ice removed?

EXTENSION: In Science class, they learned that water expands by 9% when it freezes. How much less water would be needed so that the ice will still be 10 cm deep? (One way to find 9% of a number is to multiply the number by 9 and divide the product by 100.)
Problem of the Week
Problem B and Solution
Sh-Rinking

Problem

Over the summer, Randy and Sarah have built the base for a rectangular outdoor skating rink which is 25 m by 10 m, surrounded by boards. Now it’s winter, and time to make the rink!

a) How many litres of water will they need to fill the rink to a depth of 10 cm, assuming the ground is level?

b) To resurface the rink, their Dad gets out his 4-wheeler and blade and scrapes off 3 mm of ice. What is the volume of ice removed?

EXTENSION: In Science class, they learned that water expands by 9% when it freezes. How much less water would be needed so that the ice will still be 10 cm deep?

Solution

a) Converting 25 m and 10 m to centimetres, we obtain 2500 cm and 1000 cm, respectively. The volume of water required, in cubic centimetres, is $10 \times 2500 \times 1000 = 25 000 000 \text{ cm}^3$. Since 1 litre is equal to $1 000 \text{ cm}^3$, this volume of water will be equivalent to $25 000 000 \div 1000 = 25 000 \text{ litres}$.

b) Converting 3 mm to centimetres, we obtain 0.3 cm. The volume of ice removed is $0.3 \times 2500 \times 1000 = 750 000 \text{ cm}^3$ or 750 litres.

EXTENSION: Whatever amount of water we require must be such that this amount plus 9% of this amount sums to 25 000 litres. That is, 109% of the amount must be 25 000 litres. This means that 1.09 times the required amount of water should be 25 000 litres.

If we divide 25 000 by 1.09, we obtain approximately 22 936 litres.
It follows that approximately 25 000 – 22 936 = 2064 fewer litres are needed.

NOTE: It is tempting to think as follows: We don’t need 9% of the 25 000 litres, or 2 250 litres. But if we follow that to a conclusion, we’d use $25 000 – 2 250 = 22 750$ litres of water, which would give an amount of ice equal to only $1.09 \times 22 750 = 24 797.5$ litres, not quite enough.

Many solvers would have used some sort of guess and check approach to obtain the answer. A possible guess and check approach is shown on the following page.
Finding the amount of water needed using a guess and check approach is tedious. We want an amount of water such that when we increase that amount by 9% we have 25000 litres of water.

<table>
<thead>
<tr>
<th>Amount of Water (in litres)</th>
<th>9% of Amount of Water (in litres)</th>
<th>Total Amount of Water (in litres)</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>20000</td>
<td>1800</td>
<td>21800</td>
<td>not enough</td>
</tr>
<tr>
<td>22000</td>
<td>1980</td>
<td>23980</td>
<td>not enough</td>
</tr>
<tr>
<td>23000</td>
<td>2070</td>
<td>25070</td>
<td>too much, but close</td>
</tr>
<tr>
<td>22900</td>
<td>2061</td>
<td>24961</td>
<td>close but under</td>
</tr>
<tr>
<td>22950</td>
<td>2065.5</td>
<td>25015.5</td>
<td>a bit too much</td>
</tr>
<tr>
<td>22940</td>
<td>2064.6</td>
<td>25004.6</td>
<td>so close</td>
</tr>
<tr>
<td>22935</td>
<td>2064.15</td>
<td>24999.15</td>
<td>close but under</td>
</tr>
<tr>
<td>22936</td>
<td>2064.24</td>
<td>25000.24</td>
<td>close but over</td>
</tr>
</tbody>
</table>

We will restrict our search to finding a whole number amount which when increased by 9% gives an amount closest to 25000 litres. We just need to determine which amount is closest to 25000 litres when it expands by 9%.

Looking at the last two rows of the table we see that 25000.24 is closer to 25000 than 24999.15. It follows that approximately $25000 - 22936 = 2064$ fewer litres are needed.
Problem of the Week
Problem B
A Walk in the Woods

Cassie takes a walk through the woods behind her house. She notices that there is a row of trees which seem to follow a pattern. There are pines, junipers, and maples, all 2 m apart from one another. There are 70 pines in total. The junipers are placed one after every 3\textsuperscript{rd} pine, and the maples are located one after every third juniper. At the end of the row, there is a line of 10 birch saplings which start 75 cm from the other trees, and are spaced 75 cm apart.

a) Form a row of the symbols P, J, M, and B which illustrates the pattern of Pines, Junipers, Maples, and Birch saplings.

b) How long is this row of trees?

c) Once they mature, the pines are 25 m high, and the maples are 17 m high. What is the total height of the pines and maples?
Problem of the Week
Problem B and Solution
A Walk in the Woods

Problem
Cassie takes a walk through the woods behind her house. She notices that there is a row of trees which seem to follow a pattern. There are pines, junipers, and maples, all 2 m apart from one another. There are 70 pines in total. The junipers are placed one after every 3rd pine, and the maples are located one after every third juniper. At the end of the row, there is a line of 10 birch saplings which start 75 cm from the other trees, and are spaced 75 cm apart.

a) Form a row of the symbols P, J, M, and B which illustrates the pattern of Pines, Junipers, Maples, and Birch saplings.

b) How long is this row of trees?

c) Once they mature, the pines are 25 m high, and the maples are 17 m high. What is the total height of the pines and maples?

Solution

a) The pattern up to the first maple is: P P P J P P P J P P P J M. After 7 repeats of this pattern, there will be $7 \times 9 = 63$ pines, $7 \times 3 = 21$ junipers, and 7 maples. Since there are 70 pines, this will be followed by P P P J P P P J P B B B B B B B B B B to account for the remaining 7 pines and the 10 birch saplings. In this final segment, there are 2 more junipers and no more maples.

b) There are $63 + 21 + 7 + 7 + 2 = 100$ pines, junipers, and maples, so there are 99 spaces of 2 m, or 198 m of those trees (plus the width of one tree). Since there is a 75 cm space before the first birch sapling, and 9 such spaces between them, this will add a total of $0.75 \times 10 = 7.5$ m for the saplings. Thus the total length of the row is about $198 + 7.5 = 205.5$ m.

c) At maturity, the total measure of the heights of the pines is $25 \times 70 = 1750$ m, and of the maples is $17 \times 7 = 119$ m. Thus the total length of standing lumber is $1750 + 119 = 1869$ m.
Problem of the Week

Problem B

No Business Like Snow Business

The snow is deep and it’s perfect snow fort weather. You’ve decided to build a wall 1.5 m high, 9 m long and 30 cm deep. You find that, by packing snow into your trusty plastic bin, you can make cubic ‘snow bricks’ which have dimensions 30 cm by 30 cm by 30 cm.

a) How many bricks will you need to build the wall?

b) Suppose you had a bin in the shape of a rectangular prism that was 60 cm long by 30 cm wide by 15 cm high. How many rectangular prism bricks would you need to build a wall with the same dimensions?

c) In the end, you decide to use a 2 : 1 ratio of rectangular bricks to cubic bricks. How many cubic bricks will you need?
Problem of the Week
Problem B and Solution
No Business Like Snow Business

Problem

The snow is deep and it’s perfect snow fort weather. You’ve decided to build a wall 1.5 m high, 9 m long and 30 cm deep. You find that, by packing snow into your trusty plastic bin, you can make cubic ‘snow bricks’ which have dimensions 30 cm \times 30 cm \times 30 cm.

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b) Suppose you had a rectangular prism bin 60 cm long by 30 cm wide by 15 cm high. How many rectangular prism bricks would you need to build a wall with the same dimensions?

c) In the end, you decide to use a 2:1 ratio of rectangular bricks to cubic bricks. How many cubic bricks will you need?

Solution

a) Since each brick is a 30 cm cube, it has height and length equal to 0.3 m. Thus a wall 1.5 m high and 9 m long will need to be $1.5 \div 0.3 = 5$ bricks high, and $9 \div 0.3 = 30$ bricks long. So the total number of bricks required will be $5 \times 30 = 150$ bricks.

b) The number of blocks needed is 150. This can be done in many ways. Here is one solution. Since the wall is 30 cm deep (or thick), you must use that dimension of the rectangular prism bricks as the depth. Using the 15 cm (or 0.15 m) dimension for the height, a 1.5 m high wall would need to be $1.5 \div 0.15 = 10$ bricks high. Using the 60 cm (or 0.6 m) dimension for the length, the 9 m length would require $9 \div 0.6 = 15$ bricks. Thus the total number of rectangular prism bricks required would be $10 \times 15 = 150$ bricks.

c) For a 2:1 ratio of rectangular bricks to cubic bricks, you want half as many cubic bricks as rectangular (i.e., $\frac{2}{3}$ rectangular and $\frac{1}{3}$ cubic). We see from a) and b) that 150 of either type of snow brick is needed to completely build the wall. So we can use 100 rectangular bricks (10 high and 10 wide) to cover 6 m of the length, and 50 cubic bricks (5 high by 10 wide) to cover the remaining 3 m of length.
Problem of the Week

Problem B

Doc and Friends Go for a Walk

Doc takes his menagerie on a long walk, parading along a 2 km loop around the park. Doc walks at a leisurely 4 km per hour, as do Sneeky, Yappy, and Doodle. But Grumpy and Dopey walk, in the same direction, at 6 km per hour.

a) If everyone starts at the entrance, walking in the same direction, where will Doc and his group be after $\frac{1}{2}$ hour? Where will Grumpy and Dopey be?

b) If they all walk for $2 \frac{1}{2}$ hours, how many times will Grumpy and Dopey pass by Doc and his friends Sneeky, Yappy, and Doodle?
Problem of the Week
Problem B and Solution
Doc and Friends Go for a Walk

Problem
Doc takes his menagerie on a long walk, parading along a 2 km loop around the park. Doc walks at a leisurely 4 km per hour, as do Sneeky, Yappy, and Doodle. But Grumpy and Dopey walk, in the same direction, at 6 km per hour.

Solution
a) After $\frac{1}{2}$ hour, Doc, Sneeky, Yappy, and Doodle will have travelled $\frac{1}{2} \times 4 = 2$ km, and thus will be back at the entrance A. But Grumpy and Dopey will have travelled $\frac{1}{2} \times 6 = 3$ km, so they will already be at B, half-way around the loop for the second time.

b) At the end of 1 hour, Doc’s group will be back at A, and so will Grumpy and Dopey, so they will pass by. The second hour will just be a repeat of the first hour, so Grumpy and Dopey will pass Doc’s group again at A after 2 hours. At the end of 2$\frac{1}{2}$ hours, the positions will be as they were in part a).

So Grumpy and Dopey pass by Doc’s group twice in 2$\frac{1}{2}$ hours.

For Further Thought
How would your solution change if they two groups were walking in opposite directions along the path?
Problem of the Week
Problem B
Creating Problems

Here are some answers...now exercise your math imagination to invent some word problems. Try to use more than one operation in some of your problems.

a) Create a problem that yields an answer of $3.45.
b) Create a problem for which the solution is 45 degrees.
c) Create a problem for which the solution is 17.3 metres.
d) Create a problem that yields an answer of $\frac{2}{5}$.

Strands: Number Sense and Numeration, Measurement
Problem of the Week

Problem B and Solution

Creating Problems

Problem

Here are some answers...now exercise your math imagination to invent some word problems. Try to use more than one operation in some of your problems.

a) Create a problem that yields an answer of $3.45.

b) Create a problem for which the solution is 45 degrees.

c) Create a problem for which the solution is 17.3 metres.

d) Create a problem that yields an answer of $\frac{2}{5}$.

Solution

Answers will vary widely. Here are some samples for each situation.

a) Pat sold 13 cups of lemonade at $0.25 each. She also got 20 cents in tips. How much did she make? ($13 \times 0.25 = 3.25, \text{ plus } 20 \text{ cents gives } 3.45$)

b) A circular pizza is cut along four diameters to yield 8 pieces of equal size. What is the measure of the angle at the vertex of each piece? ($\frac{1}{8} \text{ of } 360° \text{ equals } 45°$)

Ali folded a square sheet of paper in half along a diagonal. What is the measure of the angle at the corner with the fold? ($\frac{1}{2} \text{ of } 90° \text{ equals } 45°$)

c) The fence around a field in the shape of a quadrilateral is 80 metres long. Three of the sides are 20, 15.8 and 26.9 metres in length. What is the length of the fourth side? ($20 + 15.8 + 26.9 + \text{unknown side length} = 80\text{m}, \text{ so the unknown side length is } 80 - 62.7 = 17.3\text{m.}$)

Five students compete in long jump, with jumps of 3.5 m, 3.3 m, 3.1 m, 4.1 m and 3.3 m. What is the total of the distances jumped? ($\text{Sum } = 17.3\text{m.}$)

d) Mark has a circular spinner divided into equal segments coloured red, brown, black, green, and orange. What is the probability that Mark will land on a colour which starts with b? ($2 \text{ of the five segment colours start with } b, \text{ so the probability is } \frac{2}{5}.$)
Number Sense
&
Numeration
Problem of the Week  
Problem B  
Happy Birthd’eh, Canada

In July of 2017, Canada turned 150 years old, and is looking pretty good for its age. Even though Canadians think Canada is old, it is actually quite young. How old do you think the oldest civilization is? Let’s do some age comparisons with countries older than Canada. Round your final answers to one decimal.

a) How many times older than Canada is the United States of America? Spain?

b) How many times older than Canada is China? ancient India?

c) Find other countries which are older than Canada, and figure out how many times older they are than Canada.

Comment:
How the ‘age’ of a country is defined may vary widely. For example, a narrow definition might be to use the date when a country became a ‘sovereign state’; using this definition, India has only existed since 1950 (when it became independent of the British Empire), and China since 1949. A broader definition would be to ask for how long there has been an established civilization; this leads to much older ages. Have fun deciding on the ‘birth’ dates for each country!

**Strand**  Number Sense and Numeration
Problem of the Week
Problem B and Solution
Happy Birthd’eh, Canada

Problem
In July of 2017, Canada turned 150 years old, and is looking pretty good for its age. Even though Canadians think Canada is old, it is actually quite young. How old do you think the oldest civilization is? Let’s do some age comparisons with countries older than Canada. Round your final answers to one decimal.

a) How many times older than Canada is the United States of America? Spain?

b) How many times older than Canada is China? ancient India?

c) Find other countries which are older than Canada, and figure out how many times older they are than Canada.

Solution
The answers to parts a), b), c) are given in the table below; dates may vary. After the date you will find either CE or BCE. The Common Era, CE, started in the year 0. BCE is the number of years before year 0.

<table>
<thead>
<tr>
<th>Country</th>
<th>Date</th>
<th>Age (yrs.)</th>
<th>Older by (Age ÷150)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.A.</td>
<td>1776 CE</td>
<td>241</td>
<td>1.6 x</td>
</tr>
<tr>
<td>Spain</td>
<td>1492 CE</td>
<td>525</td>
<td>3.5 x</td>
</tr>
<tr>
<td>China</td>
<td>1500 BCE</td>
<td>3517</td>
<td>23.4 x</td>
</tr>
<tr>
<td>India</td>
<td>3300 BCE</td>
<td>5317</td>
<td>35.4 x</td>
</tr>
<tr>
<td>Argentina</td>
<td>1853 CE</td>
<td>164</td>
<td>1.1 x</td>
</tr>
<tr>
<td>Bolivia</td>
<td>1809 CE</td>
<td>208</td>
<td>1.4 x</td>
</tr>
<tr>
<td>Morocco</td>
<td>1666 CE</td>
<td>351</td>
<td>2.3 x</td>
</tr>
<tr>
<td>Russia</td>
<td>1480 CE</td>
<td>537</td>
<td>3.6 x</td>
</tr>
<tr>
<td>Ethiopia</td>
<td>980 BCE</td>
<td>2997</td>
<td>20.0 x</td>
</tr>
</tbody>
</table>

Students may have found other countries for part c) than the five shown here.
Patrick is reading a book about forest animals. He has read $\frac{3}{4}$ of the 100 pages in his book. Nicholas has read $\frac{2}{3}$ of the 120 pages of his book about alpine flowers.

a) Who has more pages left to read in his book?

b) To finish his book, Patrick reads five full pages per day, every day. If Nicholas also reads a certain number of full pages every day, how many pages per day must he read in order to finish his book on the same day as Patrick finishes?
Problem of the Week
Problem B and Solution
Bookity-Book!

Problem

Patrick is reading a book about forest animals. He has read $\frac{3}{4}$ of the 100 pages in his book. Nicholas has read $\frac{2}{3}$ of the 120 pages of his book about alpine flowers.

a) Who has more pages left to read in his book?

b) To finish his book, Patrick reads five full pages per day, every day. If Nicholas also reads a certain number of full pages every day, how many pages per day must he read in order to finish his book on the same day as Patrick finishes?

Solution

a) Since Patrick has read $\frac{3}{4}$ of his book, he has $\frac{1}{4}$ left to read, or $\frac{1}{4}$ of 100 = 25 pages left.

Since Nicholas has read $\frac{2}{3}$ of his book, he has $\frac{1}{3}$ left to read, or $\frac{1}{3}$ of 120 = 40 pages left.

Thus Nicholas has more pages left to read.

b) Reading at 5 pages per day, it will take Patrick $25 \div 5 = 5$ days to finish his book.

To finish his 40 pages in 5 days, Nicholas will have to read $40 \div 5 = 8$ pages per day in order to finish his book on the same day.
Problem of the Week
Problem B
We’ve Got Your Number!

I am a nine-digit number. I contain each digit from 1 to 9 except the digit 8, and I contain two appearances of the digit 5. Discover what number I am by using the following clues.

- I am less than 500 000 000.
- My ten millions digit and my ones digit are the same.
- The sum of my hundred millions, ten millions, and millions digits is 18.
- My thousands digit is 1.
- My ten thousands digit is one more than my hundred thousands digit.
- My ones digit is equal to the sum of my hundreds digit and my tens digit.
- My hundreds digit is 3.

---

EXTENSION: Create your own mystery number and clues.
Problem of the Week
Problem B and Solution
We’ve Got Your Number!

Problem
I am a nine-digit number. I contain each digit from 1 to 9 except the digit 8, and I contain two
appearances of the digit 5. Discover what number I am by using the following clues.

• I am less than 500 000 000.
• My ten millions digit and my ones digit are the same.
• The sum of my hundred millions, ten millions, and millions digits is 18.
• My thousands digit is 1.
• My ten thousands digit is one more than my hundred thousands digit.
• My ones digit is equal to the sum of my hundreds digit and my tens digit.
• My hundreds digit is 3.

Solution
The mystery number is 459 671 325.

This can be reasoned from the clues in the following steps.

• Since the digit 5 occurs twice and the ten millions digit and the units digit are the same,
  then they are both 5. The number looks like __ 5 __ __ __ __ __ __ 5

• Since the number is less than 500 000 000, the hundred millions digit must be 4 or less.

• Since the sum of the hundred millions, ten millions, and millions digits is 18, and the ten
  millions digit is 5, then the sum of the hundred millions digit and the millions digit must
  be 13. And since the hundred millions digit is 4 or less, the only combination that will
  work for the first 3 digits is 459. The number looks like 4 5 9 __ __ __ __ __ 5

• The hundreds digit is 3, and the 5 in the ones digit is the sum of the hundreds and tens
digits; thus the tens digit is 2. The number now looks like 4 5 9 __ __ __ 3 2 5

• The thousands digit is 1. The number now looks like 4 5 9 __ __ 1 __ __ __ 3 2 5

• All the digits have been used now except 6 and 7. Since the ten thousands digit is one
  more than the hundred thousands digit, they must be 7 and 6 respectively. The number
  we are looking for is 4 5 9 6 7 1 __ __ __ 3 2 5

You may find it interesting to try this alternative approach. Start at the first clue with the
greatest possible number, namely 497 655 321. Then work your way through the clues in order,
adjusting this number to fit the given information. You’ll discover that you have the answer
after only five clues!
Kelsie recently started earning money by walking dogs in her neighbourhood. She earns $8.00 each time she walks a dog. Kelsie walks two dogs per day for the month of November.

Kelsie uses her money to support an animal shelter. Starting on December 1, she donates $18.00 on odd numbered days and $22.00 on even numbered days. Will Kelsie have donated all of her earned money before Christmas Day, December 25th?
Problem

Kelsie recently started earning money by walking dogs in her neighbourhood. She earns $8.00 each time she walks a dog. Kelsie walks two dogs per day for the month of November. Kelsie uses her money to support an animal shelter. Starting on December 1, she donates $18.00 on odd numbered days and $22.00 on even numbered days. Will Kelsie have donated all of her earned money before Christmas Day, December 25th?

Solution

During the 30 days of November, Kelsie earned $2 \times 8 \times 30 = $480.

Starting December 1st, she will donate $18 for each of the odd numbered days from December 1 to December 24, 12 odd numbered days in total. In that same time period, she will donate $22 on each of the 12 even numbered days.

We will determine the final answer using two different methods.

Method 1:

Since there were 12 odd numbered days and she donated $18 on each of these days, she donated a total of $12 \times 18 = $216 on the odd numbered days.

Since there were 12 even numbered days and she donated $22 on each of these days, she donated a total of $12 \times 22 = $264 on the even numbered days.

In total, on the first 24 days of December she would donate $216 + $264 = $480. Kelsie will have donated all of her earnings before Christmas Day on December 25.

Method 2:

Since there is an equal number of odd numbered days and even numbered days in the first 24 days of December, we can determine the total that she donated every 2 days and multiply the result by 12.

Kelsie donates $18 on an odd numbered day and $22 on an even numbered day. So each 2 day combination she donates $18 + $22 = $40. There are 12 pairs of two-day combinations so she donates a total of $12 \times 40 = $480 over the first 24 days of December.

Kelsie will have donated all of her earnings before Christmas Day on December 25.
Problem of the Week
Problem B
Stop Motion - an Oxymoron

Alysia has decided to shoot a stop motion video using some of her toy cartoon characters.

a) If she wants her video to be 30 seconds long with each photo running for 0.3 seconds, how many photos will she need to take?

b) She discovers that it is taking her about 2 minutes and 20 seconds to take each photo. How many minutes will it take her to shoot all the photos for her video?

c) After researching why her video seems so jerky, she discovers that stop motion movies are shot at 20 frames per second. How many photos will she need to take to make a 30 second (half a minute) stop motion movie?
Problem

Alysia has decided to shoot a stop motion video using some of her toy cartoon characters.

a) If she wants her video to be 30 seconds long with each photo running for 0.3 seconds, how many photos will she need to take?

b) She discovers that it is taking her about 2 minutes and 20 seconds to take each photo. How many minutes will it take her to shoot all the photos for her video?

c) After researching why her video seems so jerky, she discovers that stop motion movies are shot at 20 frames per second. How many photos will she need to take to make a 30 second (half a minute) stop motion movie?

Solution

a) For a 30 second video, Alysia will need to take $30 \div 0.3 = 100$ photos.

b) To take 100 photos at 2 minutes and 20 seconds or $120 + 20 = 140$ seconds each, it will take Alysia $100 \times 140 = 14000$ seconds, or $14000 \div 60 \approx 233.3$ minutes. (The exact answer is $233\frac{1}{3}$ minutes, or 233 minutes and 20 seconds.)

c) To prevent jerkiness in her 30 second video, Alysia will need 20 photos for each second. So for 30 seconds she will need $20 \times 30 = 600$ photos.

Another way to obtain the same answer is to determine the required running time per photo to obtain 20 frames per second. Each photo runs for $1 \div 20 = 0.05$ seconds. She now needs $0.3 \div 0.05 = 6$ times as many photos as her first attempt. Therefore, she requires $6 \times 100 = 600$ photos to shoot her 30 second video at 20 frames per second.
Problem of the Week

Problem B

Right On Target!

Samina likes to practice darts using a board similar to the one shown. She shoots six darts and they all land on the board. Her score is the sum of the numbers on the rings in which her darts land. For the six darts shown on the target, her score would be $1 + 3 + 3 + 5 + 5 + 9 = 26$.

a) Could Samina’s score ever be 56? If yes, show how it can be done. If no, explain why the sum 56 cannot be obtained.

b) Show how Samina could score 12? Show how she could score 8?

c) Is there more than one possible way to score 28? If yes, list at least two different ways. If there is only one way to obtain 28, show it. If 28 cannot be obtained, explain why.

d) Is it possible to get an odd score? If yes, show a possibility. If no, explain why an odd sum cannot be obtained.
Problem

Samina likes to practice darts using a board similar to the one shown. She shoots six darts and they all land on the board. Her score is the sum of the numbers on the rings in which her darts land.

a) Could Samina’s score ever be 56? If yes, show how it can be done. If no, explain why the sum 56 cannot be obtained.

b) Show how Samina could score 12? Show how she could score 8?

c) There is more than one possible way to score 28? Find as many different ways as you can to score 28.

d) Is it possible to get an odd score? If yes, show a possibility. If no, explain why an odd sum cannot be obtained.

Solution

a) Since the maximum possible score is $6 \times 9 = 54$, Samina could not score 56.

b) Samina could score 12 in six shots in three different ways:

\[ 1 + 1 + 1 + 3 + 3 + 3, \text{ or } 1 + 1 + 1 + 1 + 3 + 5, \text{ or } 1 + 1 + 1 + 1 + 1 + 7. \]

Samina could score 8 in six shots by getting \( 1 + 1 + 1 + 1 + 3 + 5 \).

c) Yes. In fact, a score of 28 can be achieved in at least sixteen ways. Did you find any more?

<table>
<thead>
<tr>
<th>9s</th>
<th>7s</th>
<th>5s</th>
<th>3s</th>
<th>1s</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>( 9 + 9 + 7 + 1 + 1 + 1 = 28 )</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>( 9 + 9 + 5 + 3 + 1 + 1 = 28 )</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>( 9 + 9 + 3 + 3 + 3 + 1 = 28 )</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>0</td>
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<td>2</td>
<td>( 9 + 7 + 7 + 3 + 1 + 1 = 28 )</td>
</tr>
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<td>1</td>
<td>2</td>
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<td>2</td>
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</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>( 9 + 7 + 5 + 3 + 3 + 1 = 28 )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>( 9 + 7 + 3 + 3 + 3 + 3 = 28 )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>( 9 + 5 + 5 + 5 + 3 + 1 = 28 )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>( 9 + 5 + 5 + 3 + 3 + 3 = 28 )</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>( 7 + 7 + 7 + 5 + 1 + 1 = 28 )</td>
</tr>
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<td>0</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>1</td>
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<td>2</td>
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<td>1</td>
<td>( 7 + 7 + 5 + 5 + 3 + 1 = 28 )</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>( 7 + 7 + 5 + 3 + 3 + 3 = 28 )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>( 7 + 5 + 5 + 5 + 5 + 1 = 28 )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>0</td>
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</tr>
<tr>
<td>0</td>
<td>0</td>
<td>5</td>
<td>1</td>
<td>0</td>
<td>( 5 + 5 + 5 + 5 + 5 + 3 = 28 )</td>
</tr>
</tbody>
</table>

d) If all darts land in the board, an odd score is not possible, since the sum of six odd numbers must be an even number.
Problem of the Week
Problem B
Olympic Proportions

Olympic Games are held every two years, alternating between summer and winter Olympics. For example, Rio de Janeiro held the Summer Olympics in 2016, while Pyeongchang, South Korea will host the Winter Olympics this month.

a) Using the website https://www.olympic.org/olympic-games, make a list of all the countries which have hosted Olympic games from 1994 to 2016.

b) Look up the number of countries on each continent. Then complete the table below to demonstrate the fraction of countries on each continent which have hosted Olympic games from 1994 to 2016.
(http://www.enchantedlearning.com/geography/continents/Extremes.shtml)

<table>
<thead>
<tr>
<th>Continent</th>
<th>No. of Countries</th>
<th>No. of Host Countries</th>
<th>Fraction (%) Hosts/Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asia</td>
<td>47</td>
<td>3</td>
<td>$\frac{3}{47}$ (6.4%)</td>
</tr>
<tr>
<td>Europe</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. America</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S. America</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Africa</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c) If you were trying to distribute hosting countries evenly, which continent should be next to host either games? Justify your choice.

**Strands**  Number Sense and Numeration, Data Management and Probability
Problem

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<td>$\frac{3}{47}$ (6.4%)</td>
</tr>
<tr>
<td>Europe</td>
<td>43</td>
<td>4</td>
<td>$\frac{4}{43}$ (9.3%)</td>
</tr>
<tr>
<td>N. America</td>
<td>23</td>
<td>2 (3)</td>
<td>$\frac{3}{23}$ (13.0%)</td>
</tr>
<tr>
<td>S. America</td>
<td>12</td>
<td>1</td>
<td>$\frac{1}{12}$ (8.3%)</td>
</tr>
<tr>
<td>Australia</td>
<td>14</td>
<td>1</td>
<td>$\frac{1}{14}$ (7.1%)</td>
</tr>
<tr>
<td>Africa</td>
<td>54</td>
<td>0</td>
<td>$\frac{0}{54}$ (0%)</td>
</tr>
</tbody>
</table>

c) If you were trying to distribute hosting countries evenly, which continent should be next to host either games? Justify your choice.

Solution

a) Host countries are as follows:

1994 Lillehammer, Norway  2002 Salt Lake City, USA  2010 Vancouver, Canada  
1996 Atlanta, USA        2004 Athens, Greece   2012 London, UK       
1998 Nagano, Japan       2006 Turin, France    2014 Sochi, Russia     
2000 Sydney, Australia   2008 Beijing, China   2016 Rio de Janeiro, Brazil

b) See the completed table above. Source
    http://www.enchantedlearning.com/geography/continents/Extremes.shtml

c) Ideally, Africa should host; however, both the hot climate and a lack financial resources work against this.

Given the percentages of host countries on each continent, Asia should be the next host (which is to happen in Pyeonchang, Korea).
Problem of the Week

Problem B

Citius, Altius, Fortius

Consider the winter Olympic sports of downhill skiing and 1500 metre speed skating, which have recorded numeric results. In the table below are the gold medalist times for men and women in these events in the ten Winter Olympics since 1980 (from the website https://www.olympic.org/sports).

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<th>Year</th>
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<tr>
<td>2014</td>
<td>2:06.23</td>
<td>1:41.57</td>
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</table>

a) How many times has the winning time improved in each sport from one Olympics to the next, and what was the greatest improvement?

b) How much faster is the most recent record-breaking time than the 1980 winning time for each sport? Who achieved this fastest time?

c) Create two graphs (one for each sport) of the winning times over these ten Winter Olympics. Use whichever type of graph you think will best display the data.

**STRANDS**  Data Management and Probability, Number Sense and Numeration
Problem of the Week
Problem B and Solution
Citius, Altius, Fortius

Problem
Consider the winter Olympic sports of downhill skiing and 1500 metre speed skating, which have recorded numeric results. In the table below are the gold medalist times for men and women in these events in the ten Winter Olympics since 1980 (from the website https://www.olympic.org/sports).

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</table>

a) How many times has the winning time improved in each sport from one Olympics to the next, and what was the greatest improvement?
b) How much faster is the most recent record-breaking time than the 1980 winning time for each sport? Who achieved this fastest time?
c) Create two graphs (one for each sport) of the winning times over these ten Winter Olympics. Use whichever type of graph you think will best display the data.

Solution
a) In downhill skiing, the men’s times improved three times, the greatest being 10.98 seconds from 1998 to 2002; the women’s times improved five times, the greatest being 24.16 seconds from 1980 to 1984.
In speed skating, the men’s times improved six times, the greatest being 6.30 seconds from 1984 to 1988; the women’s times improved six times, the greatest being 7.53 seconds from 1980 to 1984.
b) In downhill skiing, the men’s most recent record-breaking time, achieved by Fritz Strobl of Austria in 2002, was 1:39.13, 6.37 seconds faster than 1980; the women’s most recent record-breaking time, achieved by Michela Figini of Switzerland in 1984, was 1:13.36, 24.16 seconds faster than 1980.
In speed skating, the men’s most recent record-breaking time, achieved by Derek Parra of the USA in 2002, was 1:43.95, 11.49 seconds faster than 1980; the women’s most recent record-breaking time, achieved by Jorien Ter Mors of the Netherlands in 2014, was 1:53.51, 17.44 seconds faster than 1980.
c) Here are the required graphs

![Olympic Downhill Skiing Results](image1)

**Olympic Downhill Skiing Results**
- Men's Results
- Women's Results

![Olympic 1500m Speed Skating Results](image2)

**Olympic 1500m Speed Skating Results**
- Men's Results
- Women's Results

**Extension:**
Do some research and write a paragraph about why these changes in winning times have occurred.
Four students in Miss Noether’s class are trying to solve the problem of subtracting 498 from 1397. Their attempts are shown below.

**Samira:** 
\[
\begin{array}{c}
1397 \\
-498 \\
101
\end{array}
\]

**Roger:** 
\[
\begin{array}{c}
1000 + 300 + 90 + 7 \\
- 400 + 90 + 8 \\
700 + 100 + 90 + 9 = 899
\end{array}
\]

**Beth:** 
\[
\begin{array}{c}
498 \\
500 \\
1000 \\
300 \\
90 \\
7 \\
= 899
\end{array}
\]

**Mandeep:** 
\[
\begin{array}{c}
0 \\
498 \\
500 \\
1397 \\
1399 \\
-500 \\
899
\end{array}
\]

a) Explain what you think each student did to solve the problem. Explain any errors and show them in a different colour.

b) What is the correct answer for this problem?

c) How would you solve this problem?
Problem of the Week
Problem B and Solution
Subtraction Actions

Problem
Four students in Miss Noether’s class are trying to solve the problem of subtracting 498 from 1397. Their attempts are shown below.

Samira: \[
\begin{array}{c}
1397 \\
-498 \\
\hline
101
\end{array}
\]

Roger: \[
\begin{array}{c}
1000 + 300 + 90 + 7 \\
- 400 + 90 + 8 \\
\hline
600 + 100 + 90 + 9 = 899
\end{array}
\]

Beth:

Mandeep:

a) Explain what you think each student did to solve the problem. Explain any errors and show them in a different colour.

b) What is the correct answer for this problem?

c) How would you solve this problem?

Solution

a) Samira subtracted the upper number (1)397 from the lower number 498, ignoring the 1 in the thousands column. Thus her answer is incorrect.

Roger wrote the larger number as a sum of the thousands + hundreds + tens + ones, and the smaller number as the sum of the hundreds + tens + ones. He wrote the 400 in the smaller number as 300 + 100 in order to be able to subtract 300 from 1000. He also wrote 300 + 90 + 7 as 200 + 180 + 17 to be able to subtract the hundreds, tens and ones columns. Finally, he added each of the differences, 700 + 100 + 90 + 9, to obtain the overall difference, 899.

Beth ‘added up’ from 498 to 1397, adding 2 (to get to 500), then 500 (to get to 1000), then 300 (to get to 1300), then 90 (to get to 1390), then 7, to reach 1397. Then she summed these numbers (2+500+300+90+7) to get the difference, 899.

Mandeep wrote 1397 - 498 = 1397 + 2 - 498 - 2 = 1399 - 500 = 899. The difference between 1397 and 498 is the same as the difference between 1399 and 500. Each of the two new numbers is two larger than the original two numbers so the difference between them will be the same.

b) The correct answer is 899, as shown by Roger, Beth, and Mandeep.

c) Answers will vary.
Problem of the Week
Problem B
Amazing Grids

If possible, for each maze, find your way from the top left square to the bottom right square, moving only horizontally or vertically to achieve the set of numbers specified in the title.

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**Composite numbers**

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Problem of the Week
Problem B and Solution
Amazing Grids

Problem
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<th>Multiples of 7, in order</th>
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<table>
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<th>Composite numbers</th>
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Solution
Solutions are given as continuous lines on the first three mazes. The final maze has no solution; two (of many possible) attempts are shown with dotted lines, both working forward and backward, but both dead-end.
Problem of the Week
Problem B
One of These is Not Like the Others...

a) For each of the following sets, decide which number doesn’t belong and should be kicked off the island. Give reasons for your decision.

b) Can you also find something that is common to each of the numbers on each island and hence justify why they all should stay?

i) ![Island 1]

ii) ![Island 2]

EXTENSION: See http://wodb.ca for other samples of challenges to try.
Problem of the Week
Problem B and Solution
One of These is Not Like the Others...

Problem

a) For each of the following sets, decide which number doesn’t belong and should be kicked off the island. Give reasons for your decision.

b) Can you also find something that is common to each of the numbers on each island and hence justify why they all should stay?

i)  

ii) 

EXTENSION: See http://wodb.ca for other samples of challenges to try.

Solution

a) Note that many correct answers are possible in both cases.

i) Some possible choices and reasons are:
   - 4, because it is a single digit;
   - 16, because its tens digit is a 1;
   - 25, because it is odd;
   - 36, because its tens digit is half its ones digit.
   - 64, because the sum of its digit is a two-digit number.

ii) Some possible choices and reasons are:
   - 37, because it is a prime number;
   - 44, because the sum of its digits is 8 while the others sum to 10;
   - 55, because it is the only multiple of 5;
   - 82, because its tens digit is four times its ones digit.

b) Note there are many possible answers for both cases. For example,

i) All of the numbers on the first island can be written as the product of two factors that are the same, $4 = 2 \times 2; 16 = 4 \times 4; 25 = 5 \times 5; 36 = 6 \times 6; 64 = 8 \times 8$.

ii) All of the numbers on the second island are two-digit numbers.
StatsCan is doing a census. Jayne has been hired to enumerate 330 households.

a) If Jayne can deliver, on average, one form every 8 minutes, how many hours will she need to deliver all 330 forms?

b) On average, she has to travel 0.14 km between households to deliver the forms, and she is paid 92 cents per kilometre. If she is also paid $14.42 per hour for enumerating, how much money will she earn?

c) An average household has 1.7 adults and 1.2 children. Pick an appropriate type of graph to show how many adults and how many children we could expect to see in this region.
Problem of the Week
Problem B and Solution
Number Census and Enumeration

Problem
StatsCan is doing a census. Jayne has been hired to enumerate 330 households.

a) If Jayne can deliver, on average, one form every 8 minutes, how many hours will she need to deliver all 330 forms?

b) On average, she has to travel 0.14 km between households to deliver the forms, and she is paid 92 cents per kilometre. If she is also paid $14.42 per hour for enumerating, how much money will she earn?

c) An average household has 1.7 adults and 1.2 children. Pick an appropriate type of graph to show how many adults and how many children we could expect to see in this region.

Solution

a) Jayne will need $8 \times 330 = 2640$ minutes, or $2640 \div 60 = 44$ hours to deliver all the forms.

b) There will be 329 spaces of 0.14 km between the 330 households, so Jayne will travel a total distance of $0.14 \times 329 = 46.06$ km. Thus she will be paid $46.06 \times 0.92 \approx $42.38 for travel.

In addition, she will receive $44 \times 14.42 = $634.48 for enumerating. Thus she will earn $634.48 + 42.38 = $676.86 in total.

c) This region will have, on average, $330 \times 1.7 = 561$ adults, and $330 \times 1.2 = 396$ children. Below are two types of graphs which illustrate this.
Madeline wants to trade in her gas-guzzling SUV (which uses gas at a rate of 20 litres per 100 km) for a hybrid which uses only 4.5 litres per 100 km. Allowing for her trade-in, she will pay $31 000 for the new hybrid.

If gas remains stable at $1.00 per litre, and Madeline drives 2 000 km per month, after how many months will Madeine have saved enough on gas (compared to her SUV) to compensate her for the cost of the hybrid?
Problem of the Week
Problem B and Solution
Charge or Pump?

Problem
Madeline wants to trade in her gas-guzzling SUV (which uses gas at a rate of 20 litres per 100 km) for a hybrid which uses only 4.5 litres per 100 km. Allowing for her trade-in, she will pay $31 000 for the new hybrid.

If gas remains stable at $1.00 per litre, and Madeline drives 2000 km per month, after how many months will Madeine have saved enough on gas (compared to her SUV) to compensate her for the cost of the hybrid?

Solution
Let’s look at the monthly fuel consumption for each vehicle

For the SUV:
20 litres for 100km
×20 400 litres for 2000km
Therefore the SUV will use 400 litres per month.

For the hybrid:
4.5 litres for 100km
×20 90 litres for 2000km
Therefore the hybrid will use 90 litres per month.

The hybrid will use 400 - 90 = 310 fewer litres per month.
Since gas is $1.00 per litre, she will save $1.00 \times 310 = $310 per month.
The number of months will be 31 000 \div 310 = 100 months. (This is equivalent to 8 years 4 months.)
Problem of the Week

Problem B

Creating Problems

Here are some answers...now exercise your math imagination to invent some word problems. Try to use more than one operation in some of your problems.

a) Create a problem that yields an answer of $3.45.

b) Create a problem for which the solution is 45 degrees.

c) Create a problem for which the solution is 17.3 metres.

d) Create a problem that yields an answer of $\frac{2}{5}$.

Strands: Number Sense and Numeration, Measurement
Problem of the Week
Problem B and Solution
Creating Problems

Problem
Here are some answers...now exercise your math imagination to invent some word problems. Try to use more than one operation in some of your problems.

a) Create a problem that yields an answer of $3.45.

b) Create a problem for which the solution is 45 degrees.

c) Create a problem for which the solution is 17.3 metres.

d) Create a problem that yields an answer of $\frac{2}{5}$.

Solution
Answers will vary widely. Here are some samples for each situation.

a) Pat sold 13 cups of lemonade at $0.25 each. She also got 20 cents in tips. How much did she make? ($13 \times 0.25 = 3.25$, plus 20 cents gives $3.45$)

b) A circular pizza is cut along four diameters to yield 8 pieces of equal size. What is the measure of the angle at the vertex of each piece? ($\frac{1}{8}$ of $360^\circ$ equals $45^\circ$)

Ali folded a square sheet of paper in half along a diagonal. What is the measure of the angle at the corner with the fold? ($\frac{1}{2}$ of $90^\circ$ equals $45^\circ$)

c) The fence around a field in the shape of a quadrilateral is 80 metres long. Three of the sides are 20, 15.8 and 26.9 metres in length. What is the length of the fourth side? ($20 + 15.8 + 26.9 + \text{unknown side length} = 80\text{m}$, so the unknown side length is $80 - 62.7 = 17.3\text{m}$.)

Five students compete in long jump, with jumps of 3.5 m, 3.3 m, 3.1 m, 4.1 m and 3.3 m. What is the total of the distances jumped? (Sum = 17.3 m.)

d) Mark has a circular spinner divided into equal segments coloured red, brown, black, green, and orange. What is the probability that Mark will land on a colour which starts with b? (Two of the five segment colours start with b, so the probability is $\frac{2}{5}$.)
A family has one set of twin children, one set of triplets, and two parents. The youngest children are five years old.

If the mean of the family members’ ages is 18 years, the mode is 10 years, and the parents are the same age, how old are the parents?
Problem of the Week
Problem B and Solution
A Family Matter

Problem
A family has one set of twin children, one set of triplets, and two parents. The youngest children are five years old.

If the mean of the family members’ ages is 18 years, the mode is 10 years, and the parents are the same age, how old are the parents?

Solution
Since the mode is the age which occurs most frequently, the triplets must be 10 years old. It follows that the youngest children must be the twins, each aged 5 years.

For the average age to be 18 years, the sum of the ages of all 7 people must be $18 \times 7 = 126$ since $126 \div 7 = 18$. So the total age of the twins plus the total age of the triples plus the age of the two parents must be 126. Symbolically, this can be written

$$\underbrace{(2 \times 5)}_{\text{twins}} + \underbrace{(3 \times 10)}_{\text{triplets}} + \text{(two parents’ ages)} = 126.$$

The total age of the twins is $2 \times 5 = 10$ years. The total age of the triplets is $3 \times 10 = 30$ years. The total age of all five children is $10 + 30 = 40$ years.

Removing the children’s ages from the total 126 leaves the 2 parents with a total age of 86. Since both parents are the same age, each parent’s age must be $\frac{1}{2}$ of 86. It follows that the age of each parent is 43 years.
Problem of the Week

Problem B

Nein!

In the table, the first column contains a number $n$, where $n$ goes from 2 to 10.

Complete the table as follows:

- in the next two columns, write the tens digit and the ones digit of the product $p$ where $p = 9 \times n$;
- in the fourth column, write the difference, $d$, between the number $n$ and the tens digit of $p$. That is, $d = n - \text{tens digit of } p$; and
- in the last column, write the sum, $S$, of the tens and ones digits of the product $p$.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$p = 9 \times n$</th>
<th>difference</th>
<th>sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>18</td>
<td>2 - 1 = 1</td>
<td>1 + 8 = 9</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>5</td>
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<td>8</td>
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<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Answer the following questions by making observations from the completed table.

a) What is a good way to find the tens digit of the product $p$?

b) What is a good way to determine the ones digit of the product $p$?

c) What other interesting patterns do you see in the table?
Problem of the Week
Problem B and Solution

Nein!

Problem
In the table, the first column contains a number $n$, where $n$ goes from 2 to 10.

Complete the table as follows:

- in the next two columns, write the tens digit and the ones digit of the product $p$ where $p = 9 \times n$;
- in the fourth column, write the difference, $d$, between the number $n$ and the tens digit of $p$. That is, $d = n - \text{tens digit of } p$; and
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<td>2 - 1 = 1</td>
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</tr>
<tr>
<td>3</td>
<td>27</td>
<td>3 - 2 = 1</td>
<td>2 + 7 = 9</td>
</tr>
<tr>
<td>4</td>
<td>36</td>
<td>4 - 3 = 1</td>
<td>3 + 6 = 9</td>
</tr>
<tr>
<td>5</td>
<td>45</td>
<td>5 - 4 = 1</td>
<td>4 + 5 = 9</td>
</tr>
<tr>
<td>6</td>
<td>54</td>
<td>6 - 5 = 1</td>
<td>5 + 4 = 9</td>
</tr>
<tr>
<td>7</td>
<td>63</td>
<td>7 - 6 = 1</td>
<td>6 + 3 = 9</td>
</tr>
<tr>
<td>8</td>
<td>72</td>
<td>8 - 7 = 1</td>
<td>7 + 2 = 9</td>
</tr>
<tr>
<td>9</td>
<td>81</td>
<td>9 - 8 = 1</td>
<td>8 + 1 = 9</td>
</tr>
<tr>
<td>10</td>
<td>90</td>
<td>10 - 9 = 1</td>
<td>9 + 0 = 9</td>
</tr>
</tbody>
</table>

Answer the following questions by making observations from the completed table.

a) What is a good way to find the tens digit of the product $p$?
b) What is a good way to determine the ones digit of the product $p$?
c) What other interesting patterns do you see in the table?

Solution

a) The completed table above reveals that the tens digit of a multiple of 9 is one less than the number by which you are multiplying 9. For example, $9 \times 3 = 27$, has the tens digit of $3 - 1 = 2$.

b) To find the ones digit of a multiple of 9, once you have found the tens digit, just subtract that tens digit from 9. For example, $9 \times 3 = 27$ has a tens digit of 2 and thus the ones digit is $9 - 2 = 7$.

c) All the differences $d = n - \text{tens digit}$ are equal to 1. Also, the sum tens digit + ones digit is equal to 9 in all cases.

To think about: What happens if you look at further products of 9? Do any of these patterns persist? Which ones?
Problem of the Week

Problem B

A Walk in the Woods

Cassie takes a walk through the woods behind her house. She notices that there is a row of trees which seem to follow a pattern. There are pines, junipers, and maples, all 2 m apart from one another. There are 70 pines in total. The junipers are placed one after every 3rd pine, and the maples are located one after every third juniper. At the end of the row, there is a line of 10 birch saplings which start 75 cm from the other trees, and are spaced 75 cm apart.

a) Form a row of the symbols P, J, M, and B which illustrates the pattern of Pines, Junipers, Maples, and Birch saplings.

b) How long is this row of trees?

c) Once they mature, the pines are 25 m high, and the maples are 17 m high. What is the total height of the pines and maples?
Problem of the Week
Problem B and Solution
A Walk in the Woods

Problem
Cassie takes a walk through the woods behind her house. She notices that there is a row of trees which seem to follow a pattern. There are pines, junipers, and maples, all 2 m apart from one another. There are 70 pines in total. The junipers are placed one after every 3rd pine, and the maples are located one after every third juniper. At the end of the row, there is a line of 10 birch saplings which start 75 cm from the other trees, and are spaced 75 cm apart.

a) Form a row of the symbols P, J, M, and B which illustrates the pattern of Pines, Junipers, Maples, and Birch saplings.

b) How long is this row of trees?

c) Once they mature, the pines are 25 m high, and the maples are 17 m high. What is the total height of the pines and maples?

Solution

a) The pattern up to the first maple is: P P P J P P P J P P P J M. After 7 repeats of this pattern, there will be $7 \times 9 = 63$ pines, $7 \times 3 = 21$ junipers, and 7 maples. Since there are 70 pines, this will be followed by P P P J P P P J P B B B B B B B B to account for the remaining 7 pines and the 10 birch saplings. In this final segment, there are 2 more junipers and no more maples.

b) There are $63 + 21 + 7 + 7 + 2 = 100$ pines, junipers, and maples, so there are 99 spaces of 2 m, or 198 m of those trees (plus the width of one tree). Since there is a 75 cm space before the first birch sapling, and 9 such spaces between them, this will add a total of $0.75 \times 10 = 7.5$ m for the saplings. Thus the total length of the row is about $198 + 7.5 = 205.5$ m.

c) At maturity, the total measure of the heights of the pines is $25 \times 70 = 1750$ m, and of the maples is $17 \times 7 = 119$ m. Thus the total length of standing lumber is $1750 + 119 = 1869$ m.
Problem of the Week
Problem B
This Strikes a Chord

A line segment that has its endpoints on a circle is called a *chord*. The diameter of a circle is a special chord which passes through the centre of the circle.

Two chords can divide a circle in two different ways:

1. non-intersecting chords divide the circle into three pieces, or
2. the two chords can also intersect, giving four pieces.

NOTE: An intersection only refers to chords crossing *inside* the circle, and for this problem, no more than two chords can intersect at one point.

a) Three chords offer more possibilities. What is the maximum number of pieces into which a circle can be divided by three chords?

b) Sketch all the ways a circle can be subdivided by three chords. How is the number of intersections related to the number of pieces?

c) Find all the ways a circle can be subdivided by four chords. Is the number of intersections related to the number of pieces in the same way as in part b)?

d) If you used six chords, what would you predict to be the maximum number of pieces? Explain your reasoning.

**Strands**  Geometry and Spatial Sense, Patterning and Algebra
Problem of the Week
Problem B and Solution
This Strikes a Chord

Problem
A line segment that has its endpoints on a circle is called a chord.

a) What is the maximum number of pieces into which a circle can be divided by three chords?
b) Sketch all the ways a circle can be subdivided by three chords. How is the number of intersections related to the number of pieces?
c) Find all the ways a circle can be subdivided by four chords. Is the number of intersections related to the number of pieces in the same way as in part b)?
d) If you used six chords, what would you predict to be the maximum number of pieces? Explain your reasoning.

Solution
In the following, the letter \( i \) refers to the number of intersections, and the letter \( p \) refers to the corresponding number of pieces into which the circle is divided.

a),b) The ways a circle can be divided by three chords is shown below. The maximum number of pieces is 7. Note that in each case, the number of pieces is 4 more than the number of intersections.

\[
\begin{align*}
\text{i=0, p=4} & \quad \text{i=1, p=5} & \quad \text{i=2, p=6} & \quad \text{i=3, p=7} \\
\end{align*}
\]

\( c \) For four chords, the possibilities are shown below. In this case the number of pieces exceeds the number of intersections by 5 in each case. Thus we see that for 2, 3, and 4 chords, if \( n \) is the number of chords, then \( p = i + n + 1 \).

\[
\begin{align*}
\text{i=0, p=5} & \quad \text{i=1, p=6} & \quad \text{i=2, p=7} & \quad \text{i=3, p=8} & \quad \text{i=4, p=9} & \quad \text{i=5, p=10} & \quad \text{i=6, p=11} \\
\end{align*}
\]

d) Think of placing the chords one after the other. Since each chord can intersect all the previous chords, the maximum number of intersections for six chords will be \( 1 + 2 + 3 + 4 + 5 = 15 \). Observing the results of parts a), b), c), we predict that the number of pieces here will be \( p = i + 6 + 1 \). Thus, for six chords, the maximum number of pieces will be \( 15 + 6 + 1 = 22 \), as shown below.