The problems in this booklet are organized into strands. A problem often appears in multiple strands. The problems are suitable for most students in Grade 3 or higher.
Data Management & Probability
Problem of the Week
Problem A
Rolling Dice

Asha and Naomi are rolling a six-sided die to determine who gets the larger half of a cookie that they are sharing. If Asha rolls a number less than 4, she will get the larger piece of the cookie. If Naomi rolls the number 4 or greater, she will get the larger piece. Who is more likely to win the larger piece of cookie?

Explain your thinking.
Problem of the Week
Problem A and Solution
Rolling Dice

Problem
Asha and Naomi are rolling a six-sided die to determine who gets the larger half of a cookie that they are sharing. If Asha rolls a number less than 4, she will get the larger piece of the cookie. If Naomi rolls the number 4 or greater, she will get the larger piece. Who is more likely to win the larger piece of cookie?

Explain your thinking.

Solution
On a fair, six-sided die, we expect each of the numbers (1, 2, 3, 4, 5, 6) are equally likely to appear on any given roll. Asha is hoping that the numbers 1, 2, or 3 will appear; Naomi is hoping that the numbers 4, 5, or 6 will appear. Each is hoping for 3 different outcomes. Since they are both hoping for the same number of outcomes, and each of the outcomes is equally likely to happen, each has an equal chance of getting the larger piece of the cookie.
Teacher’s Notes

Assuming there is no effort to cheat, rolling dice provides random outcomes. We can predict that, in the long term, we expect the numbers 1, 2, and 3 will appear approximately half of the time. However, we can not accurately predict the outcome of a single roll; we can only make a guess.

Computers use random numbers in many applications such as games, simulations, and cryptography. In most cases, these applications would use a pseudorandom number generator. Numbers generated this way would appear random, but they are actually generated by a function. This can be useful in software development, because the programmer could repeatedly use the same sequence of random values during the testing process. The random number generator often requires a seed, to get it started. Starting with the same seed would produce the same sequence of numbers.

It is possible that someone could determine a pattern in pseudorandom numbers. This is not a serious problem in gaming or simulations, but could be important for cryptography applications that require true randomness to send secure messages. Computers can generate truly random numbers using hardware components. These components may measure naturally occurring phenomenon that has a random nature, and then convert that data into random numbers. Another technique samples human input values through the keyboard or mouse movements as a person uses another program, and records values that can be used as random numbers themselves, or as seeds for psuedorandom number generators.

Generally speaking, people are not good at generating random numbers. Ask a large group of people to pick a number between 1 and 100. How many picked even numbers? How many picked odd numbers? What are the most frequently picked numbers? Which numbers were not picked? See if you can find any patterns in the numbers people chose.
Problem of the Week
Problem A
Different Continents

Najeep surveyed some grade 3 and 4 students at school to see in which continents they were born. Her results are recorded in the chart below.

<table>
<thead>
<tr>
<th>Where Students Were Born</th>
</tr>
</thead>
<tbody>
<tr>
<td>North America</td>
</tr>
<tr>
<td>13</td>
</tr>
</tbody>
</table>

A) Create a bar graph to display Najeep’s data.

B) How many students completed Najeep’s survey?

C) *Range* is the difference between the greatest and smallest values in a set of data. What is the range of the data that Najeep collected?

D) What conclusions can you draw from this data?

E) How does this data compare to the data from your own school?
Problem of the Week
Problem A and Solution
Different Continents

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<table>
<thead>
<tr>
<th>Where Students Were Born</th>
<th>North America</th>
<th>Europe</th>
<th>Africa</th>
<th>Asia</th>
<th>South America</th>
<th>Australia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>13</td>
<td>12</td>
<td>8</td>
<td>17</td>
<td>9</td>
<td>4</td>
</tr>
</tbody>
</table>

A) Create a bar graph to display Najeep’s data.

B) How many students completed Najeep’s survey?

C) \textit{Range} is the difference between the greatest and smallest values in a set of data. What is the range of the data that Najeep collected?

D) What conclusions can you draw from this data?

E) How does this data compare to the data from your own school?

Solution

A)
B) The total number of students who completed the survey was:

\[ 17 + 12 + 8 + 13 + 9 + 4 = 63 \]

C) The smallest value is 4. The largest value is 17. The range is 17 - 4 = 13.

D) There are many ways to interpret the data. Here are a few to consider. We can group the continents by geography and compare the total number of people from the northern and southern hemispheres or the eastern and western hemispheres. A bar chart easily distinguishes the largest and the smallest values. We could create a pie chart to see the percentages from each continent more easily.

E) This will depend on the data collected.
Teacher’s Notes

In this problem, we are working with discrete data. In statistics, some problems involve discrete data and others involve continuous data.

Countable data, such as a number of people born on a particular continent, the number of coins saved in a piggy bank, or how many credits someone has earned at school, is discrete. We often represent this type of data in the form of a bar graph.

Values such as the time it takes someone to run a race, or the height of a person, or the weight of food that has been purchased, are all measured on a continuous scale. We are limited in how accurately we can measure these values by the tools at our disposal. For example, when people run the 100 m dash, their times are recorded to the nearest millisecond. However, this result is likely to have been rounded off. If we had more precise measuring tools, the time could be measured to the nearest microsecond. In fact, there are an infinite number of times between 9.58 seconds and 9.59 seconds; time intervals cannot be counted.

With continuous data we often use scatter plots to record the information. In some cases, we may take the resulting graph and find a line of best fit. This line can show trends that can be inferred by the specific data that has been gathered.
Problem of the Week
Problem A
Spinner

A spinner is a math tool that can be used to demonstrate the probability of a particular outcome. For example if you design a spinner that has a background divided into two equal parts: red and blue, there is a 50% probability of landing on red and a 50% probability of landing on blue.

Design a spinner to match the following requirements:

1) The probability of landing on blue is greater than landing on red.
2) The probability of landing on yellow is the same as landing on blue.
3) You are more likely to land on green than any other colour.

Is it possible to create a different spinner using the same clues? If it is possible design another spinner.

Strand Data Management and Probability
Problem of the Week
Problem A and Solution
Spinner

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Is it possible to create a different spinner using the same clues? If it is possible design another spinner.

Solution
There are many correct solutions to this problem. Here are two designs that satisfy the requirements.
The spinner on the left has four sections. The green section takes up half of the spinner’s area. The other three sections fill the other half of the spinner, so each of those sections must be less than half of the spinner’s area. This means that the probability of landing on green is higher than any other colour. The yellow and blue areas are the same size. This means that the probability of landing on yellow is equal to the probability of landing on blue. The red section is smaller than any of the other areas of the spinner, so the probability of landing on red is less than the probability of landing on blue. This means all of the requirements have been met.

The spinner on the right has been divided into eight equal sections. The sections are filled with the following distribution of colours:

- 1 section of red
- 2 sections of yellow
- 2 sections of blue
- 3 sections of green

Since the sections are the same size, where there are more sections of a colour on the spinner the probability of landing on that colour is higher. So, there is a greater probability of landing on green than any other colour. The probability of landing on yellow is equal to the probability of landing on blue. The probability of landing on blue is greater than the probability of landing on red. This means all of the requirements have been met.
Teacher’s Notes

Many people misunderstand probability. Describing an outcome as having a 50%, 75%, or even 99% probability of occurring does not provide any guarantees, especially in the short term. This is simply an outcome we expect to observe over the long term.

For example, with our 50/50 spinner, there is 100% guarantee that the pointer does not land on one of the colours on each spin. So even though we expect the spinner to land on red half of the time and blue the other half of the time, we cannot know the outcome of any particular spin.

We can make better guesses about the outcome of many spins. For example, if we spin four times, we would guess that two times we would land on blue and two times we would land on red. However, there are actually 16 possible outcomes:

```
RRRR  RRRB  RRBR  RRBB  RBRR  RBRB  RBBR  RBBB
BRRR  BRRB  BRBR  BRBB  BBRR  BBRB  BBBR  BBBB
```

all of which are equally likely sequences, and only 6 of the 16 outcomes have the spinner landing on red twice and blue twice. So more than half the time we actually expect that we will not get an equal number of spins landing on red and landing on blue.

Over a larger number of spins, we do expect to see approximately as many red spins as blue spins. However, it would still be possible to get all red or all blue outcomes. The chance is very small, but all things being equal, we expect that

2 out of $2^{100}$ or 2 out of 1267650600228229401496703205376 times

the spinner would land on the same colour for each of the 100 spins.
Problem of the Week
Problem A
Winter Fun

Ben asked his classmates what their favourite activities were during the winter. The following chart shows the results.

Survey Results

<table>
<thead>
<tr>
<th>Activity</th>
<th>Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building a Snowman</td>
<td>🌟🌟🌟</td>
</tr>
<tr>
<td>Sledding</td>
<td>🌟🌟🌟🌟</td>
</tr>
<tr>
<td>Skiing</td>
<td>🌟🌟🌟🌟</td>
</tr>
<tr>
<td>Skating</td>
<td>🌟🌟🌟</td>
</tr>
</tbody>
</table>

Key

🌟 = 3 students

Which of the following statements are true about the chart?

a) Six fewer students like skating than like sledding.

b) One more student likes building a snowman than likes skating.

c) One student likes skiing and four students like sledding.

d) Nine more students like sledding than like skiing.

Justify your answers.
Problem

Ben asked his classmates what their favourite activities were during the winter. The following chart shows the results.

Survey Results

<table>
<thead>
<tr>
<th>Activity</th>
<th>Snowflakes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building a Snowman</td>
<td>✿ ✿ ✿ ✿</td>
</tr>
<tr>
<td>Sledding</td>
<td>✿ ✿ ✿ ✿</td>
</tr>
<tr>
<td>Skiing</td>
<td>✿</td>
</tr>
<tr>
<td>Skating</td>
<td>✿ ✿</td>
</tr>
</tbody>
</table>

Key

✿ ✿ ✿ ✿ = 3 students

Which of the following statements are true about the chart?

a) Six fewer students like skating than like sledding.

b) One more student likes building a snowman than likes skating.

c) One student likes skiing and four students like sledding.

d) Nine more students like sledding than like skiing.

Justify your answers.

Solution

To answer these questions, we can use the chart to determine the actual number of people that answered the questions in the survey. From the key we know that each snowflake represents 3 students.

Survey Results

<table>
<thead>
<tr>
<th>Activity</th>
<th>Snowflakes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building a Snowman</td>
<td>3 + 3 + 3 = 9 students</td>
</tr>
<tr>
<td>Sledding</td>
<td>3 + 3 + 3 = 12 students</td>
</tr>
<tr>
<td>Skiing</td>
<td>3 students</td>
</tr>
<tr>
<td>Skating</td>
<td>3 + 3 = 6 students</td>
</tr>
</tbody>
</table>

Based on the information from the table,

a) Is true, since 12 - 6 = 6 fewer students like skating.

b) Is not true, since 9 - 6 = 3 more students like building a snowman than like skating.

c) Is not true, since 3 students like skiing and 12 students like sledding.

d) Is true, since 12 - 3 = 9 more students like sledding than like skiing.
Teacher’s Notes

The key in a pictograph gives us specific information about the survey data. Similarly the numbers on an axis of a bar chart or line chart provide more information about the data being represented in these graphical formats. However, there are several conclusions we can reach based on the pictograph, without knowing how much each snowflake represents. Here are some observations that would be the same even if we changed the value of the key:

- Sledding is the most popular activity.
- The majority of the students enjoy either sledding or building a snowman the most.
- Twice as many students say sledding is their favourite compared to those who say skating is their favourite activity.
- Three times as many students say building a snowman is their favourite compared to those who say skiing is their favourite activity.
- When asked what their favourite activity is, 30% of the students said building a snowman, 40% said sledding, 10% said skiing, and 20% said skating.
- Without knowing the key, we could use the pictograph information to draw a pie chart of the survey results.

Can you think of other observations that are independent of the fact that one snowflake represents three students?
Problem of the Week
Problem A
Party Games

Arezoo is having a birthday party and she invited five friends: Gerry, Jason, Laila, Nabil, and Lisa. Everyone at the party likes to play games, so Arezoo plans to have two stations. At Station A, four people will play table tennis; at Station B two people will play tether ball. Arezoo wants to make sure everyone has a chance to play at each station at least once with each person at the party. Make a schedule so that everyone at the party plays both games at some point with every other person at the party.
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Solution
If we decide who is playing tether ball, then the other four people will automatically be scheduled to play table tennis. We can make a schedule that guarantees each person plays tether ball against each other person at the party.

<table>
<thead>
<tr>
<th>Playing Tether Ball</th>
<th>Playing Table Tennis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arezoo and Gerry</td>
<td>Jason, Laila, Nabil, and Lisa</td>
</tr>
<tr>
<td>Arezoo and Jason</td>
<td>Gerry, Laila, Nabil, and Lisa</td>
</tr>
<tr>
<td>Arezoo and Laila</td>
<td>Gerry, Jason, Nabil, and Lisa</td>
</tr>
<tr>
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</tr>
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<td>Jason and Lisa</td>
<td>Arezoo, Gerry, Laila, and Nabil</td>
</tr>
<tr>
<td>Laila and Nabil</td>
<td>Arezoo, Gerry, Jason, and Lisa</td>
</tr>
<tr>
<td>Laila and Lisa</td>
<td>Arezoo, Gerry, Jason, and Nabil</td>
</tr>
<tr>
<td>Nabil and Lisa</td>
<td>Arezoo, Gerry, Jason, and Laila</td>
</tr>
</tbody>
</table>

We can double check that each of the pairs playing tether ball, plays table tennis together at some other time during the party. If we search through the table, we will see that each pair does play table tennis at the same time together multiple times. So, as long as we have listed all possible pairs playing tether ball, we know that everyone plays every other person at the party at least once at each station.

Arezoo will need to schedule 15 separate times during her party to ensure that each person is guaranteed to play both tether ball and table tennis with every other person at the party.
Teacher’s Notes

This is classic combinatorics question. We need to be careful when listing all of the pairs of people who are going to play tether ball. We want to make sure we have included every pair, without duplication.

This solution organized the pairs by listing:

- all pairs that include Arezoo,
- followed by all pairs that include Gerry, that do not include Arezoo,
- followed by all pairs that include Jason, that do not include Arezoo or Gerry,
- followed by all pairs that include Laila, that do not include Arezoo, Gerry, or Jason,
- followed by the pair that includes Nabil, that does not include Arezoo, Gerry, Jason, or Laila.

We have to stop the pattern here because there are no pairs for Lisa that do not include at least one of the other five people at the party. Hence, we have covered all pairs.

Here is another way of looking at the order of the pairs. Suppose we assigned each person a number: Arezoo (1), Gerry (2), Jason (3), Laila (4), Nabil (5), and Lisa (6). Now count (in order) the two-digit numbers that only use the digits 1 through 6, and have a second digit that is greater than the first digit. So you would include numbers like 25 and 46, but you would not include numbers like 31 or 55.

You will create a list of 15 numbers that describe the pairs at our party playing tether ball. The numbers in this list can never have the same two digits. This means we never count someone being paired with themselves. Since we always make the second digit greater than the first digit in this list, we never see two numbers with the same digits. This means the list does not count pairs like “Arezoo and Jason” and “Jason and Arezoo” separately. This counting technique ensures we have a complete list of the possible pairs, without any duplication.
Problem of the Week
Problem A
Smart Saver

James saves all of the coins he can find. He is very organized, so he has a separate container for each type of coin. In Canada a toonie is worth $2 (2 dollars), a loonie is worth $1, a quarter is worth 25¢ (25 cents), a dime is worth 10¢, a nickel is worth 5¢, and $1 is equal to 100¢.

Each container James has holds 80 coins. Here is a picture representing the containers. The shaded part represents how many coins are in each container.

A) Which container holds the most money? Justify your answer.

B) Approximately what is the difference between the amount James has saved in quarters and the amount he has saved in dimes?

C) Approximately what is the total amount of money James has saved so far?
Problem of the Week
Problem A and Solution
Smart Saver

Problem
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Solution
We need to estimate what fraction of each container is filled with coins.

Container A appears to be approximately \(\frac{1}{4}\) full.

Container B appears to be approximately \(\frac{3}{4}\) full.

Container C appears to be approximately \(\frac{1}{2}\) full.

Container D appears to be approximately \(\frac{1}{2}\) full.

Container E appears to be completely full.
Half of 80 is 40, since \(40 + 40 = 80\).

Half of 40 is 20, since \(20 + 20 = 40\).

Since half of \(\frac{1}{2}\) is \(\frac{1}{4}\), then \(\frac{1}{4}\) of 80 is 20.

Since \(\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\), then \(\frac{3}{4}\) of 80 is \(20 + 20 + 20 = 60\).

With this information we can estimate how many coins are in each container, as well as the value of the coins that have been collected. To determine the amount of money in each container, we can use skip counting, or multiplication.

<table>
<thead>
<tr>
<th>Container</th>
<th>Number of Coins</th>
<th>Value of Coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20</td>
<td>(20 \times 2 = $40)</td>
</tr>
<tr>
<td>B</td>
<td>60</td>
<td>(60 \times 1 = $60)</td>
</tr>
<tr>
<td>C</td>
<td>40</td>
<td>(40 \times 25 = 1000) = $10</td>
</tr>
<tr>
<td>D</td>
<td>40</td>
<td>(40 \times 10 = 400) = $4</td>
</tr>
<tr>
<td>E</td>
<td>80</td>
<td>(80 \times 5 = 400) = $4</td>
</tr>
</tbody>
</table>

It may be easier to calculate the values of the quarters, dimes, and nickels by noting how many of each coin we need to make $1. It takes 4 quarters, 10 dimes, or 20 nickels to make $1. To calculate the total value of the quarters, we can skip count by 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40. This means that 40 is equal to 10 groups of 4. Since each of these groups of 4 is worth $1, then the 40 quarters are worth $10. Similarly, if we skip count by 10: 10, 20, 30, 40, we see that the 4 groups of 10 dimes James has saved are worth $4, and if we skip count by 20: 20, 40, 60, 80, we see that the 4 groups of 20 nickels James has saved are worth $4.

A) Based on the calculations shown in the table, it appears that the container holding the loonies has the most money.

B) Based on the calculations shown in the table, it appears that James has saved \(10 - 4 = \$6\) more in quarters than he has in dimes.

C) James has saved approximately, \(40 + 60 + 10 + 4 + 4 = \$118\), in his containers.
Teacher’s Notes

Here is a question that you might want to consider. Which answers would change if the size of the containers changed? For example, suppose the containers hold 200 coins instead of 80 coins. The answer to part A) would not change, but the answers to part B) and part C) would change.

We could describe the amount of money each container holds algebraically. Let \( x \) represent the maximum number of coins each container can hold. Then,

\[
\text{number of cents in Toonies} = \frac{200 \cdot x}{4} = 50x
\]

\[
\text{number of cents in Loonies} = \frac{100 \cdot 3x}{4} = 75x
\]

\[
\text{number of cents in Quarters} = \frac{25 \cdot x}{2} = 12.5x
\]

\[
\text{number of cents in Dimes} = \frac{10 \cdot x}{2} = 5x
\]

\[
\text{number of cents in Nickels} = 5x
\]

So, no matter what the size of the container is, the most amount of money saved is in the Loonies container. Interestingly, there is an equal amount of money saved in the container with dimes when compared to the container with nickels.

The difference between the amount of money saved in quarters and the amount of money saved in dimes would depend on the size of the container. We can describe that difference as: \( 12.5x - 5x = 7.5x \)

We can also generally describe the total amount of money saved so far, with an unknown container size, as an equation: \( 50x + 75x + 12.5x + 5x + 5x = 147.5x \)
Geometry

&

Spatial Sense

TAKE ME TO THE COVER
Problem of the Week
Problem A
Following Directions

Yeni is playing a game that uses a $12 \times 12$ grid like the one shown below. She moves pieces on the grid by giving a sequence of steps. Each step is a direction indicated by an arrow: ↑ (up one block), ↓ (down one block), ← (left one block), and → (right one block).

Yeni has a special move which combines six steps: → → ↓ ↓ ← ↑

A) Yeni starts by putting a marker (X) at position A1.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>6</td>
<td></td>
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What is the position of the marker after making the special move three times in a row?

B) How many more times can she repeat the special move before the marker will move off the grid?

Strands  Patterning and Algebra, Geometry and Spatial Sense
Problem of the Week
Problem A and Solution
Following Directions

Problem
Yeni is playing a game that uses a $12 \times 12$ grid like the one shown below. She moves pieces on the grid by giving a sequence of steps. Each step is a direction indicated by an arrow: $\uparrow$ (up one block), $\downarrow$ (down one block), $\leftarrow$ (left one block), and $\rightarrow$ (right one block).

Yeni has a special move which combines six steps: $\rightarrow \rightarrow \downarrow \downarrow \leftarrow \uparrow$

A) Yeni starts by putting a marker (X) at position A1.

What is the position of the marker after making the special move three times in a row?

B) How many more times can she repeat the special move before the marker will move off the grid?

Solution

A) Starting at position A1, after making the special move once, the marker ends up at position B2.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
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Each time Yeni uses the special move, the marker ends up one row below and one column to the right of its starting position. So after the second time she executes the move, the marker moves to position C3, and after the third time she executes the move, the marker moves to position D4.

B) If we continue repeating this pattern, after the 10th time she repeats the move, the marker moves to position K11. From that position, if Yeni tries the special move she can move to the right once safely. However if she tries to move to the right a second time that would mean the marker moves off the edge of the grid. This means that starting from position A1 Yeni can repeat the special move 10 times safely, or 7 more times after using the special move 3 times.
Teacher’s Notes

This question was inspired by turtle graphics and Karel the Robot. In the 1960s, the programming language Logo allowed coders to use a turtle robot to draw patterns on the screen. The turtle normally appeared as a simple triangle, which indicated a direction it would move next. Programmers could easily write programs that would make the turtle move forward, turn, and draw lines. With a small set of instructions, people could create very interesting patterns and geometric images on the screen. Karel the Robot was designed to teach students how to code by having them control a simple, graphical robot that could move around a grid on the screen. Karel only responded to a small set of instructions, but this was enough to teach and learn the essential concepts of computer programming. Both the Logo turtle and Karel the Robot have been replicated in more modern programming languages.

This problem demonstrates two of those essential programming concepts: modularization and repetition. When writing programs of any size, it is important to be able to break up the problem into smaller subproblems. Programmers will bundle instructions together into subprograms, that are often referred to as functions, procedures, subroutines, or methods. Identifying these smaller pieces of the problem makes them easier to solve, and the code used to solve them more flexible.

Having the ability to repeat instructions in code, leads to more and better solutions to the problems that can be solved by computers. It can be tricky for new programmers to use repetition properly. Knowing when and how to stop the repetition is essential to success in coding.
Problem of the Week
Problem A
Sorting Solids

Use the Venn diagram below to classify the solids labelled with the letters A - L.

Strand  Geometry and Spatial Sense
Problem of the Week
Problem A and Solution
Sorting Solids

Problem
Use the Venn diagram below to classify the solids labelled with the letters A - K.

Solution
Triangular Faces

Rectangular Faces

Odd Number of Faces
Teacher’s Notes

You might notice in the solution that there are no examples of solids that have both triangular faces and rectangular faces but have an even number of faces. This is because all of the solids shown that have triangular faces and rectangular are either prisms or pyramids. When these shapes are formed, we either need to connect triangles to the sides of a single rectangle or connect two triangles using rectangles. In the first case we have one face from the rectangle and four faces from the triangles on each side. This is a total of five faces which is an odd number. In the second case, we have two faces from the triangles at the ends of the prism connected by three rectangles that are attached to the sides of the triangles. Again this is a total of five faces which is an odd number. Any prism or pyramid formed by using only triangles and rectangles must have an odd number of faces.

However, it is possible to form a solid that has an even number of faces with triangular and rectangular faces by introducing another polygon. An example of this is an antiprism. For more information about antiprisms, you can look at the Wikipedia page [https://en.wikipedia.org/wiki/Antiprism](https://en.wikipedia.org/wiki/Antiprism).
Problem of the Week
Problem A
Ship Flip

A transformation is the result of moving a shape according to a rule. There are three basic transformations: translate (slide), rotate (turn) and reflect (flip).

A sailing ship that is shown in the diagram on the right has one side that is simply an outline of the image. The other side of the ship, which is not shown, is filled in (black).

Identify which of the following images are a result of one or more transformations of our original image? Explain your reasoning.

A. 
B. 
C. 
D. 

Strand: Geometry and Spatial Sense
Problem of the Week
Problem A and Solution
Ship Flip

Problem
A transformation is the result of moving a shape according to a rule. There are three basic transformations: translate (slide), rotate (turn) and reflect (flip).

A sailing ship that is shown in the diagram on the right has one side that is simply an outline of the image. The other side of the ship, which is not shown, is filled in (black).

Identify which of the following images are a result of one or more transformations of our original image? Explain your reasoning.

Solution
Note that a flip of the original image would cause the black side to show.

- Image A is possible by rotating the original image by a half turn \((180^\circ)\).
- It is not possible to transform the original image into image B.
- Image C is possible by flipping the original image vertically, and rotating it by quarter turn \((90^\circ)\) counterclockwise.
- It is not possible to transform the original image into image D.

So images A and C result from transforming our original image.
Teacher’s Notes

There is more than one way to transform the original image to the images A and C. Generally, we should expect that there are multiple paths from a starting image to a transformed image. For example, it is possible to generate image C by doing a clockwise quarter turn rotation first and then a vertical flip. We could also get to that final image by doing a counterclockwise quarter turn followed by a horizontal flip.

We always have two choices when rotating images to get the same result. A rotation of $x^\circ$ clockwise is equal to $(360 - x^\circ)$ counterclockwise. So image C can be generated by doing a vertical flip followed by a $270^\circ$ turn clockwise. Also, a rotation of $180^\circ$ is equivalent to a horizontal flip followed by a vertical flip or a vertical flip followed by a horizontal flip. So we can see that there are multiple ways to transform the original image to image A.
Problem of the Week
Problem A
Painting Planning

Jack and Farah want to paint some of the walls in their house. They need to calculate the area that will be painted. They have a floor plan of the part of the house they want to paint. It shows which walls have doorways, as well as the dimensions of the rooms. The walls are all 3 metres high and the doorways are all 2 metres high and 1 metre wide. They do not need to paint the doorways.

A) Draw a diagram for each of the walls in the area that needs to be painted. Show the dimensions of the walls and the doorways on each diagram.

B) Calculate the total area that needs to be painted.

**Strands** Measurement, Geometry and Spatial Sense
Problem of the Week
Problem A and Solution
Painting Planning

Problem
Jack and Farah want to paint some of the walls in their house. They need to calculate the area that will be painted. They have a floor plan of the part of the house they want to paint. It shows which walls have doorways, as well as the dimensions of the rooms. The walls are all 3 metres high and the doorways are all 2 metres high and 1 metre wide. They do not need to paint the doorways.

A) Draw a diagram for each of the walls in the area that needs to be painted. Show the dimensions of the walls and the doorways on each diagram.

B) Calculate the total area that needs to be painted.
Solution

A) To keep things organized, consider this diagram that labels each of the walls in the floor plan of the house.

Here are diagrams showing the dimensions for each of the six walls:

- **A**: 3 m x 3 m, 2 m x 1 m
- **B**: 7 m x 3 m
- **C**: 6 m x 3 m
- **D**: 3 m x 3 m
- **E**: 3 m x 3 m
- **F**: 4 m x 3 m, 2 m x 1 m
B) Note that it is not necessary to know the exact location of the doors in order to calculate how much surface area requires paint. For the walls that have doors we can calculate the area of the wall, ignoring the door, and then subtract the area of the door to determine how much paint is required. The area of any of the doors is $2 \times 1 = 2 \text{ m}^2$.

Here are the calculated surface areas that need to be painted:

<table>
<thead>
<tr>
<th>Wall</th>
<th>Wall Area</th>
<th>Door Area</th>
<th>Painted Area</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>$3 \times 3 = 9 \text{ m}^2$</td>
<td>$2 \text{ m}^2$</td>
<td>$9 - 2 = 7 \text{ m}^2$</td>
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<tr>
<td>B</td>
<td>$3 \times 7 = 21 \text{ m}^2$</td>
<td>$0 \text{ m}^2$</td>
<td>$21 \text{ m}^2$</td>
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<td>C</td>
<td>$3 \times 6 = 18 \text{ m}^2$</td>
<td>$0 \text{ m}^2$</td>
<td>$18 \text{ m}^2$</td>
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<tr>
<td>D</td>
<td>$3 \times 3 = 9 \text{ m}^2$</td>
<td>$0 \text{ m}^2$</td>
<td>$9 \text{ m}^2$</td>
</tr>
<tr>
<td>E</td>
<td>$3 \times 3 = 9 \text{ m}^2$</td>
<td>$0 \text{ m}^2$</td>
<td>$9 \text{ m}^2$</td>
</tr>
<tr>
<td>F</td>
<td>$3 \times 4 = 12 \text{ m}^2$</td>
<td>$2 \text{ m}^2$</td>
<td>$12 - 2 = 10 \text{ m}^2$</td>
</tr>
</tbody>
</table>

So this means there is a total of $7 + 21 + 18 + 9 + 9 + 10 = 74 \text{ m}^2$ to paint.

Here is another way to calculate the surface area that requires painting. Imagine that we moved the wall labelled F so that it lines up with the wall labelled D and now the room is a rectangle with dimensions $6 \text{ m} \times 7 \text{ m}$.

When we do that, we lose the surface area of the wall labelled E, but gain an area on at the top of the newly formed rectangle with exactly the same surface area.

Ignoring the doors, the surface area of the walls of this rectangular room would be: $3 \times 6 = 18 \text{ m}^2$, $3 \times 7 = 21 \text{ m}^2$, $3 \times 6 = 18 \text{ m}^2$, and $3 \times 7 = 21 \text{ m}^2$. This is a total of $18 + 21 + 18 + 21 = 78 \text{ m}^2$. However, we still need to consider the area of the two doors that will not be painted. That total area of the doors is $2 \times 2 = 4 \text{ m}^2$. If we subtract this area from the previous total we have a surface area of $78 - 4 = 74 \text{ m}^2$ that needs to be painted.
Teacher’s Notes

Abstraction is an essential part of mathematics. Mathematicians use various models to represent real world information. This problem uses a diagram to represent the information about a physical building.

In general, when creating a mathematical model we have to think carefully about what information needs to be included, and what can be ignored. In this case, we need to know the dimensions of the walls that are to be painted, we need to know which walls have doors, and we need to know the dimensions of the doors. However, we do not need to know exactly where the doors are located on the walls of the rooms. We could have included the details of where the doors were located, but as a general principle we want to keep the abstraction as simple as possible.

This problem allows students to practise extrapolating information from the model back to the real world situation. They are required to imagine the 3-dimensional reality that is represented by the 2-dimensional diagram. It is the job of many mathematicians to create models of the real world that they can use to solve problems in the real world.
Problem of the Week
Problem A
Fancy Pattern

Elisabeth drew this fancy pattern with black, white and grey shading.

A) If the smallest black squares are 1 cm by 1 cm, what are the dimensions of the whole drawing?

B) What is the total area of the grey part of the drawing?
Problem of the Week  
Problem A and Solution  
Fancy Pattern  

Problem  
Elisabeth drew this fancy pattern with black, white and grey shading.  

A) If the smallest black squares are 1 cm by 1 cm, what are the dimensions of the whole drawing?  
B) What is the total area of the grey part of the drawing?  

Solution  
A) To determine the dimensions of the whole drawing, let’s focus on one corner section.  

Since the white squares align horizontally and vertically with the black squares in the checkerboard pattern, then we know that the white squares are also 1 cm by 1 cm. This can also be written $1 \text{ cm} \times 1 \text{ cm}$. We will use this convention throughout the rest of the solution. In a single checkerboard, there are 3 squares across and 3 squares down. This means that the checkerboard sections themselves are $3 \text{ cm} \times 3 \text{ cm}$. And since the checkerboard sections are aligned, and there are 3 of these sections across and 3 down, then this whole corner section must be $9 \text{ cm} \times 9 \text{ cm}$.  

This section is a repeated pattern in the whole drawing, and the sections are aligned horizontally and vertically. Since there are three sections across and three sections down, each of the dimensions of the whole drawing is 3 times the dimensions of this section. Therefore the dimensions of the whole drawing are $27 \text{ cm} \times 27 \text{ cm}$. 


B) One way to determine the area of the grey parts of the drawing would be to draw gridlines that align with the black and white squares in the checkerboard. The gridlines can be used to identify grey squares that are also 1 cm × 1 cm. Then we can count the number of those squares to determine the grey shaded area of the drawing.

Here is another way to calculate the area. Looking at the whole drawing, it can be divided up into nine sections. Five of those sections have black and white checkerboard patterns and smaller grey squares, like this:

![Checkerboard pattern](image)

and 4 of those sections are larger, solid grey squares with the same dimensions as the square shown above.

In part (A) we determined that the dimensions of this section are 9 cm × 9 cm, which means that the dimensions of the larger grey squares are also 9 cm × 9 cm. The area of one of those squares would be 9 × 9 = 81 cm². This means the total area of those four squares is 81 + 81 + 81 + 81 = 81 × 4 = 324 cm².

Within a section that contains the checkerboard patterns, there are nine subsections: five subsections that each contain the checkerboard pattern, and four subsections that are small, solid grey squares. In part (A) we determined that checkerboard subsection has dimensions 3 cm × 3 cm. So these small grey squares also have dimensions 3 cm × 3 cm, which means the area of one of those squares is 3 × 3 = 9 cm². Since there are four smaller grey squares in one of these sections, then the grey part of one of these sections is 9 + 9 + 9 + 9 = 9 × 4 = 36 cm². Since the section shown above is repeated five times in the entire drawing, the grey squares of these sections have a total area of 36 × 5 = 180 cm².

Therefore the total area of the grey part of the drawing is 180 + 324 = 504 cm².

Another way to calculate the area of the grey part is to start with the area of the whole drawing and subtract the area of the black and white squares. The area of the whole drawing is 27 × 27 = 729 cm². We know the dimensions of the black and white squares are 1 cm × 1 cm, so their areas are each 1 cm². We can count the total number of black and white squares and see that their are 225 of them. So the total area of the grey part of the drawing is 729 – 225 = 504 cm².
Teacher’s Notes

The fancy pattern Elisabeth drew is an example of a fractal. Fractals occur in mathematics, art, and nature. The essential characteristic of a fractal is that it has a repeating pattern, within its pattern. In this example, we can break down the whole image into a square containing nine squares. The basic pattern of the whole image has the following structure:

<table>
<thead>
<tr>
<th>pattern</th>
<th>solid</th>
<th>pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>solid</td>
<td>pattern</td>
<td>solid</td>
</tr>
<tr>
<td>pattern</td>
<td>solid</td>
<td>pattern</td>
</tr>
</tbody>
</table>

If you look at the pattern sections of the whole image, they can also be described as a square, containing nine squares, with the structure described above. Finally, each of these patterned sections can be described as a square that contains nine squares with the following similar structure:

<table>
<thead>
<tr>
<th>black</th>
<th>white</th>
<th>black</th>
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</thead>
<tbody>
<tr>
<td>white</td>
<td>black</td>
<td>white</td>
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<tr>
<td>black</td>
<td>white</td>
<td>black</td>
</tr>
</tbody>
</table>

The beauty of fractals, is that you can continue this structured repetition to more and more levels. You can also represent the fractal with a mathematical model. The next couple of pages show the fractal from this problem repeated to deeper levels. All of the images in this problem were actually generated by a relatively short computer program. We used the same code to create different images, by just changing the value of one variable to indicate how many levels of repetition we wanted.
Fractal with Four Levels
Fractal with Five Levels
Problem of the Week
Problem A
Blind Soccer

Blind people play soccer in the Paralympics. Blind soccer is just like soccer for
the sighted, except that each half lasts 25 minutes, rather than 45 minutes.
Jordan plays blind soccer on the Screaming Eagles team. In order to win a
match, Jordan’s team must win 3 of 5 games. Her team won the match.

A) What is the minimum number of minutes her team must play to win a match?

B) How many hours is this?

C) What is the maximum number of minutes her team could play to win a
   match?
Problem of the Week
Problem A and Solution
Blind Soccer

Problem
Blind people play soccer in the Paralympics. Blind soccer is just like soccer for the sighted, except that each half lasts 25 minutes, rather than 45 minutes.

Jordan plays blind soccer on the Screaming Eagles team. In order to win a match, Jordan’s team must win 3 of 5 games. Her team won the match.

A) What is the minimum number of minutes her team must play to win a match?
B) How many hours is this?
C) What is the maximum number of minutes her team could play to win a match?

Solution
A) Jordan’s team would need to play a minimum of three games to win the match. Since each game has 25 minute halves, the total playing time in a game is $25 + 25 = 50$ minutes. This means that in three games, the team plays a total of $50 + 50 + 50 = 3 \times 50 = 150$ minutes.

B) Since,
1 hour = 60 minutes, then
2 hours = $2 \times 60 = 120$ minutes, and
3 hours = $3 \times 60 = 180$ minutes.

So 150 minutes is more than 2 hours, but less than 3 hours. In fact, it is $150 - 120 = 30$ minutes more than 2 hours. So, the minimum amount of time Jordan’s team must play is 2 hours and 30 minutes. Since 30 minutes is half of an hour, we could also say they play for $2\frac{1}{2}$ hours.

C) Jordan’s team could play a maximum of five games in a match. This would be a total of $50 + 50 + 50 + 50 + 50 = 5 \times 50 = 250$ minutes of playing time.
Teacher’s Notes

We could also determine the number of hours that is equal to a given number of minutes by using division. We could calculate the quotient and remainder when we divide by 60 which is the number of minutes in an hour. For example:

\[ 150 \div 60 = 2 \text{ remainder } 30 \]

So 150 minutes is equal to 2 hours and 30 minutes.

Converting from one unit of measurement to another can often be done by multiplying one unit’s measurement by a conversion factor. For example, if we wanted to convert hours to seconds, the conversion factor is 3600, since there are 3600 seconds in one hour. If we want to convert from seconds to hours, the conversion factor is \( \frac{1}{3600} \) since \( \frac{1}{3600} \) of an hour is one second. In the case where the conversion factor is a fraction, we can either multiply by the fraction or break up the calculation into two parts where we multiply by the numerator of the fraction and divide the result by the denominator of the fraction.

Sometimes conversion factors, such as the number of minutes in an hour or the number of metres in a kilometre, are easy to remember. Other times, conversion factors, such as the number of feet in a mile or the number of minutes in a week, are more difficult to remember.

Conversion factors are not always helpful. For example, converting between Celsius and Fahrenheit is tricky. This conversion is unlike the previous examples, because 0° Celsius is not equal to 0° Fahrenheit. In all of the previous examples, the measurement value at 0 is the same. For example, 0 minutes is equal to 0 hours and 0 metres is equal to 0 kilometres. You can figure out how to convert between two different units like Celsius and Fahrenheit if you know two pairs of values that are equal to each other (e.g. the boiling point of water is 100° Celsius and 212° Fahrenheit, and the freezing point of water is 0° Celsius and -32° Fahrenheit), and you know how to use these two points to derive the equation of a line. This kind of calculation is done in later mathematics, but as a point of information you can convert Celsius to Fahrenheit using this formula:

\[ F = \left( C \times \frac{9}{5} \right) + 32 \]

and you can convert Fahrenheit to Celsius using this formula:

\[ C = \left( F - 32 \right) \times \frac{5}{9} \]
Problem of the Week
Problem A
Holiday Lights

Hunter needs exactly 6 metres of twinkle lights to decorate the roof-line of his house for the holidays. He has 3 strings of lights. The first string of lights covers 250 cm.

Which of the following options could represent the lengths of the other two strings of lights? Explain your thinking.

a) 175 cm and 175 cm
b) 150 cm and 150 cm
c) 180 cm and 170 cm
d) 150 cm and 200 cm
e) More than two of the above options are possibilities.
f) None of the options above are possibilities.
Problem of the Week
Problem A and Solution
Holiday Lights

Problem
Hunter needs exactly 6 metres of twinkle lights to decorate the roof-line of his house for the holidays. He has 3 strings of lights. The first string of lights covers 250 cm.

Which of the following options could represent the lengths of the other two strings of lights? Explain your thinking.

a) 175 cm and 175 cm
b) 150 cm and 150 cm
c) 180 cm and 170 cm
d) 150 cm and 200 cm
e) More than two of the above options are possibilities.
f) None of the options above are possibilities.

Solution
We know that 1 m = 100 cm, so 6 m = 6 \times 100 = 600 cm. Since Hunter already has a 250 cm string of lights, he needs the other two strings to cover the remaining 600 − 250 = 350 cm of his roof-line. We can check the four pairs of lengths to see how many of them total 350 cm.

a) 175 + 175 = 350 cm
b) 150 + 150 = 300 cm
c) 180 + 170 = 350 cm
d) 150 + 200 = 350 cm

From these calculations, we see that the total for three of these choices give us the required 350 cm. So the correct answer to the original question is e), since three of the first four options would work.

Alternatively, we could add 250 cm to each of the pairs of lengths listed in the problem to see which triples would give us a total 600 cm.

a) 175 + 175 + 250 = 600 cm
b) 150 + 150 + 250 = 550 cm
c) 180 + 170 + 250 = 600 cm
d) 150 + 200 + 250 = 600 cm

We see that three of the four possibilities provide the required 6 m. So the correct answer to the original question is e), since three of the first four options would work.
Teacher’s Notes

We can break up numbers into smaller units in a variety of ways. A factorization of an integer is a combination of positive integers (factors) that can be multiplied together to produce the original value. For example, we can factor 6 in the following ways:

\[
\begin{align*}
1 \times 6 \\
2 \times 3
\end{align*}
\]

The Holiday Lights problem essentially asks students to identify partitions of 600. A partition is a combination of positive integers (parts) that can be added together to produce the original value. For example, we can partition 6 in the following ways:

\[
\begin{align*}
1 + 1 + 1 + 1 + 1 + 1 & \quad 1 + 1 + 4 \\
1 + 1 + 1 + 1 + 2 & \quad 1 + 5 \\
1 + 1 + 1 + 1 + 3 & \quad 1 + 1 + 2 + 2
\end{align*}
\]

\[
\begin{align*}
1 + 2 + 3 & \quad 2 + 2 + 2 \quad 6 \\
3 + 3 & \quad 2 + 4
\end{align*}
\]

Multiplication and addition are commutative operations, meaning the order of the operands does not affect the outcome of the operation. So, when we write factorizations or partitions of numbers, we do not include variations that are the same set of integers. For example, 1 + 5 and 5 + 1 are the same partition.

We can visualize a partition using circles. To represent a partition of a number \( n \), we arrange the \( n \) circles into rows. Each row represents a part in the partition. This is called a Ferrers diagram. For example here is a diagram of one partition of the number 11:

\[
\begin{align*}
\circ \circ \circ \\
\circ \circ \circ \\
\circ \circ \\
\circ
\end{align*}
\]

This represents the partition 5 + 3 + 2 + 1 since the first row has 5 circles, the second has 3 circles, the third has 2 circles, and the fourth has 1 circle. We can easily find another partition of 11 by rotating this diagram by 90 degrees.

\[
\begin{align*}
\circ \\
\circ \\
\circ \circ \\
\circ \circ \circ \\
\circ \circ \circ \circ
\end{align*}
\]

This diagram represents the partition 1 + 1 + 2 + 3 + 4. These two partitions are said to be conjugates of one another.

We can ask many different questions about partitions. We could ask for all of the partitions of a number or some subset. For example, in our problem the partitions we considered were restricted to those that included 250. We could ask for partitions that did not include any duplicate parts or partitions that only included odd numbers. The possibilities are endless.
Kunik takes swimming lessons at the community pool. A lap is the length of the pool, which is 25 m. During the lessons Kunik swims laps, from one end of the pool to the other end and back to the starting point. Kunik can swim 10 laps of the pool in 5 minutes if she is doing the front crawl, and she can swim 6 laps of the pool in 4 minutes if she is doing the backstroke.

A) How many metres can Kunik swim in 10 minutes, using the front crawl?

B) Approximately how long will it take Kunik to swim 60 laps of the pool if she uses the front crawl for half the laps and the backstroke for half the laps?

C) If Kunik wants to swim faster when she is in the pool, should she do the front crawl or the backstroke? Justify your answer.
Problem
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C) If Kunik wants to swim faster when she is in the pool, should she do the front crawl or the backstroke? Justify your answer.

Solution

A) Notice that 10 minutes is twice as much as 5 minutes. Since Kunik can swim 10 laps in 5 minutes using the front crawl, if we double the amount of time in the pool, we expect her to swim double the number of laps. Since double the time is $2 \times 5 = 10$ minutes, then we expect her to swim $2 \times 10 = 20$ laps in that time. Since each lap is 25 m, Kunik can swim $25 \times 20 = 500$ m in 10 minutes.
B) Half of 60 is 30 since $30 + 30 = 60$, so Kunik will be swimming 30 laps using the front crawl and 30 laps using the backstroke. For each stroke, we can use a table to determine how long it takes Kunik to swim 30 laps.

<table>
<thead>
<tr>
<th>Minutes</th>
<th>Laps</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>15</td>
<td>30</td>
</tr>
</tbody>
</table>

Table for front crawl

<table>
<thead>
<tr>
<th>Minutes</th>
<th>Laps</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>16</td>
<td>24</td>
</tr>
<tr>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

Table for backstroke

From the tables, we expect it will take Kunik $15 + 20 = 35$ minutes to swim 60 laps if she uses each stroke half of the time.

C) From the tables we notice that it takes Kunik 15 minutes to swim 30 laps using the front crawl, and it takes her 20 minutes to swim the 30 laps using the backstroke. Since it takes longer for her to swim the same number of laps using the backstroke, she should do the front crawl if she wants to swim faster.
Teacher’s Notes

This problem could be solved using the rates of speed. The formula for calculating speed is:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

Since Kunik swims 10 laps of the pool in 5 minutes, then her front crawl speed is:

$$\frac{250}{5} = 50 \text{ metres/min}$$

Since she swims 6 laps of the pool in 4 minutes, then her backstroke speed is:

$$\frac{150}{4} = 37.5 \text{ metres/min}$$

Based on these calculations, we can see that Kunik’s rate of speed is faster doing the front crawl.

If we want to find out the time of an activity given the speed and distance, we can rearrange the formula this way:

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

To calculate her time in part B) of the question we can substitute the distances and speeds into our formula for finding the time. The distance she swims using each stroke is 750 m.

$$\text{time} = \frac{750 \text{ metres}}{50 \text{ metres/min}} + \frac{750 \text{ metres}}{37.5 \text{ metres/min}} = 15 \text{ minutes} + 20 \text{ minutes} = 35 \text{ minutes}$$

Notice the units in the calculation of the time. The distance unit (metres) appears in the numerator and denominator of the fraction. These units cancel each other. The time unit (min) appears in the denominator of the unit of speed in the denominator of the fraction. This means that the units we are calculating in the end are minutes. Keeping track of the units in your calculations that involve speed, time, and distance is a good way to check that you are working with the proper formula.
Problem of the Week
Problem A
Weighing Brown Bats

“Brown bats, common in Canada, weigh as much as two nickels and a dime.”

Riya, who has always been curious about bats, decided to count the number of bats that flew through her yard. Riya counted 15 bats in one hour. She wondered what the total weight of these bats would be using the information from the quotation above. She made a table to figure out how much 15 bats would weigh, if we actually measured weight in terms of nickels and dimes.

A) Finish her table.

<table>
<thead>
<tr>
<th>Number of Bats</th>
<th>Number of Nickels</th>
<th>Number of Dimes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

B) In Canada, a nickel is worth 5¢ (5 cents), a dime is worth 10¢, and $1 (one dollar) is equal to 100¢. What would the total value of 15 bats be if each bat was actually worth “two nickels and a dime”?

Strands Patterning and Algebra, Measurement
Problem of the Week
Problem A and Solution
Weighing Brown Bats

Problem
“Brown bats, common in Canada, weigh as much as two nickels and a dime.”

Riya, who has always been curious about bats, decided to count the number of bats that flew through her yard. Riya counted 15 bats in one hour. She wondered what the total weight of these bats would be using the information from the quotation above. She made a table to figure out how much 15 bats would weigh, if we actually measured weight in terms of nickels and dimes.

A) Finish her table.

B) In Canada, a nickel is worth 5¢ (5 cents), a dime is worth 10¢, and $1 (one dollar) is equal to 100¢. What would the total value of 15 bats be if each bat was actually worth “two nickels and a dime”?

Solution

A) Here is the completed table:

<table>
<thead>
<tr>
<th>Number of Bats</th>
<th>Number of Nickels</th>
<th>Number of Dimes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>18</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>22</td>
<td>11</td>
</tr>
<tr>
<td>12</td>
<td>24</td>
<td>12</td>
</tr>
<tr>
<td>13</td>
<td>26</td>
<td>13</td>
</tr>
<tr>
<td>14</td>
<td>28</td>
<td>14</td>
</tr>
<tr>
<td>15</td>
<td>30</td>
<td>15</td>
</tr>
</tbody>
</table>
B) According to the table, 15 bats are equal to 30 nickels and 15 dimes.

- The value of the nickels is: \(30 \times 5 = 150\)¢.
- The value of the dimes is: \(15 \times 10 = 150\)¢.
- Total value of 15 bats would be: \(150 + 150 = 300\)¢ or $3.

Another way to calculate the total monetary value of the bats would be to add a column to the table. The extra column in the table can be used to keep track of how much the bats are worth.

Since 2 nickels are equal to: \(5 + 5 = 2 \times 5 = 10\)¢, then the value of 2 nickels and a dime would be: \(10 + 10 = 20\)¢.

<table>
<thead>
<tr>
<th>Number of Bats</th>
<th>Number of Nickels</th>
<th>Number of Dimes</th>
<th>Value in ¢</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>3</td>
<td>60</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>5</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>6</td>
<td>120</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
<td>7</td>
<td>140</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
<td>8</td>
<td>160</td>
</tr>
<tr>
<td>9</td>
<td>18</td>
<td>9</td>
<td>180</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>10</td>
<td>200</td>
</tr>
<tr>
<td>11</td>
<td>22</td>
<td>11</td>
<td>220</td>
</tr>
<tr>
<td>12</td>
<td>24</td>
<td>12</td>
<td>240</td>
</tr>
<tr>
<td>13</td>
<td>26</td>
<td>13</td>
<td>260</td>
</tr>
<tr>
<td>14</td>
<td>28</td>
<td>14</td>
<td>280</td>
</tr>
<tr>
<td>15</td>
<td>30</td>
<td>15</td>
<td>300</td>
</tr>
</tbody>
</table>
Teacher’s Notes

The solution for part B) of this problem can be determined algebraically. If we let $x$ represent the number of bats, and let $y$ represent their monetary worth in cents, we can write the following equation: $y = ((2 \times 5) + 10) \times x$

This equation can be simplified to: $y = 20x$

This equation shows a *linear relationship* between the number of bats and their monetary value. We can see that it is a linear relationship in the equation because it reflects one of the standard equations for a line: $y = mx + b$

This standard equation shows a relationship between variables $x$ and $y$, given the constants $m$ and $b$. The constants in our equation $y = 20x$ are:

- 20 for the value of $m$
- 0 for the value of $b$

The constant $m$ in this form of the equation of a line represents the *slope* of the line. The slope is defined as the rise (the amount $y$ changes) over the run (the amount $x$ changes) between two points on the line. The slope of a line is the same for every pair of points on the line. Using our table, we can check the slope using two random pairs of values. For example, according to the table, when we have 12 bats, the monetary value is 240¢, and when we have 7 bats, the monetary value is 140¢. So the rise over run is:

$$\frac{(240 - 140)}{(12 - 7)} = \frac{100}{5} = 20$$

The constant $b$ in this form represents the *y-intercept* of the line. The y-intercept is the place where the line crosses the Y-axis if we draw a graph the equation. It is also the value of $y$ when $x$ is 0. In our equation, when the number of bats is 0 then the monetary value is 0.
Problem of the Week
Problem A
Screen Time vs. Play Time

Xiao wanted to figure out how much time she spends doing activities outside compared to how much time she spends using a computer. She kept track of her activities over two days. This was what she recorded:

Day 1
A 8:30 a.m. to 9:00 a.m. Played outside before school.
B 9:45 a.m. to 10:10 a.m. Used the computer during math class.
C 10:30 a.m. to 10:45 a.m. Played outside during recess.
D 12:30 p.m. to 1:00 p.m. Played outside during lunch break.
E 1:20 p.m. to 1:40 p.m. Used the computer during French class.
F 2:00 p.m. to 2:15 p.m. Played outside during recess.
G 5:15 p.m. to 6:15 p.m. Did homework on the computer

Day 2
H 10:00 a.m. to 10:45 a.m. Went for a bike ride with her friends.
I 1:10 p.m. to 2:20 p.m. Went for a hike with her aunt.
J 3:30 p.m. to 5:15 p.m. Watched a movie on the computer.

Did Xiao spend more time outside or using the computer over the two days? Justify your answer.

Strands Number Sense and Numeration, Measurement
Problem

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**Day 1**
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- B 9:45 a.m. to 10:10 a.m. Used the computer during math class.
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- D 12:30 p.m. to 1:00 p.m. Played outside during lunch break.
- E 1:20 p.m. to 1:40 p.m. Used the computer during French class.
- F 2:00 p.m. to 2:15 p.m. Played outside during recess.
- G 5:15 p.m. to 6:15 p.m. Did homework on the computer.

**Day 2**
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- I 1:10 p.m. to 2:20 p.m. Went for a hike with her aunt.
- J 3:30 p.m. to 5:15 p.m. Watched a movie on the computer.

Did Xiao spend more time outside or using the computer over the two days? Justify your answer.

Solution

One way to determine the answer is to calculate the total number of minutes Xiao spent on each kind of activity. We can calculate the amount of time each activity takes by using a number line.

For example, to calculate how much time elapsed when Xiao used the computer during math class, we can create a number line representing the times from 9:00 a.m. until 11:00 a.m., showing intervals of 5 minutes each.

From this number line, we skip count by five: 5, 10, 15, 20, 25, to see that she spends 25 minutes on the computer during that time.
We could draw a number line to calculate each of the intervals that Xiao recorded. Here is a table that shows how many minutes she spent on the computer:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Minutes Elapsed</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>25</td>
</tr>
<tr>
<td>E</td>
<td>20</td>
</tr>
<tr>
<td>G</td>
<td>60</td>
</tr>
<tr>
<td>J</td>
<td>105</td>
</tr>
</tbody>
</table>

This is a total of $25 + 20 + 60 + 105 = 210$ minutes.

Here is a table that shows how many minutes she spent outside:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Minutes Elapsed</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>30</td>
</tr>
<tr>
<td>C</td>
<td>15</td>
</tr>
<tr>
<td>D</td>
<td>30</td>
</tr>
<tr>
<td>F</td>
<td>15</td>
</tr>
<tr>
<td>H</td>
<td>45</td>
</tr>
<tr>
<td>I</td>
<td>70</td>
</tr>
</tbody>
</table>

This is a total of $30 + 15 + 30 + 15 + 45 + 70 = 205$ minutes.

So Xiao spent 5 more minutes on the computer than she spent outside during those two days.
Teacher’s Notes

When we think about calculating elapsed time, we normally count the minutes from the start time until the end time. When we think about calculating the difference between two numbers we usually subtract the smaller number from the larger number. If we use a number line to calculate the time spent on each activity, we can do this by determining the length of the line segment between the time the activity started and the time it ended. It does not matter if we measure this length from the start to the end or from the end to the start. We refer to this length as the magnitude of the number.

In mathematics, if we only care about the size of a number, and not its sign, we would be interested in the absolute value of that number. The absolute value of a number is denoted with vertical bars like this: $|x|$. So $|-4| = 4$ and $|12| = 12$.

In general, if we want to know an interval between two numbers $x$ and $y$ we would calculate $|x - y|$. The result will always be a non-negative number whether $x \geq y$ or $y \geq x$. 
Problem of the Week
Problem A
Baking for a Bunch

Samir wants to make treats for his class. There are 24 people in the class including Samir and his teacher. He has two recipes: one for brownies and one for cookies. The brownie recipe makes enough for 12 people to eat. The cookie recipe makes 48 cookies.

***Brownies***

- 1 cup white sugar
- 1/2 cup butter
- 1/4 cup cocoa powder
- 1/4 teaspoon vanilla extract
- 3/4 cup all-purpose flour
- 1/2 teaspoon salt
- 2 eggs

***Chocolate Chip Cookies***

- 2 cups all-purpose flour
- 1 teaspoon baking soda
- 1 teaspoon salt
- 1 cup butter
- 1 cup white sugar
- 1/2 cup brown sugar
- 1 teaspoon vanilla extract
- 2 eggs
- 2 cups chocolate chips

Samir wants to bake exactly enough so each person gets one brownie and one cookie. Make a list showing the total amount of each ingredient he needs to do his baking.

**Strands**  Measurement, Number Sense and Numeration
# Problem of the Week

## Problem A and Solution

### Baking for a Bunch

#### Problem

Samir wants to make treats for his class. There are 24 people in the class including Samir and his teacher. He has two recipes: one for brownies and one for cookies. The brownie recipe makes enough for 12 people to eat. The cookie recipe makes 48 cookies.

<table>
<thead>
<tr>
<th>Brownies</th>
<th>Chocolate Chip Cookies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cup white sugar</td>
<td>2 cups all-purpose flour</td>
</tr>
<tr>
<td>½ cup butter</td>
<td>1 teaspoon baking soda</td>
</tr>
<tr>
<td>¼ cup cocoa powder</td>
<td>1 teaspoon salt</td>
</tr>
<tr>
<td>¼ teaspoon vanilla extract</td>
<td>1 cup butter</td>
</tr>
<tr>
<td>¾ cup all-purpose flour</td>
<td>1 cup white sugar</td>
</tr>
<tr>
<td>½ teaspoon salt</td>
<td>½ cup brown sugar</td>
</tr>
<tr>
<td>2 eggs</td>
<td>1 teaspoon vanilla extract</td>
</tr>
<tr>
<td></td>
<td>2 eggs</td>
</tr>
<tr>
<td></td>
<td>2 cups chocolate chips</td>
</tr>
</tbody>
</table>

Samir wants to bake exactly enough so each person gets one brownie and one cookie. Make a list showing the total amount of each ingredient he needs to do his baking.

#### Solution

Since Samir wants to make 24 brownies, and the recipe makes enough for 12, then he must double the brownie recipe, because $12 \times 2 = 24$. To double a number, you can either multiply it by 2 or you can add the number to itself.

Since he wants to make 24 cookies, and the recipe makes enough for 48, then he must cut the cookie recipe in half, because $48 \div 2 = 24$.

The most difficult calculation is probably determining the required amount of flour. You will need to double the amount from the brownie recipe ($\frac{3}{4}$ cup) and add it to half the amount from the cookie recipe (2 cups). There are many ways to determine the result of $\frac{3}{4} \times 2$ or $\frac{3}{4} + \frac{3}{4}$. For example, you can use fractions of geometric shapes such as circles or squares, or use money such as Canadian quarters. You could also literally use measuring cups with flour, sand, or some other measurable material.
Here is another way to think about the calculation.

Break up $\frac{3}{4}$ into 3 separate quarters: $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

Now, double this amount using addition: $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

Group the first four quarters together and the last two quarters together:

$\left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{4} + \frac{1}{4}\right)$

When you add four quarters together you get a whole, or 1. When you add two quarters together you get a half ($\frac{1}{2}$). So in this case the total is $1\frac{1}{2}$ cups of flour required for the brownies. You also need half of the 2 cups of flour for the cookies. Half of 2 cups is 1 cup. So altogether you need $1\frac{1}{2} + 2 = 2\frac{1}{2}$ cups of flour.

Here is a summary of the totals required for the rest of the ingredients:

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>Brownie Amount</th>
<th>Cookie Amount</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>flour</td>
<td>$\frac{3}{4} \times 2 = 1\frac{1}{2}$ cups</td>
<td>$2 \div 2 = 1$ cup</td>
<td>$1\frac{1}{2} + 1 = 2\frac{1}{2}$ cups</td>
</tr>
<tr>
<td>white sugar</td>
<td>$1 \times 2 = 2$ cups</td>
<td>$1 \div 2 = \frac{1}{2}$ cup</td>
<td>$2 + \frac{1}{2} = 2\frac{1}{2}$ cups</td>
</tr>
<tr>
<td>butter</td>
<td>$\frac{1}{2} + \frac{1}{2} = 1$ cup</td>
<td>$1 \div 2 = \frac{1}{2}$ cup</td>
<td>$1 + \frac{1}{2} = 1\frac{1}{2}$ cups</td>
</tr>
<tr>
<td>cocoa powder</td>
<td>$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ cup</td>
<td></td>
<td>$\frac{1}{2}$ cup</td>
</tr>
<tr>
<td>vanilla extract</td>
<td>$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ teaspoon</td>
<td>$1 \div 2 = \frac{1}{2}$ teaspoon</td>
<td>$\frac{1}{2} + \frac{1}{2} = 1$ teaspoon</td>
</tr>
<tr>
<td>salt</td>
<td>$\frac{1}{2} + \frac{1}{2} = 1$ teaspoon</td>
<td>$1 \div 2 = \frac{1}{2}$ teaspoon</td>
<td>$1 + \frac{1}{2} = 1\frac{1}{2}$ teaspoons</td>
</tr>
<tr>
<td>eggs</td>
<td>$2 \times 2 = 4$ eggs</td>
<td>$2 \div 2 = 1$ egg</td>
<td>$4 + 1 = 5$ eggs</td>
</tr>
<tr>
<td>baking soda</td>
<td></td>
<td>$1 \div 2 = \frac{1}{2}$ teaspoon</td>
<td>$\frac{1}{2}$ teaspoon</td>
</tr>
<tr>
<td>brown sugar</td>
<td>$\frac{1}{2} \div 2 = \frac{1}{4}$ cup</td>
<td></td>
<td>$\frac{1}{4}$ cup</td>
</tr>
<tr>
<td>chocolate chips</td>
<td></td>
<td>$2 \div 2 = 1$ cup</td>
<td></td>
</tr>
</tbody>
</table>
Teacher’s Notes

It is very important for students to have a solid understanding of fractions and how to do calculations with them. Their ability to work with fractions tends to diminish once they are given a calculator to work with, and this can be problematic in later mathematics classes.

A calculator is a great tool for computing results given numbers. However, when students are working with algebraic expressions that involve fractions, a calculator may not be able to help. For example, a calculator cannot help you simplify an expression like:

\[
\frac{(x + 1)}{3} + \frac{(x - 5)}{2}
\]

Understanding and practising adding, subtracting, multiplying and dividing fractions early will make a difference for students in high school and beyond.
Problem of the Week
Problem A
Biking Around the Lake

Jessie lives on Island Lake. There is a road all the way around the lake called Ring Road. There is one road from town to the lake, called Town Road (not shown in the picture). The distance around Island Lake is 10 km.

Measuring the distance clockwise, the distance from Jessie’s house to the intersection of Ring Road and Town Road is 3 km, and the distance from the beach to that intersection is 9 km.

Jessie is biking to the beach, to meet her friend Brody who is already there. She wants to take the shortest route to the beach. Jessie rides 2 km, then remembers she promised to bring a water bottle for Brody. She cycles back to her house, picks up the water bottle, then bikes to the beach to meet her friend.

How many kilometres did Jessie travel to reach her final destination?

Strands  Number Sense and Numeration,  Measurement
Problem of the Week
Problem A and Solution
Biking Around the Lake

Problem
Jessie lives on Island Lake. There is a road all the way around the lake called Ring Road. There is one road from town to the lake, called Town Road (not shown in the picture). The distance around Island Lake is 10 km.

Measuring the distance clockwise, the distance from Jessie’s house to the intersection of Ring Road and Town Road is 3 km, and the distance from the beach to that intersection is 9 km.

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How many kilometres did Jessie travel to reach her final destination?
Solution

The first thing we need to do is determine the shortest distance from Jessie’s house to the beach. It would be helpful to identify exactly where Town Road is relative to the location of the house and the beach. We could do this by creating a diagram that includes Town Road, and shows 10 equidistant markers on Ring Road. Since the distance around Ring Road is 10 km, the distance between adjacent markers is 1 km. Now, we can determine distances between points on Ring Road by counting markers. After deciding where the intersection for Ring Road and Town Road is on the diagram, add Jessie’s house and the beach. We can measure 3 km counter-clockwise from the intersection to identify the location of the house, and measure 9 km counter-clockwise from the intersection to the location of the beach.

From this diagram, it is clear that it would be shorter for Jessie to ride clockwise on Ring Road to get from her house to the beach. Counting the markers, it is 4 km from her house to the beach if she rides clockwise on Ring Road, and it is 6 km if she rides counter-clockwise to the beach. Now we can make a number line to track how far Jessie rode.

\[ \text{In total, Jessie has ridden } 2 + 2 + 4 = 8 \text{ km.} \]
Teacher’s Notes

We used a diagram in the solution of this problem. A diagram does not need to be an exact representation of the problem, nor does it even need to be a scaled representation. A diagram needs to include all known, and important information in an organized way that will help us find a solution. In this case, the important information is the relative location of the house to Town Road, the relative location of the beach to Town Road, and the length of Ring Road. Details such as the fact that Ring Road surrounds a lake and the exact shape of the road are irrelevant. A diagram is an abstraction of the actual problem. The process of creating a mathematical model from a real-world situation is an essential part of advanced mathematics and computer science.

Once we have created a good diagram, we can use deductive reasoning to determine the distance between Jessie’s house and the beach. As students continue to study mathematics, they will be introduced to Euclidean geometry. They will be asked to solve problems where they are required to determine an unknown value given some starting information about angles, lines, triangles, circles, and other polygons. Creating a useful diagram is often the first step to solving these types of problems. Students will practice using deductive reasoning to find the answers. Both of these skills can be applied to solving many other problems.
Problem of the Week

Problem A

Painting Planning

Jack and Farah want to paint some of the walls in their house. They need to calculate the area that will be painted. They have a floor plan of the part of the house they want to paint. It shows which walls have doorways, as well as the dimensions of the rooms. The walls are all 3 metres high and the doorways are all 2 metres high and 1 metre wide. They do not need to paint the doorways.

A) Draw a diagram for each of the walls in the area that needs to be painted. Show the dimensions of the walls and the doorways on each diagram.

B) Calculate the total area that needs to be painted.

**Strands**  Measurement, Geometry and Spatial Sense
Problem of the Week
Problem A and Solution
Painting Planning

Problem
Jack and Farah want to paint some of the walls in their house. They need to calculate the area that will be painted. They have a floor plan of the part of the house they want to paint. It shows which walls have doorways, as well as the dimensions of the rooms. The walls are all 3 metres high and the doorways are all 2 metres high and 1 metre wide. They do not need to paint the doorways.

A) Draw a diagram for each of the walls in the area that needs to be painted. Show the dimensions of the walls and the doorways on each diagram.

B) Calculate the total area that needs to be painted.
Solution

A) To keep things organized, consider this diagram that labels each of the walls in the floor plan of the house.

Here are diagrams showing the dimensions for each of the six walls:

- **A**
  - Dimensions: 3 m x 3 m, 2 m x 1 m

- **B**
  - Dimensions: 7 m x 3 m

- **C**
  - Dimensions: 6 m x 3 m

- **D**
  - Dimensions: 3 m x 3 m

- **E**
  - Dimensions: 3 m x 3 m

- **F**
  - Dimensions: 4 m x 3 m, 2 m x 1 m
B) Note that it is not necessary to know the exact location of the doors in order to calculate how much surface area requires paint. For the walls that have doors we can calculate the area of the wall, ignoring the door, and then subtract the area of the door to determine how much paint is required. The area of any of the doors is $2 \times 1 = 2 \ \text{m}^2$.

Here are the calculated surface areas that need to be painted:

<table>
<thead>
<tr>
<th>Wall</th>
<th>Wall Area</th>
<th>Door Area</th>
<th>Painted Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$3 \times 3 = 9 \ \text{m}^2$</td>
<td>$2 \ \text{m}^2$</td>
<td>$9 - 2 = 7 \ \text{m}^2$</td>
</tr>
<tr>
<td>B</td>
<td>$3 \times 7 = 21 \ \text{m}^2$</td>
<td>$0 \ \text{m}^2$</td>
<td>$21 \ \text{m}^2$</td>
</tr>
<tr>
<td>C</td>
<td>$3 \times 6 = 18 \ \text{m}^2$</td>
<td>$0 \ \text{m}^2$</td>
<td>$18 \ \text{m}^2$</td>
</tr>
<tr>
<td>D</td>
<td>$3 \times 3 = 9 \ \text{m}^2$</td>
<td>$0 \ \text{m}^2$</td>
<td>$9 \ \text{m}^2$</td>
</tr>
<tr>
<td>E</td>
<td>$3 \times 3 = 9 \ \text{m}^2$</td>
<td>$0 \ \text{m}^2$</td>
<td>$9 \ \text{m}^2$</td>
</tr>
<tr>
<td>F</td>
<td>$3 \times 4 = 12 \ \text{m}^2$</td>
<td>$2 \ \text{m}^2$</td>
<td>$12 - 2 = 10 \ \text{m}^2$</td>
</tr>
</tbody>
</table>

So this means there is a total of $7 + 21 + 18 + 9 + 9 + 10 = 74 \ \text{m}^2$ to paint.

Here is another way to calculate the surface area that requires painting. Imagine that we moved the wall labelled F so that it lines up with the wall labelled D and now the room is a rectangle with dimensions $6 \ \text{m} \times 7 \ \text{m}$.

When we do that, we lose the surface area of the wall labelled E, but gain an area on at the top of the newly formed rectangle with exactly the same surface area.

Ignoring the doors, the surface area of the walls of this rectangular room would be: $3 \times 6 = 18 \ \text{m}^2$, $3 \times 7 = 21 \ \text{m}^2$, $3 \times 6 = 18 \ \text{m}^2$, and $3 \times 7 = 21 \ \text{m}^2$. This is a total of $18 + 21 + 18 + 21 = 78 \ \text{m}^2$. However, we still need to consider the area of the two doors that will not be painted. That total area of the doors is $2 \times 2 = 4 \ \text{m}^2$. If we subtract this area from the previous total we have a surface area of $78 - 4 = 74 \ \text{m}^2$ that needs to be painted.
Teacher’s Notes

Abstraction is an essential part of mathematics. Mathematicians use various models to represent real world information. This problem uses a diagram to represent the information about a physical building.

In general, when creating a mathematical model we have to think carefully about what information needs to be included, and what can be ignored. In this case, we need to know the dimensions of the walls that are to be painted, we need to know which walls have doors, and we need to know the dimensions of the doors. However, we do not need to know exactly where the doors are located on the walls of the rooms. We could have included the details of where the doors were located, but as a general principle we want to keep the abstraction as simple as possible.

This problem allows students to practise extrapolating information from the model back to the real world situation. They are required to imagine the 3-dimensional reality that is represented by the 2-dimensional diagram. It is the job of many mathematicians to create models of the real world that they can use to solve problems in the real world.
Problem of the Week
Problem A
Fuel Frenzy

Different vehicles are more fuel efficient than others, meaning some vehicles can go further on a tank of gas than others. Our car uses 6 litres for every 100 km of driving.

A) The car has a 42 litre tank. How many kilometres can our car drive on one tank of gas?

B) We are going on a trip to visit our family, 900 kilometres away. How much fuel will we use to get to our destination?

C) Fuel costs $2.00 per litre. How much will the fuel cost for the trip?
Problem of the Week
Problem A and Solution
Fuel Frenzy

Problem
Different vehicles are more fuel efficient than others, meaning some vehicles can go further on a tank of gas than others. Our car uses 6 litres for every 100 km of driving.

A) The car has a 42 litre tank. How many kilometres can our car drive on one tank of gas?

B) We are going on a trip to visit our family, 900 kilometres away. How much fuel will we use to get to our destination?

C) Fuel costs $2.00 per litre. How much will the fuel cost for the trip?

Solution
A) We can use a table to see the relationship between the number of litres of gas used and how far the car can travel. On each row we increase the litres used by 6 and the km travelled by 100.

<table>
<thead>
<tr>
<th>Litres Used</th>
<th>km Travelled</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>100</td>
</tr>
<tr>
<td>12</td>
<td>200</td>
</tr>
<tr>
<td>18</td>
<td>300</td>
</tr>
<tr>
<td>24</td>
<td>400</td>
</tr>
<tr>
<td>30</td>
<td>500</td>
</tr>
<tr>
<td>46</td>
<td>600</td>
</tr>
<tr>
<td>42</td>
<td>700</td>
</tr>
<tr>
<td>48</td>
<td>800</td>
</tr>
<tr>
<td>54</td>
<td>900</td>
</tr>
</tbody>
</table>

From this table we see that you can travel 700 km on 42 litres of fuel.

B) From the table in part A) we can also see that we use 54 litres of fuel to travel 900 km.

Alternatively, we can calculate how much fuel is required by using division and multiplication. We see that $900 \div 100 = 9$. This means we require 9 times as much fuel as it would take to travel 100 km. Since we know it takes 6 litres to travel 100 km, then it will take $9 \times 6 = 54$ litres of fuel to travel 900 km.

C) It will cost $2.00 \times 54 = $108.00 to buy the fuel necessary for the trip.
Teacher’s Notes

This problem is analogous to questions that deal with the relationship between velocity, distance and time. We could use an equation to describe the relationship between the rate of fuel consumption \( R \), the volume of fuel \( V \), and the distance travelled \( d \):

\[
R = \frac{V}{d}
\]

Given any two of the values for \( R \), \( V \), and \( d \) we can calculate the third value. Rearranging the equation we get:

\[
d = \frac{V}{R}
\]

or

\[
V = R \cdot d
\]

In this problem, we are given the rate \( \frac{6}{100} = 0.06 \).

In part A) we are given the volume and are asked to find the distance travelled. We can calculate that by substituting the known values into our equation for \( d \):

\[
d = \frac{42}{0.06} = 700
\]

In part B) we are given the distance travelled and asked to find the volume of fuel consumed. We can calculate that by substituting the known values into our equation for \( V \):

\[
V = 0.06 \cdot 900 = 54
\]

When we are able to describe these kinds of relationships with equations, we can calculate values quickly. Equations also allows us to use tools such as spreadsheets for quick calculations. However, understanding the relationship between these values is important. Creating a table can help solidify understanding this kind of relationship.
Problem of the Week

Problem A

Ups and Downs

Graham rides his bike to school. He takes a different route home. On the way to school he rides up a hill for 500 metres, then he rides on a flat section for 3 kilometres, and then he rides downhill for 1 kilometre. On his way home he rides on a flat section for 250 metres, then he rides uphill for 750 metres, followed by another flat section for 2 kilometres, then downhill for 2 kilometres, and finally on a flat section for 500 metres.

A) How far does Graham ride to and from school every day?

B) What fraction of the total distance travelled in one day is Graham riding downhill?

Strands  Number Sense and Numeration,  Measurement
Problem of the Week
Problem A and Solution
Ups and Downs

Problem
Graham rides his bike to school. He takes a different route home. On the way to school he rides up a hill for 500 metres, then he rides on a flat section for 3 kilometres, and then he rides downhill for 1 kilometre. On his way home he rides on a flat section for 250 metres, then he rides uphill for 750 metres, followed by another flat section for 2 kilometres, then downhill for 2 kilometres, and finally on a flat section for 500 metres.

A) How far does Graham ride to and from school every day?

B) What fraction of the total distance travelled in one day is Graham riding downhill?

Solution

A) We can use a number line to calculate the total distance Graham travelled. Each interval on this line represents 250 m.

From the number line we can see that Graham travels a total of 10 km.

We could also add the individual distances together to determine the total. It is probably easier to do this if all of the distances were measured with the same unit. So, converting all of the distances to metres, we get:

\[ 500 + 3000 + 1000 + 250 + 750 + 2000 + 2000 + 2000 + 500 = 10000 \]

and 10000 m is equal to 10 km.

B) Graham travels 1 km downhill on the way to school and 2 km downhill on the way home. This is a total of \( 1 + 2 = 3 \) km.

Since he travels a total of 10 km in one day, the downhill portion of the distance he travels is: \( \frac{3}{10} \).
Teacher’s Notes

In this solution we use a number line to accumulate the distance Graham travels. To make the number line a useful tool, we must make a good choice for the size of its intervals. A simple number line might have each interval be equal to one unit, however that scale does not work in this case. If we had each interval represent 1 km, then it would be hard to accurately accumulate fractional values like 250 m or 500 m. If we had each interval represent 1 m, then the number line would either be extremely long or the spaces between intervals would be extremely small. Even choosing 100 m as the distance between intervals would not be the best choice, since some of the accumulated values would land between the tick marks on the number line.

One way to pick the interval size is to consider the greatest common divisor or $GCD$ of the numbers that we are accumulating. The $GCD$ of a set of numbers is the largest integer that divides evenly into each of the numbers in the set. In this case, we have the numbers 500, 3000, 1000, 250, 750, and 2000. For all of these numbers, we get an integer result when we divide them by 250. By choosing an interval size that is a divisor of each of the numbers we are accumulating, we guarantee that accumulated values will always land on one of the tick marks of our number line. Choosing the interval size to be the greatest common divisor, means we have the fewest intervals necessary to guarantee accumulated values will always end up landing on a tick mark.

In this case, the $GCD$ also happens to be the smallest number in our set. However, that is not always the case. For example, the $GCD$ of the numbers 400, 2000, and 900 is 100. The $GCD$ will always be less than or equal to the smallest number in the set and greater than or equal to 1.
Problem of the Week
Problem A
Fancy Pattern

Elisabeth drew this fancy pattern with black, white and grey shading.

A) If the smallest black squares are 1 cm by 1 cm, what are the dimensions of the whole drawing?

B) What is the total area of the grey part of the drawing?
Problem of the Week
Problem A and Solution
Fancy Pattern

Problem
Elisabeth drew this fancy pattern with black, white and grey shading.

A) If the smallest black squares are 1 cm by 1 cm, what are the dimensions of the whole drawing?

B) What is the total area of the grey part of the drawing?

Solution

A) To determine the dimensions of the whole drawing, let’s focus on one corner section.

Since the white squares align horizontally and vertically with the black squares in the checkerboard pattern, then we know that the white squares are also 1 cm by 1 cm. This can also be written 1 cm × 1 cm. We will use this convention throughout the rest of the solution. In a single checkerboard, there are 3 squares across and 3 squares down. This means that the checkerboard sections themselves are 3 cm × 3 cm. And since the checkerboard sections are aligned, and there are 3 of these sections across and 3 down, then this whole corner section must be 9 cm × 9 cm.

This section is a repeated pattern in the whole drawing, and the sections are aligned horizontally and vertically. Since there are three sections across and three sections down, each of the dimensions of the whole drawing is 3 times the dimensions of this section.

Therefore the dimensions of the whole drawing are 27 cm × 27 cm.
B) One way to determine the area of the grey parts of the drawing would be to draw gridlines that align with the black and white squares in the checkerboard. The gridlines can be used to identify grey squares that are also 1 cm × 1 cm. Then we can count the number of those squares to determine the grey shaded area of the drawing.

Here is another way to calculate the area. Looking at the whole drawing, it can be divided up into nine sections. Five of those sections have black and white checkerboard patterns and smaller grey squares, like this:

and 4 of those sections are larger, solid grey squares with the same dimensions as the square shown above.

In part (A) we determined that the dimensions of this section are 9 cm × 9 cm, which means that the dimensions of the larger grey squares are also 9 cm × 9 cm. The area of one of those squares would be $9 \times 9 = 81 \text{ cm}^2$. This means the total area of those four squares is $81 + 81 + 81 + 81 = 81 \times 4 = 324 \text{ cm}^2$.

Within a section that contains the checkerboard patterns, there are nine subsections: five subsections that each contain the checkerboard pattern, and four subsections that are small, solid grey squares. In part (A) we determined that checkerboard subsection has dimensions $3 \text{ cm} \times 3 \text{ cm}$. So these small grey squares also have dimensions $3 \text{ cm} \times 3 \text{ cm}$, which means the area of one of those squares is $3 \times 3 = 9 \text{ cm}^2$. Since there are four smaller grey squares in one of these sections, then the grey part of one of these sections is $9 + 9 + 9 + 9 = 9 \times 4 = 36 \text{ cm}^2$. Since the section shown above is repeated five times in the entire drawing, the grey squares of these sections have a total area of $36 \times 5 = 180 \text{ cm}^2$.

Therefore the total area of the grey part of the drawing is $180 + 324 = 504 \text{ cm}^2$.

Another way to calculate the area of the grey part is to start with the area of the whole drawing and subtract the area of the black and white squares. The area of the whole drawing is $27 \times 27 = 729 \text{ cm}^2$. We know the dimensions of the black and white squares are 1 cm × 1 cm, so their areas are each 1 cm$^2$. We can count the total number of black and white squares and see that their are 225 of them. So the total area of the grey part of the drawing is $729 - 225 = 504 \text{ cm}^2$. 
Teacher’s Notes

The fancy pattern Elisabeth drew is an example of a fractal. Fractals occur in mathematics, art, and nature. The essential characteristic of a fractal is that it has a repeating pattern, within its pattern. In this example, we can break down the whole image into a square containing nine squares. The basic pattern of the whole image has the following structure:

<table>
<thead>
<tr>
<th>pattern</th>
<th>solid</th>
<th>pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>solid</td>
<td>pattern</td>
<td>solid</td>
</tr>
<tr>
<td>pattern</td>
<td>solid</td>
<td>pattern</td>
</tr>
</tbody>
</table>

If you look at the pattern sections of the whole image, they can also be described as a square, containing nine squares, with the structure described above. Finally, each of these patterned sections can be described as a square that contains nine squares with the following similar structure:

<table>
<thead>
<tr>
<th>black</th>
<th>white</th>
<th>black</th>
</tr>
</thead>
<tbody>
<tr>
<td>white</td>
<td>black</td>
<td>white</td>
</tr>
<tr>
<td>black</td>
<td>white</td>
<td>black</td>
</tr>
</tbody>
</table>

The beauty of fractals, is that you can continue this structured repetition to more and more levels. You can also represent the fractal with a mathematical model. The next couple of pages show the fractal from this problem repeated to deeper levels. All of the images in this problem were actually generated by a relatively short computer program. We used the same code to create different images, by just changing the value of one variable to indicate how many levels of repetition we wanted.
Fractal with Four Levels
Fractal with Five Levels
Number Sense & Numeration
Problem of the Week
Problem A
Circle of Calculation

Determine the numbers that go in the boxes to make this diagram correct.
Problem of the Week
Problem A and Solution
Circle of Calculation

Problem
Determine the numbers that go in the boxes to make this diagram correct.

Solution
Since the circle started with the number 8, this is the solution.
Teacher’s Notes

Although there is only one correct solution using the starting number 8, this circle will work given any rational number in any one of the boxes as a starting value. In other words, no matter what number you start with, if you follow the operations in order, the application of the last operation will result in the starting number. For example, suppose you start with the number 50 in the box on the left side of the circle. The calculations would be as follows:

\[
\begin{align*}
50 \div 5 &= 10 \\
10 \times 3 &= 30 \\
30 \div 6 &= 5 \\
5 \times 10 &= 50
\end{align*}
\]

The circle will work with non-integer numbers as well. For example, if you start with the number 5 at the top of the circle, here are the calculations:

\[
\begin{align*}
5 \times 3 &= 15 \\
15 \div 6 &= \frac{5}{2} \\
\frac{5}{2} \times 10 &= 25 \\
25 \div 5 &= 5
\end{align*}
\]

The reason this works is that multiplication and division are inverse functions. This means that if you multiply a number by some value and divide the result by the same value you end up with the original number. Also, if you divide a number by some value and then multiply the result by the same value you end up with the original number. (The exception here is if you try to divide a number by 0, since the result of dividing by 0 is undefined.)

It is a bit harder to explain why the circle works, because it involves more than one multiplication and division operation. We can see that overall we multiply by 3 and 10 which results in a multiple of 30 and we divide by 6 and 5 which results in a fraction of 30. However, we should be careful about the order in which we apply operations, especially division, The circle could be considered a composition of functions, which means we apply a function to the result of another function. It would be possible to show that the circle works by showing the result of a composition of four functions. Informally, we can see that the circle works by using a variable, rather than a number as a starting value. If we start with a variable \( k \) at the top of the circle, here are the calculations:

\[
\begin{align*}
k \times 3 &= 3k \\
3k \div 6 &= \frac{k}{2} \\
\frac{k}{2} \times 10 &= 5k \\
5k \div 5 &= k
\end{align*}
\]
Problem of the Week
Problem A
Repairing Computers

In Eris’ repair shop, she is usually able to fix seven computers in a regular 8-hour day. Her shop is open Monday through Friday, 9:00 a.m. until 5:00 p.m., and Saturday 9:00 a.m. until 1:00 p.m..

Estimate how many computers is Eris able to fix each week. Justify your thinking.
Problem of the Week
Problem A and Solution
Repairing Computers

Problem
In Eris’ repair shop, she is usually able to fix seven computers in a regular 8-hour day. Her shop is open Monday through Friday, 9:00 a.m. until 5:00 p.m., and Saturday 9:00 a.m. until 1:00 p.m..

Estimate how many computers is Eris able to fix each week. Justify your thinking.

Solution
During the weekdays, Eris is working 8 hours. On Monday through Friday, she would be expected to fix: $7 + 7 + 7 + 7 + 7 = 7 \times 5 = 35$ computers. On Saturday she works from 9:00 a.m. until 1:00 p.m., which is a total of 4 hours. Since $4 \div 8 = \frac{1}{2}$, we expect Eris to be able to fix approximately half as many computers on Saturday as she does on a weekday. Unfortunately, half of 7 is not a whole number, and she can only fix whole computers. The closest whole numbers to “half of 7” are 3 and 4. So we expect Eris to fix either 3 or 4 computers that day. We add this to the number she is expected to fix during the week. This means we estimate she fixes $35 + 3 = 38$ computers or $35 + 4 = 39$ computers each week.

There are many factors which may be considered that might make us think it is more likely that she is more likely to fix 3 computers rather than 4 computers or vice versa on a Saturday. For example, we have calculated that Eris is expected to fix $3\frac{1}{2}$ computers in a 4-hour day. However, fixing a $\frac{1}{2}$ a computer means the computer is still broken. This means we should think of this computer as not fixed, so it should not be counted.

Another factor to consider is that we assume that Eris takes a lunch break during an 8-hour day. If she does not take that break on Saturday, that means she has more time to fix computers. If she does take that break on a Saturday, then that is a larger portion of the time her shop is open that is not being used for fixing computers.
Teacher’s Notes

The problem statement, “... she is usually able to fix seven computers in a regular 8-hour day” is essentially describing an *expected value*. In statistics, an expected value is the average that has been determined based on data gathered over a long period of time. On any given day, Eris may fix more or less than 7 computers. Notice that the expected value for the number of computers that she can fix on a Saturday is 3.5 machines. Since computers being fixed is a binary state, (they are either broken or they are fixed), Eris cannot actually fix half a computer. So an expected value is not necessary an actual value that can be achieved in the real life situation.

We often use expected values to make decisions. However, when the decision is related to a short-term outcome we may not get the result we want. For example, suppose there is a good strategy for playing a game where you are expected to win 75% of the time if you use it. This means if you play the game 1000 times, you will probably win approximately 750 times. However, when you are playing a single one of those games, there is no guarantee that choosing the good strategy will lead you to a win. When people evaluate results like this in the short term, they may incorrectly conclude that the strategy is ineffective or incorrect. However, mathematicians know that the real payoff is in the long term application of that strategy.
Stella wants to plant some vegetables in her garden so her family can enjoy them. She has five types of plants in the garden: beans, peas, carrots, lettuce, and tomatoes. She has four more bean plants than tomato plants. She has twice as many pea plants as lettuce plants. She has 10 fewer lettuce plants than carrot plants. She has the same number of bean plants as pea plants. Stella planted 16 carrot plants. How many of each type of plant does Stella have in her garden?
Problem of the Week
Problem A and Solution
Stella’s Vegetable Garden

Problem
Stella wants to plant some vegetables in her garden so her family can enjoy them. She has five types of plants in the garden: beans, peas, carrots, lettuce, and tomatoes. She has four more bean plants than tomato plants. She has twice as many pea plants as lettuce plants. She has 10 fewer lettuce plants than carrot plants. She has the same number of bean plants as pea plants. Stella planted 16 carrot plants. How many of each type of plant does Stella have in her garden?

Solution
We know the number of carrot plants without knowing any other information about the garden. All other numbers of plants are related to other the information about other plants. The number of lettuce plants can be determined given the number of carrots, so we can calculate it relatively easily. There are $16 - 10 = 6$ lettuce plants.

Knowing the number of lettuce plants, we can calculate the number of pea plants. There are $6 \times 2 = 12$ pea plants. Since she has an equal number of pea plants and bean plants, she also has 12 bean plants.

If Stella has four more bean plants than tomato plants, that is the same as saying she has four fewer tomato plants than bean plants. Knowing that she has 12 bean plants, she must have $12 - 4 = 8$ tomato plants.

In summary, Stella’s garden contains:

- 12 bean plants
- 12 pea plants
- 16 carrot plants
- 6 lettuce plants
- 8 tomato plants
Teacher’s Notes

It is possible to solve this problem algebraically. We could assign variables to each of the types of plants and then set up a system of equations that reflects all of the relationships between the numbers of each of the plants. Then we could solve the equations to determine how many of each plant are in the garden. For example if we use variables $b, p, c, l, t$ for the number of beans, peas, carrots, lettuce, and tomatoes, we can describe the relationships as follows:

$$
\begin{align*}
    t + 4 &= b \\
    2 \times l &= p \\
    c - 10 &= l \\
    b &= p \\
    c &= 16
\end{align*}
$$

There are many algebraic techniques we could use to solve the equations and determine the values for each of the variables. We could also describe the relationships in such a way so that each different variable appears by itself on one side of the equation as follows:

$$
\begin{align*}
    t &= b - 4 \\
    p &= 2 \times l \\
    l &= c - 10 \\
    b &= p \\
    c &= 16
\end{align*}
$$

Using these relationships we can set up formulae in a spreadsheet like this:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Type</td>
<td>Number</td>
</tr>
<tr>
<td>2</td>
<td>Tomatoes</td>
<td>=B5 - 4</td>
</tr>
<tr>
<td>3</td>
<td>Peas</td>
<td>=2 * B4</td>
</tr>
<tr>
<td>4</td>
<td>Lettuce</td>
<td>=B6 - 10</td>
</tr>
<tr>
<td>5</td>
<td>Beans</td>
<td>=B3</td>
</tr>
<tr>
<td>6</td>
<td>Carrots</td>
<td>16</td>
</tr>
</tbody>
</table>

Using this setup, a spreadsheet will show you the same results for each type of plant that we calculated in our solution.
Problem of the Week
Problem A
Dining Dilemma

We want to start a new restaurant. We have square tables that allow one chair on each side. Therefore, we can arrange four chairs around each table.

A) If the restaurant has 32 tables, how many chairs do we need to buy?

B) As we set up the restaurant, we put out one table at a time with its full set of chairs surrounding it. If we have put out 36 chairs, how many tables have been set up so far?

C) When we have banquets we sometimes need to push the tables together. This changes the amount of chairs we can put around the table grouping, as shown in the following picture.

How many chairs are required if we set up the 32 tables in pairs?

D) How would the answer to part (C) change if we group 8 tables end to end, and still use all 32 tables?

**Strands**  Patterning and Algebra, Number Sense and Numeration
Problem of the Week
Problem A and Solution
Dining Dilemma

Problem
We want to start a new restaurant. We have square tables that allow one chair on each side. Therefore, we can arrange four chairs around each table.

A) If the restaurant has 32 tables, how many chairs do we need to buy?

B) As we set up the restaurant, we put out one table at a time with its full set of chairs surrounding it. If we have put out 36 chairs, how many tables have been set up so far?

C) When we have banquets we sometimes need to push the tables together. This changes the amount of chairs we can put around the table grouping, as shown in the following picture.

How many chairs are required if we set up the 32 tables in pairs?

D) How would the answer to part (C) change if we group 8 tables end to end, and still use all 32 tables?
Solution

A) We can use multiplication to calculate the number of chairs: $32 \times 4 = 128$.

We might also notice that multiplying by 4 is the same as doubling a number and then doubling the answer. For example, we can find the answer to $32 \times 4$ by calculating $32 \times 2 = 64$ and then calculating $64 \times 2 = 128$.

B) We can use skip counting to find out how many tables are out.
We skip count by 4 to count the chairs: 4, 8, 12, 16, 20, 24, 28, 32, 36. This means there are 9 tables set up so far. We could have also used division to calculate this answer: $36 \div 4 = 9$.

C) We can make a table showing the relationship between the number of tables and the number of chairs using this configuration. Each cluster of 2 tables is surrounded by 6 chairs. So we add 2 to the number of tables from one row to the next, and we add 6 to the number of chairs from one row to the next.

<table>
<thead>
<tr>
<th>Number of Tables</th>
<th>Number of Chairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>12</td>
<td>36</td>
</tr>
<tr>
<td>14</td>
<td>42</td>
</tr>
<tr>
<td>16</td>
<td>48</td>
</tr>
<tr>
<td>18</td>
<td>54</td>
</tr>
<tr>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>22</td>
<td>66</td>
</tr>
<tr>
<td>24</td>
<td>72</td>
</tr>
<tr>
<td>26</td>
<td>78</td>
</tr>
<tr>
<td>28</td>
<td>84</td>
</tr>
<tr>
<td>30</td>
<td>90</td>
</tr>
<tr>
<td>32</td>
<td>96</td>
</tr>
</tbody>
</table>

Another way to calculate this result is as follows:

Since there are 32 tables in total, there are $32 \div 2 = 16$ pairs of tables. In this configuration, there 6 chairs around each pair of tables. Therefore, there are a total of $16 \times 6 = 96$ chairs required for this set up.
D) With 8 tables arranged end to end, there will be one chair on each end, and eight chairs on each side. This is a total of $1 + 1 + 8 + 8 = 18$ chairs around the tables.

We can make a new table showing the relationship between the number of tables and the number of chairs using this configuration. In this case we add 8 to the number of tables from one row to the next, and we add 18 to the number of chairs from one row to the next.

<table>
<thead>
<tr>
<th>Number of Tables</th>
<th>Number of Chairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>18</td>
</tr>
<tr>
<td>16</td>
<td>36</td>
</tr>
<tr>
<td>24</td>
<td>54</td>
</tr>
<tr>
<td>32</td>
<td>72</td>
</tr>
</tbody>
</table>

Another way to calculate this result is as follows:
Since there are 32 tables in total, there are $32 \div 8 = 4$ sets of 8 tables. In this configuration, there 18 chairs around each pair of tables. Therefore, there are a total of $18 \times 4 = 72$ chairs required for this set up.
Teacher’s Notes

Part of the solutions for parts C and D can be described as a table of values (no pun intended). A table of values can be used to list specific values that are described by a function. We write functions in a variety of ways. For example, we can write a function describing the values in part D in the form of an equation that uses a variable \( c \) to represent the number of chairs and a variable \( t \) that represents the number of tables. This equation would be:

\[
c = \frac{t}{8} \times 18
\]

Another way of describing this function uses \( x \) to represent the number of tables and \( f(x) \) to represent the number of chairs. The function that uses this notation would be:

\[
f(x) = \frac{x}{8} \times 18
\]

Normally when we use these formats to describe functions, we expect that the variables in the equations represent many possible numbers - sometimes infinitely many numbers. However, for this problem there is a finite set of possible values for the number of tables we arrange in the restaurant. In particular, we only consider groupings of 8, 16, 24, or 32 tables. So numbers like 1, 2, 3, ..., 7, 9, 10 and so on are not actual values that would be used for \( t \) or \( x \), in the functions we have written.

Another way we describe functions is called a mapping. This is a visual representation of the relationship between two sets of numbers. For example, we use a mapping like this to describe the values in part D of this problem:

![Mapping Diagram]

In a mapping, an oval represents a set of values. In this example, the oval on the left represents the set of values representing the number of tables we may group together in this problem. The oval on the right represents the set of values that are the related number of chairs. Values in the set on the left are connected, using an arrow, to a particular value in the set on the right. Altogether, this picture represents a function that describes the relationship between 8, 16, 24, or 32 tables and the number of chairs required in each case. A mapping is a nice way to represent a function with a finite set of values.
Problem of the Week
Problem A
Purchasing Pencil Crayons

Mrs. Zhang needs to buy pencil crayons for her classroom. A local store is running a sale where the shopper will receive a free box of pencil crayons for every 4 boxes purchased. Her teaching partner, Mr. Holland also mentioned that he needs 5 more boxes of pencil crayons for his classroom. How many boxes of pencil crayons must Mrs. Zhang purchase in order to get 5 free boxes to give to Mr. Holland?
Problem of the Week
Problem A and Solution
Purchasing Pencil Crayons

Problem
Mrs. Zhang needs to buy pencil crayons for her classroom. A local store is running a sale where the shopper will receive a free box of pencil crayons for every 4 boxes purchased. Her teaching partner, Mr. Holland also mentioned that he needs 5 more boxes of pencil crayons for his classroom. How many boxes of pencil crayons must Mrs. Zhang purchase in order to get 5 free boxes to give to Mr. Holland?

Solution
We can use a table to show how many free boxes Mrs. Zhang receives as she buys her pencil crayons. On each row, we increase the number of boxes purchased by 4 and the number of free boxes by 1.

<table>
<thead>
<tr>
<th>Boxes Purchased</th>
<th>Free Boxes</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
</tr>
</tbody>
</table>

From the table we see that Mrs. Zhang would need to buy 20 boxes of pencil crayons in order to get 5 free boxes.

Alternatively we can determine the result this way. In order to get the pencil crayons that Mr. Holland needs, Mrs. Zhang needs to buy 5 sets of 4 boxes. This means that she needs to buy a total of $5 \times 4 = 20$ boxes of pencil crayons.
Teacher’s Notes

We can describe the relationship between the number of boxes Mrs. Zhang bought and the number of boxes she got for free in a number of ways.

The ratios $4 : 1$ and $20 : 5$ show the relative number of paid boxes to the number of free boxes. Since these two ratios show the same relationship, they are said to be proportional.

The fraction of free boxes in the total that Mrs. Zhang acquired is $\frac{5}{25}$ or $\frac{1}{5}$. These are examples of equivalent fractions.

The percentage of free boxes is 20% which can also be written as the decimal number 0.2 or the fraction $\frac{20}{100}$. The term percent, literally means per 100.

The less well know term per mille is used to describe a fraction out of 1000. This is written using the symbol %₀. So, 2%₀ of the boxes acquired were free.
Problem of the Week
Problem A
Holiday Lights

Hunter needs exactly 6 metres of twinkle lights to decorate the roof-line of his house for the holidays. He has 3 strings of lights. The first string of lights covers 250 cm.

Which of the following options could represent the lengths of the other two strings of lights? Explain your thinking.

a) 175 cm and 175 cm
b) 150 cm and 150 cm
c) 180 cm and 170 cm
d) 150 cm and 200 cm
e) More than two of the above options are possibilities.
f) None of the options above are possibilities.

Strands Measurement, Number Sense and Numeration
Problem of the Week
Problem A and Solution
Holiday Lights

Problem
Hunter needs exactly 6 metres of twinkle lights to decorate the roof-line of his house for the holidays. He has 3 strings of lights. The first string of lights covers 250 cm.

Which of the following options could represent the lengths of the other two strings of lights? Explain your thinking.

a) 175 cm and 175 cm
b) 150 cm and 150 cm
c) 180 cm and 170 cm
d) 150 cm and 200 cm
e) More than two of the above options are possibilities.
f) None of the options above are possibilities.

Solution
We know that 1 m = 100 cm, so 6 m = $6 \times 100 = 600$ cm. Since Hunter already has a 250 cm string of lights, he needs the other two strings to cover the remaining $600 - 250 = 350$ cm of his roof-line. We can check the four pairs of lengths to see how many of them total 350 cm.

a) $175 + 175 = 350$ cm
b) $150 + 150 = 300$ cm
c) $180 + 170 = 350$ cm
d) $150 + 200 = 350$ cm

From these calculations, we see that the total for three of these choices give us the required 350 cm. So the correct answer to the original question is e), since three of the first four options would work.

Alternatively, we could add 250 cm to each of the pairs of lengths listed in the problem to see which triples would give us a total 600 cm.

a) $175 + 175 + 250 = 600$ cm
b) $150 + 150 + 250 = 550$ cm
c) $180 + 170 + 250 = 600$ cm
d) $150 + 200 + 250 = 600$ cm

We see that three of the four possibilities provide the required 6 m. So the correct answer to the original question is e), since three of the first four options would work.
Teacher’s Notes

We can break up numbers into smaller units in a variety of ways. A *factorization* of an integer is a combination of positive integers (*factors*) that can be multiplied together to produce the original value. For example, we can factor 6 in the following ways:

\[
\begin{align*}
1 \times 6 \\
2 \times 3
\end{align*}
\]

The Holiday Lights problem essentially asks students to identify *partitions* of 600. A partition is a combination of positive integers (*parts*) that can be added together to produce the original value. For example, we can partition 6 in the following ways:

\[
\begin{align*}
1 + 1 + 1 + 1 + 1 + 1 & \quad 1 + 1 + 4 \\
1 + 1 + 1 + 1 + 2 & \quad 1 + 5 \\
1 + 1 + 1 + 3 & \quad 1 + 1 + 2 + 2
\end{align*}
\]

\[
\begin{align*}
1 + 1 + 1 + 1 + 2 & \quad 1 + 2 + 3 \\
1 + 5 & \quad 2 + 2 + 2 \\
2 + 4 & \quad 3 + 3
\end{align*}
\]

Multiplication and addition are commutative operations, meaning the order of the operands does not affect the outcome of the operation. So, when we write factorizations or partitions of numbers, we do not include variations that are the same set of integers. For example, \(1 + 5\) and \(5 + 1\) are the same partition.

We can visualize a partition using circles. To represent a partition of a number \(n\), we arrange the \(n\) circles into rows. Each row represents a part in the partition. This is called a *Ferrers diagram*. For example here is a diagram of one partition of the number 11:

\[
\begin{align*}
\circ & \quad \circ \quad \circ \quad \circ \\
\circ & \quad \circ \\
\circ & \\
\circ
\end{align*}
\]

This represents the partition \(5 + 3 + 2 + 1\) since the first row has 5 circles, the second has 3 circles, the third has 2 circles, and the fourth has 1 circle. We can easily find another partition of 11 by rotating this diagram by 90 degrees.

\[
\begin{align*}
\circ & \\
\circ \\
\circ & \quad \circ \\
\circ & \quad \circ \\
\circ & \quad \circ & \quad \circ & \quad \circ
\end{align*}
\]

This diagram represents the partition \(1 + 1 + 2 + 3 + 4\). These two partitions are said to be *conjugates* of one another.

We can ask many different questions about partitions. We could ask for all of the partitions of a number or some subset. For example, in our problem the partitions we considered were restricted to those that included 250. We could ask for partitions that did not include any duplicate parts or partitions that only included odd numbers. The possibilities are endless.
Problem of the Week
Problem A
Flowers for Volunteers

To thank the volunteers at school, the school council would like to give 5 flowers to each volunteer. There are 15 volunteers to thank. You cannot buy individual flowers; they are only sold by the dozen.

A) How many dozens will the school council need to buy?

B) If one dozen flowers costs $10.00, how much will the school council need to spend on flowers?
Problem of the Week
Problem A and Solution
Flowers for Volunteers

Problem
To thank the volunteers at school, the school council would like to give 5 flowers to each volunteer. There are 15 volunteers to thank. You cannot buy individual flowers; they are only sold by the dozen.

A) How many dozens will the school council need to buy?
B) If one dozen flowers costs $10.00, how much will the school council need to spend on flowers?

Solution
A) Since the school council wants to give each volunteer 5 flowers, then they need a total of \(15 \times 5 = 75\) flowers. Alternatively, we can skip count by 5: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75 to calculate this total.
To calculate how many dozen flowers we need, we could use repeated subtraction of 12, starting with 75:

\[
\begin{align*}
75 - 12 &= 63 \\
63 - 12 &= 51 \\
51 - 12 &= 39 \\
39 - 12 &= 27 \\
27 - 12 &= 15 \\
15 - 12 &= 3
\end{align*}
\]

So after subtracting 12, six times, we still have 3 flowers to buy. Since the smallest number of flowers we can buy is 12, we need to buy a total of 7 dozen flowers.

Alternatively, we can skip count by 12 until we get a number greater than or equal to 75: 12, 24, 36, 48, 60, 72, 84. This also shows us that we need at least 7 dozen flowers to have enough for the volunteers.

Also we can use division to see that \(75 \div 12 = 6\) remainder 3. Again, this means that we need 7 dozen flowers.

B) Since flowers cost $10.00 per dozen, then the school council must spend \(7 \times 10.00 = $70.00\) to buy enough flowers for the volunteers.
**Teacher’s Notes**

Dividing integers results in a two part answer: the quotient and the remainder. For example, $38 \div 11 = 3$ with a remainder of 5. This is an exact result. If we use a calculator to do division, we may end up with an inexact result. For example, the result of dividing 38 by 11 appears as 3.45454545 on a calculator. This is an approximation. The exact answer has the digits 45 repeating forever.

When computers calculate the result of division with integers, the quotient and remainder are stored separately. This means that it is possible to access those two parts separately. Most computer languages have commands that allow you to access each of the parts. They also have commands that allow you to generate integer values from non-integer results. For example, you may be able to round off a number (to the nearest integer), round down a number (often referred to as finding the floor) or round up a number (often referred to as finding the ceiling).

If we want to see single result of a division where the decimal representation has infinitely repeating decimals, a computer can only display a finite number of digits. These types of results are converted into a different format, usually known as floating point numbers. Since this value is an inexact result of a calculation, small errors can appear when we work with this kind of number. You might see the result of working with inexact data when you expect to see a result like 4 after doing some calculations on your computer, but you see the number 3.9999999999 or 4.0000000001 instead.
Henri is saving money to purchase a new tablet. The tablet he wishes to buy costs $260.00. Henri started with $65.00 in his bank account. Last week, he received $100.00 from his grandmother for his birthday and an additional $25.00 from his uncle. He deposited his birthday money into his bank account.

A) After depositing his birthday gifts, how much money does Henri have in his bank account?

B) Henri earns a weekly allowance of $8.00. How many weeks will it take Henri to save enough money from his allowance to be able to purchase the tablet?

Show your thinking.
Problem of the Week
Problem A and Solution
Saving to Surf

Problem
Henri is saving money to purchase a new tablet. The tablet he wishes to buy costs $260.00. Henri started with $65.00 in his bank account. Last week, he received $100.00 from his grandmother for his birthday and an additional $25.00 from his uncle. He deposited his birthday money into his bank account.

A) After depositing his birthday gifts, how much money does Henri have in his bank account?

B) Henri earns a weekly allowance of $8.00. How many weeks will it take Henri to save enough money from his allowance to be able to purchase the tablet?

Show your thinking.

Solution

A) Henri received a total of $100.00 + $25.00 = $125.00 for his birthday. After he deposited his birthday money, Henri had $65.00 + $125.00 = $190.00 in the bank.

B) We can make a table to determine how much money Henri saves:

<table>
<thead>
<tr>
<th>Week</th>
<th>Total Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$190.00 + $8.00 = $198.00</td>
</tr>
<tr>
<td>2</td>
<td>$198.00 + $8.00 = $206.00</td>
</tr>
<tr>
<td>3</td>
<td>$206.00 + $8.00 = $214.00</td>
</tr>
<tr>
<td>4</td>
<td>$214.00 + $8.00 = $222.00</td>
</tr>
<tr>
<td>5</td>
<td>$222.00 + $8.00 = $230.00</td>
</tr>
<tr>
<td>6</td>
<td>$230.00 + $8.00 = $238.00</td>
</tr>
<tr>
<td>7</td>
<td>$238.00 + $8.00 = $246.00</td>
</tr>
<tr>
<td>8</td>
<td>$246.00 + $8.00 = $254.00</td>
</tr>
<tr>
<td>9</td>
<td>$254.00 + $8.00 = $262.00</td>
</tr>
</tbody>
</table>

After nine weeks, Henri would have saved enough of his allowance in combination with the money in his bank account so that he can buy the tablet.
This problem can be solved algebraically. We can write an equation that shows the relationship between the time that has passed and the money that has been saved. Let \( w \) represent the number of weeks Henri has been saving money since his birthday. Let \( m \) represent the total amount of money that has been saved. We have calculated that there is $190.00 in the bank after depositing his birthday gifts. This is the equation that describes the relationship between weeks passed and money saved:

\[
m = 8w + 190
\]

We know that Henri needs $260.00 to buy the tablet, so we can substitute 260 in place of \( m \) in the equation.

\[
260 = 8w + 190
\]

Now we need to find out the value of \( w \). We subtract 190 from both sides of the equation.

\[
260 - 190 = 8w + 190 - 190
70 = 8w
\]

Then we divide both sides by 8.

\[
\frac{70}{8} = \frac{8w}{8}
8.75 = \frac{8w}{8}
8.75 = w
\]

This means Henri needs more than 8 weeks to save enough money for the tablet. He will have what he needs after 9 weeks of saving.
Problem of the Week

Problem A

A Total Profit

Penny Saver found a great deal on tote bags. She bought a case of 47 bags for $517. Penny sold 19 of them online for $12 each, and the rest she sold to a store for $20 each. How much money did Penny Saver make from selling all the tote bags?
Problem of the Week

Problem A and Solution

A Total Profit

Problem

Penny Saver found a great deal on tote bags. She bought a case of 47 bags for $517. Penny sold 19 of them online for $12 each, and the rest she sold to a store for $20 each. How much money did Penny Saver make from selling all the tote bags?

Solution

To calculate the amount of money Penny gets from selling the tote bags, we can multiply the number of tote bags by the sale price. For the online sales, we need to calculate $19 \times 12$. One way to calculate this product is to use friendly numbers. For example, calculate $20 \times 12$ instead. We can skip count by 20s:

$20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240$

to determine that this product is 240.

The total for 20 bags is $12$ more (the price of 1 bag) than the total for 19. Now we can calculate the original product: $19 \times 12 = 240 - 12 = 228$.

Next we need to calculate the total Penny received from the store sales. She sold $47 - 19 = 28$ bags in the stores. So the amount of money Penny receives from the store sales is $28 \times 20$. Again, we could use skip counting to determine that the product is $560$. Alternatively, we could use the friendly number 30 in a calculation, which is 2 more than 28. Since 2 bags sold in the stores would be a total of $40$, the total amount made from selling 30 bags would be $40$ more than the amount made from selling 28 bags.

Using friendly numbers we can do the following calculations:

$30 \times 20 = 600$
$2 \times 20 = 40$
$28 \times 20 = 600 - 40 = 560$

The total amount collected by Penny in sales is: $228 + 560 = 788$. Since she invested $517$ initially, her total profit is: $788 - 517 = 271$. 
Teacher’s Notes

When we use friendly numbers to calculate products, we are taking advantage of the *distributive property* of multiplication. This property allows us to write equivalent expressions such as:

\[ a \times (b + c) = (a \times b) + (a \times c) \]

or

\[ a \times (b - c) = (a \times b) - (a \times c) \]

Keeping this property in mind, when we are multiplying two large numbers we can rewrite one of the factors into a series of terms separated by addition or subtraction operators.

For example, in this problem, we want to multiply \(19 \times 12\). We can rewrite this product in many ways.

We can replace 19 with \((20 - 1)\) and do this calculation:

\[
(20 - 1) \times 12 = (20 \times 12) - (1 \times 12) = 240 - 12 = 228
\]

We can rewrite 12 as \((10 + 2)\) and do this calculation:

\[
19 \times (10 + 2) = (19 \times 10) + (19 \times 2) = 190 + 38 = 228
\]

We can rewrite 12 as \((10 + 1 + 1)\) and do this calculation:

\[
19 \times (10 + 1 + 1) = (19 \times 10) + (19 \times 1) + (19 \times 1) = 190 + 19 + 19 = 228
\]

In all cases, we get the same result.
Alex, Ben, Gwen, and Jenna can hardly wait for the Winter Olympics. This year, they are lucky enough to attend the games in person, and they got tickets for several events. They have given each other nicknames as well. The four nicknames are Boss, Buzz, Cosmo, and Tiger. Using the following clues, determine the nickname of each of the children.

Clues:

- Jenna plans to watch the bobsled race with Tiger and drink hot chocolate with Buzz.
- Gwen will attend the ski jumping event with Tiger, and a hockey game with Boss.
- Cosmo, Tiger and Alex all have tickets for the opening ceremonies.
- Cosmo and Jenna are excited to see the figure skating competition.

A chart may be helpful in solving this problem.

<table>
<thead>
<tr>
<th>Nickname</th>
<th>Alex</th>
<th>Ben</th>
<th>Gwen</th>
<th>Jenna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boss</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Buzz</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cosmo</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tiger</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**STRAND** Logic
Problem
Alex, Ben, Gwen, and Jenna can hardly wait for the Winter Olympics. This year, they are lucky enough to attend the games in person, and they got tickets for several events. They have given each other nicknames as well. The four nicknames are Boss, Buzz, Cosmo, and Tiger. Using the following clues, determine the nickname of each of the children.

Clues:

- Jenna plans to watch the bobsled race with Tiger and drink hot chocolate with Buzz.
- Gwen will attend the ski jumping event with Tiger, and a hockey game with Boss.
- Cosmo, Tiger and Alex all have tickets for the opening ceremonies.
- Cosmo and Jenna are excited to see the figure skating competition.

A chart may be helpful in solving this problem.

Solution
Each clue eliminates at least one possible person/nickname pair. If Person A is doing something with Person B, then Person A and Person B must be different people. We can put an \( \times \) in the chart where we know someone does not match a nickname.

There are several ways to determine the correct nicknames; here is one way.

From this clue:

- Jenna plans to watch the bobsled race with Tiger and drink hot chocolate with Buzz.

We can update the table as follows:

<table>
<thead>
<tr>
<th>Nickname</th>
<th>Alex</th>
<th>Ben</th>
<th>Gwen</th>
<th>Jenna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boss</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Buzz</td>
<td></td>
<td></td>
<td>( \times )</td>
<td></td>
</tr>
<tr>
<td>Cosmo</td>
<td></td>
<td></td>
<td>( \times )</td>
<td></td>
</tr>
<tr>
<td>Tiger</td>
<td></td>
<td></td>
<td>( \times )</td>
<td></td>
</tr>
</tbody>
</table>
From these clues:

- Gwen will attend the ski jumping event with Tiger, and a hockey game with Boss.
- Cosmo, Tiger and Alex all have tickets for the opening ceremonies.
- Cosmo and Jenna are excited to see the figure skating competition.

We can update the table as follows:

<table>
<thead>
<tr>
<th>Nickname</th>
<th>Alex</th>
<th>Ben</th>
<th>Gwen</th>
<th>Jenna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boss</td>
<td>×</td>
<td></td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>Buzz</td>
<td></td>
<td></td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Cosmo</td>
<td>×</td>
<td></td>
<td></td>
<td>×</td>
</tr>
<tr>
<td>Tiger</td>
<td>×</td>
<td></td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>

At this point, we know that Jenna must have the nickname Boss, since this is the only nickname left in her column. We can put a check in that box, and eliminate Boss as the possible nickname for everyone else. Now the table looks like this:

<table>
<thead>
<tr>
<th>Nickname</th>
<th>Alex</th>
<th>Ben</th>
<th>Gwen</th>
<th>Jenna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boss</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>✔</td>
</tr>
<tr>
<td>Buzz</td>
<td>✔</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Cosmo</td>
<td>×</td>
<td></td>
<td></td>
<td>×</td>
</tr>
<tr>
<td>Tiger</td>
<td>×</td>
<td></td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>

Now we can conclude that Alex must have the nickname Buzz, since this is the only nickname left in his column. We can put a check in that box, and eliminate Buzz as the possible nickname for everyone else. Now the table looks like this:

<table>
<thead>
<tr>
<th>Nickname</th>
<th>Alex</th>
<th>Ben</th>
<th>Gwen</th>
<th>Jenna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boss</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>✔</td>
</tr>
<tr>
<td>Buzz</td>
<td>✔</td>
<td>×</td>
<td>×</td>
<td>✔</td>
</tr>
<tr>
<td>Cosmo</td>
<td>×</td>
<td></td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Tiger</td>
<td>×</td>
<td></td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>
Next we can conclude that *Gwen* must have the nickname *Cosmo*, since this is the only nickname left in her column. We can put a check in that box, and eliminate *Cosmo* as the possible nickname for *Ben*, who is the only person left. The only nickname left is *Tiger*. That must be *Ben’s* nickname. The final version of the table must be:

<table>
<thead>
<tr>
<th>Nickname</th>
<th>Alex</th>
<th>Ben</th>
<th>Gwen</th>
<th>Jenna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boss</td>
<td>✓</td>
<td>❌</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Buzz</td>
<td>✓</td>
<td>❌</td>
<td>❌</td>
<td>❌</td>
</tr>
<tr>
<td>Cosmo</td>
<td>❌</td>
<td>❌</td>
<td>✓</td>
<td>❌</td>
</tr>
<tr>
<td>Tiger</td>
<td>❌</td>
<td>✓</td>
<td>❌</td>
<td>❌</td>
</tr>
</tbody>
</table>

So Alex’s nickname is Buzz, Ben’s nickname is Tiger, Gwen’s nickname is Cosmo, and Jenna’s nickname is Boss.
Teacher’s Notes

Logical thinking is essential in the study of mathematics and computer science. Since the time of Aristotle, mathematicians have been describing formal ways of writing logical statements. One of the most commonly used systems for writing logical statements was developed by British mathematician George Boole in the mid-nineteenth century. Today we refer to AND, OR, and NOT as Boolean operators.

We use these words in English to make logical conclusions. For example in this problem, when we put an x in the chart to eliminate “Buzz” as a possible nickname for Jenna, we have recorded the logical fact that “Buzz” is NOT Jenna’s nickname.

If we revisit this version of the table:

<table>
<thead>
<tr>
<th>Nickname</th>
<th>Alex</th>
<th>Ben</th>
<th>Gwen</th>
<th>Jenna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boss</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Buzz</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Cosmo</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tiger</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
</tr>
</tbody>
</table>

we can describe Jenna’s nickname using a lesser known Boolean operator. We can say that Jenna’s nickname is “Boss” XOR “Cosmo”. This is more precise than saying Jenna’s nickname is “Boss” OR “Cosmo”, since A XOR B describes a situation where A is true or B is true but A and B cannot both be true. Since Jenna only has one nickname, it cannot be both “Boss” and “Cosmo”.


Problem of the Week
Problem A
Swimming Practice

Kunik takes swimming lessons at the community pool. A lap is the length of the pool, which is 25 m. During the lessons Kunik swims laps, from one end of the pool to the other end and back to the starting point. Kunik can swim 10 laps of the pool in 5 minutes if she is doing the front crawl, and she can swim 6 laps of the pool in 4 minutes if she is doing the backstroke.

A) How many metres can Kunik swim in 10 minutes, using the front crawl?

B) Approximately how long will it take Kunik to swim 60 laps of the pool if she uses the front crawl for half the laps and the backstroke for half the laps?

C) If Kunik wants to swim faster when she is in the pool, should she do the front crawl or the backstroke? Justify your answer.
Problem of the Week
Problem A and Solution
Swimming Practice

Problem
Kunik takes swimming lessons at the community pool. A lap is the length of the pool, which is 25 m. During the lessons Kunik swims laps, from one end of the pool to the other end and back to the starting point. Kunik can swim 10 laps of the pool in 5 minutes if she is doing the front crawl, and she can swim 6 laps of the pool in 4 minutes if she is doing the backstroke.

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C) If Kunik wants to swim faster when she is in the pool, should she do the front crawl or the backstroke? Justify your answer.

Solution

A) Notice that 10 minutes is twice as much as 5 minutes. Since Kunik can swim 10 laps in 5 minutes using the front crawl, if we double the amount of time in the pool, we expect her to swim double the number of laps. Since double the time is $2 \times 5 = 10$ minutes, then we expect her to swim $2 \times 10 = 20$ laps in that time. Since each lap is 25 m, Kunik can swim $25 \times 20 = 500$ m in 10 minutes.
B) Half of 60 is 30 since $30 + 30 = 60$, so Kunik will be swimming 30 laps using the front crawl and 30 laps using the backstroke. For each stroke, we can use a table to determine how long it takes Kunik to swim 30 laps.

**Table for front crawl**

<table>
<thead>
<tr>
<th>Minutes</th>
<th>Laps</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>15</td>
<td>30</td>
</tr>
</tbody>
</table>

**Table for backstroke**

<table>
<thead>
<tr>
<th>Minutes</th>
<th>Laps</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>16</td>
<td>24</td>
</tr>
<tr>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

From the tables, we expect it will take Kunik $15 + 20 = 35$ minutes to swim 60 laps if she uses each stroke half of the time.

C) From the tables we notice that it takes Kunik 15 minutes to swim 30 laps using the front crawl, and it takes her 20 minutes to swim the 30 laps using the backstroke. Since it takes longer for her to swim the same number of laps using the backstroke, she should do the front crawl if she wants to swim faster.
Teacher’s Notes

This problem could be solved using the rates of speed. The formula for calculating speed is:

\[ \text{speed} = \frac{\text{distance}}{\text{time}} \]

Since Kunik swims 10 laps of the pool in 5 minutes, then her front crawl speed is:

\[ \frac{250}{5} = 50 \text{ metres/min} \]

Since she swims 6 laps of the pool in 4 minutes, then her backstroke speed is:

\[ \frac{150}{4} = 37.5 \text{ metres/min} \]

Based on these calculations, we can see that Kunik’s rate of speed is faster doing the front crawl.

If we want to find out the time of an activity given the speed and distance, we can rearrange the formula this way:

\[ \text{time} = \frac{\text{distance}}{\text{speed}} \]

To calculate her time in part B) of the question we can substitute the distances and speeds into our formula for finding the time. The distance she swims using each stroke is 750 m.

\[ \text{time} = \frac{750 \text{ metres}}{50 \frac{\text{metres}}{\text{min}}} + \frac{750 \text{ metres}}{37.5 \frac{\text{metres}}{\text{min}}} = 15 \text{ minutes} + 20 \text{ minutes} = 35 \text{ minutes} \]

Notice the units in the calculation of the time. The distance unit (metres) appears in the numerator and denominator of the fraction. These units cancel each other. The time unit (min) appears in the denominator of the unit of speed in the denominator of the fraction. This means that the units we are calculating in the end are minutes. Keeping track of the units in your calculations that involve speed, time, and distance is a good way to check that you are working with the proper formula.
Problem of the Week
Problem A
How Many Players?

Robbie likes to play many different card games. A deck has 52 cards made up of: aces, twos, threes, fours, fives, sixes, sevens, eights, nines, tens, jacks, queens, and kings. There are four of each of these values in the deck. When you deal the cards, you give each player the same number of cards to start. In some games you might deal all of the cards. In other games, you might deal some of the cards and have some cards left over. Answer the following questions about different card games Robbie plays.

A) **Bridge** is a game with four players. You deal all of the cards in the deck. How many cards does each player get?

B) In the game **99**, each player starts with three cards. How many cards are left over after dealing if there are six people playing the game?

C) Each player starts with eight cards in the game of **Crazy Eights**. To play the game, you must have at least 10 cards left over after dealing. What is the maximum number of players who can play **Crazy Eights** using these rules?

D) The game of **Euchre** does not use any of the cards with numbers 2, 3, 4, 5, 6, 7, or 8. How many cards does this game use?
Problem of the Week
Problem A and Solution
How Many Players?

Problem
Robbie likes to play many different card games. A deck has 52 cards made up of: aces, twos, threes, fours, fives, sixes, sevens, eights, nines, tens, jacks, queens, and kings. There are four of each of these values in the deck. When you deal the cards, you give each player the same number of cards to start. In some games you might deal all of the cards. In other games, you might deal some of the cards and have some cards left over. Answer the following questions about different card games Robbie plays.

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C) Each player starts with eight cards in the game of **Crazy Eights**. To play the game, you must have at least 10 cards left over after dealing. What is the maximum number of players who can play **Crazy Eights** using these rules?

D) The game of **Euchre** does not use any of the cards with numbers 2, 3, 4, 5, 6, 7, or 8. How many cards does this game use?

Solution

A) We could use skip counting or repeated subtraction by fours to see how many cards each player gets from the 52 card deck. Alternatively, we can calculate the result by dividing: $52 \div 4 = 13$. Using any of these techniques, we see that each player gets 13 cards.

B) Since each of the 6 players starts with 3 cards, we can multiply $6 \times 3 = 18$ to see that there are 18 cards dealt at the beginning of the game. Since there are 52 cards in the deck, then after dealing there would be $52 - 18 = 34$ cards left over.
C) We can use repeated subtraction of 8, starting with 52, to calculate this answer:

\[
\begin{align*}
52 - 8 &= 44 \\
44 - 8 &= 36 \\
36 - 8 &= 28 \\
28 - 8 &= 20 \\
20 - 8 &= 12 \\
12 - 8 &= 4 \\
\end{align*}
\]

When we subtract 8 repeatedly 5 times, we get 12. When we subtract 8 repeatedly 6 times, we get 4. Since we need at least 10 cards left after dealing the cards, we can have a maximum of 5 players in the game.

Alternatively, since we need at least 10 cards left after dealing, then we can deal a maximum of \(52 - 10 = 42\) cards to the players. Since each player needs 8 cards, we can calculate \(42 \div 8 = 5\) with a remainder of 2. Therefore, we can deal in a maximum of 5 players to start the game.

D) Since there are seven values of cards that are not used in this game, and there are four of each value in the deck, then there is a total of \(7 \times 4 = 28\) cards not used. Since there are 52 cards in the deck, then there is a total of \(52 - 28 = 24\) cards in play.

Alternatively, we could also notice that only the aces, nines, tens, jacks, queens, and kings are used in the game. This means that there are six values of cards that are used in this game, and there are four of each value in the deck. Therefore a total of \(6 \times 4 = 24\) cards are in play.
Teacher’s Notes

In part C) of this problem, Robbie needs “at least 10 cards left after dealing”. In this context, *at least* implies the mathematical operator $\geq$ (i.e. greater than or equal to). We could have written the same information this way:

$$\text{cards left over } \geq 10$$

Although we often think of solving math problems as finding a precise result, in many cases we are interested in finding a range of values. Part C) asks for the maximum number of players, but if we asked for the possible number of players the answer would have been 2, 3, 4, or 5. There are many ways to describe this range of answers. In each case it is important to make sure we clearly indicate that 2 and 5 must be included in the range and that 1 and 6 are not included. We should also make it clear that our range in this case are only integer values, since when we use operators like $\geq$ or $<$ we could be including other real numbers such as fractions.

Here are a few different ways to describe our range of values:

- integers between 2 and 5 inclusive
- integers between 1 and 6 exclusive (meaning the boundaries of this range are not included)
- $2 \leq \text{number of players} \leq 5$, where the number of players is an integer
- Let $x$ represent the number of players. Then $x = (1, 5], x \in \mathbb{Z}$

Here is a breakdown of this mathematical expression. A round bracket at the beginning or end of a range indicates that value is not included. A square bracket at the beginning or end of a range indicates that value is included. The symbol $\in$ means “belongs to”. The symbol $\mathbb{Z}$ refers to the “set of integers”.


Problem of the Week
Problem A
Screen Time vs. Play Time

Xiao wanted to figure out how much time she spends doing activities outside compared to how much time she spends using a computer. She kept track of her activities over two days. This was what she recorded:

Day 1
A  8:30 a.m. to  9:00 a.m.  Played outside before school.
B  9:45 a.m. to 10:10 a.m. Used the computer during math class.
C  10:30 a.m. to 10:45 a.m. Played outside during recess.
D 12:30 p.m. to  1:00 p.m.  Played outside during lunch break.
E  1:20 p.m. to  1:40 p.m.  Used the computer during French class.
F  2:00 p.m. to  2:15 p.m.  Played outside during recess.
G  5:15 p.m. to  6:15 p.m. Did homework on the computer

Day 2
H  10:00 a.m. to 10:45 a.m. Went for a bike ride with her friends.
I  1:10 p.m. to  2:20 p.m. Went for a hike with her aunt.
J  3:30 p.m. to  5:15 p.m. Watched a movie on the computer.

Did Xiao spend more time outside or using the computer over the two days? Justify your answer.

Strands  Number Sense and Numeration, Measurement
Problem of the Week
Problem A and Solution
Screen Time vs. Play Time

Problem
Xiao wanted to figure out how much time she spends doing activities outside compared to how much time she spends using a computer. She kept track of her activities over two days. This was what she recorded:

**Day 1**
A 8:30 a.m. to 9:00 a.m. Played outside before school.
B 9:45 a.m. to 10:10 a.m. Used the computer during math class.
C 10:30 a.m. to 10:45 a.m. Played outside during recess.
D 12:30 p.m. to 1:00 p.m. Played outside during lunch break.
E 1:20 p.m. to 1:40 p.m. Used the computer during French class.
F 2:00 p.m. to 2:15 p.m. Played outside during recess.
G 5:15 p.m. to 6:15 p.m. Did homework on the computer

**Day 2**
H 10:00 a.m. to 10:45 a.m. Went for a bike ride with her friends.
I 1:10 p.m. to 2:20 p.m. Went for a hike with her aunt.
J 3:30 p.m. to 5:15 p.m. Watched a movie on the computer.

Did Xiao spend more time outside or using the computer over the two days? Justify your answer.

Solution
One way to determine the answer is to calculate the total number of minutes Xiao spent on each kind of activity. We can calculate the amount of time each activity takes by using a number line.

For example, to calculate how much time elapsed when Xiao used the computer during math class, we can create a number line representing the times from 9:00 a.m. until 11:00 a.m., showing intervals of 5 minutes each.

From this number line, we skip count by five: 5, 10, 15, 20, 25, to see that she spends 25 minutes on the computer during that time.
We could draw a number line to calculate each of the intervals that Xiao recorded. Here is a table that shows how many minutes she spent on the computer:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Minutes Elapsed</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>25</td>
</tr>
<tr>
<td>E</td>
<td>20</td>
</tr>
<tr>
<td>G</td>
<td>60</td>
</tr>
<tr>
<td>J</td>
<td>105</td>
</tr>
</tbody>
</table>

This is a total of $25 + 20 + 60 + 105 = 210$ minutes.

Here is a table that shows how many minutes she spent outside:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Minutes Elapsed</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>30</td>
</tr>
<tr>
<td>C</td>
<td>15</td>
</tr>
<tr>
<td>D</td>
<td>30</td>
</tr>
<tr>
<td>F</td>
<td>15</td>
</tr>
<tr>
<td>H</td>
<td>45</td>
</tr>
<tr>
<td>I</td>
<td>70</td>
</tr>
</tbody>
</table>

This is a total of $30 + 15 + 30 + 15 + 45 + 70 = 205$ minutes.

So Xiao spent 5 more minutes on the computer than she spent outside during those two days.
Teacher’s Notes

When we think about calculating elapsed time, we normally count the minutes from the start time until the end time. When we think about calculating the difference between two numbers we usually subtract the smaller number from the larger number. If we use a number line to calculate the time spent on each activity, we can do this by determining the length of the line segment between the time the activity started and the time it ended. It does not matter if we measure this length from the start to the end or from the end to the start. We refer to this length as the magnitude of the number.

In mathematics, if we only care about the size of a number, and not its sign, we would be interested in the absolute value of that number. The absolute value of a number is denoted with vertical bars like this: $|x|$. So $|-4| = 4$ and $|12| = 12$.

In general, if we want to know an interval between two numbers $x$ and $y$ we would calculate $|x - y|$. The result will always be a non-negative number whether $x \geq y$ or $y \geq x$. 
Problem of the Week

Problem A

Baking for a Bunch

Samir wants to make treats for his class. There are 24 people in the class including Samir and his teacher. He has two recipes: one for brownies and one for cookies. The brownie recipe makes enough for 12 people to eat. The cookie recipe makes 48 cookies.

<table>
<thead>
<tr>
<th>Brownies</th>
<th>Chocolate Chip Cookies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cup white sugar</td>
<td>2 cups all-purpose flour</td>
</tr>
<tr>
<td>1/2 cup butter</td>
<td>1 teaspoon baking soda</td>
</tr>
<tr>
<td>1/4 cup cocoa powder</td>
<td>1 teaspoon salt</td>
</tr>
<tr>
<td>1/4 teaspoon vanilla extract</td>
<td>1 cup butter</td>
</tr>
<tr>
<td>3/4 cup all-purpose flour</td>
<td>1 cup white sugar</td>
</tr>
<tr>
<td>1/2 teaspoon salt</td>
<td>1/2 cup brown sugar</td>
</tr>
<tr>
<td>2 eggs</td>
<td>1 teaspoon vanilla extract</td>
</tr>
<tr>
<td></td>
<td>2 eggs</td>
</tr>
<tr>
<td></td>
<td>2 cups chocolate chips</td>
</tr>
</tbody>
</table>

Samir wants to bake exactly enough so each person gets one brownie and one cookie. Make a list showing the total amount of each ingredient he needs to do his baking.

**Strands** Measurement, Number Sense and Numeration
Problem of the Week
Problem A and Solution
Baking for a Bunch

Problem
Samir wants to make treats for his class. There are 24 people in the class including Samir and his teacher. He has two recipes: one for brownies and one for cookies. The brownie recipe makes enough for 12 people to eat. The cookie recipe makes 48 cookies.

Brownies
1 cup white sugar
1/2 cup butter
1/4 cup cocoa powder
1/4 teaspoon vanilla extract
3/4 cup all-purpose flour
1/2 teaspoon salt
2 eggs

Chocolate Chip Cookies
2 cups all-purpose flour
1 teaspoon baking soda
1 teaspoon salt
1 cup butter
1 cup white sugar
1/2 cup brown sugar
1 teaspoon vanilla extract
2 eggs
2 cups chocolate chips

Samir wants to bake exactly enough so each person gets one brownie and one cookie. Make a list showing the total amount of each ingredient he needs to do his baking.

Solution
Since Samir wants to make 24 brownies, and the recipe makes enough for 12, then he must double the brownie recipe, because 12 \times 2 = 24. To double a number, you can either multiply it by 2 or you can add the number to itself.

Since he wants to make 24 cookies, and the recipe makes enough for 48, then he must cut the cookie recipe in half, because 48 \div 2 = 24.

The most difficult calculation is probably determining the required amount of flour. You will need to double the amount from the brownie recipe (\frac{3}{4} \text{ cup}) and add it to half the amount from the cookie recipe (2 \text{ cups}). There are many ways to determine the result of \frac{3}{4} \times 2 or \frac{3}{4} + \frac{3}{4}. For example, you can use fractions of geometric shapes such as circles or squares, or use money such as Canadian quarters. You could also literally use measuring cups with flour, sand, or some other measurable material.
Here is another way to think about the calculation.

Break up \( \frac{3}{4} \) into 3 separate quarters: \( \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \)

Now, double this amount using addition: \( \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \)

Group the first four quarters together and the last two quarters together:

\[
\left( \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \right) + \left( \frac{1}{4} + \frac{1}{4} \right)
\]

When you add four quarters together you get a whole, or 1. When you add two quarters together you get a half \( (\frac{1}{2}) \). So in this case the total is \( 1\frac{1}{2} \) cups of flour required for the brownies. You also need half of the 2 cups of flour for the cookies. Half of 2 cups is 1 cup. So altogether you need \( 1\frac{1}{2} + 2 = 2\frac{1}{2} \) cups of flour.

Here is a summary of the totals required for the rest of the ingredients:

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>Brownie Amount</th>
<th>Cookie Amount</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>flour</td>
<td>( \frac{3}{4} \times 2 = 1\frac{1}{2} ) cups</td>
<td>( 2 \div 2 = 1 ) cup</td>
<td>( 1\frac{1}{2} + 1 = 2\frac{1}{2} ) cups</td>
</tr>
<tr>
<td>white sugar</td>
<td>( 1 \times 2 = 2 ) cups</td>
<td>( 1 \div 2 = \frac{1}{2} ) cup</td>
<td>( 2 + \frac{1}{2} = 2\frac{1}{2} ) cups</td>
</tr>
<tr>
<td>butter</td>
<td>( \frac{1}{2} + \frac{1}{2} = 1 ) cup</td>
<td>( 1 \div 2 = \frac{1}{2} ) cup</td>
<td>( 1 + \frac{1}{2} = 1\frac{1}{2} ) cups</td>
</tr>
<tr>
<td>cocoa powder</td>
<td>( \frac{1}{4} + \frac{1}{4} = \frac{1}{2} ) cup</td>
<td></td>
<td>( \frac{1}{2} ) cup</td>
</tr>
<tr>
<td>vanilla extract</td>
<td>( \frac{1}{4} + \frac{1}{4} = \frac{1}{2} ) teaspoon</td>
<td>( 1 \div 2 = \frac{1}{2} ) teaspoon</td>
<td>( \frac{1}{2} + \frac{1}{2} = 1 ) teaspoon</td>
</tr>
<tr>
<td>salt</td>
<td>( \frac{1}{2} + \frac{1}{2} = 1 ) teaspoon</td>
<td>( 1 \div 2 = \frac{1}{2} ) teaspoon</td>
<td>( 1 + \frac{1}{2} = 1\frac{1}{2} ) teaspoons</td>
</tr>
<tr>
<td>eggs</td>
<td>( 2 \times 2 = 4 ) eggs</td>
<td>( 2 \div 2 = 1 ) egg</td>
<td>( 4 + 1 = 5 ) eggs</td>
</tr>
<tr>
<td>baking soda</td>
<td></td>
<td>( 1 \div 2 = \frac{1}{2} ) teaspoon</td>
<td>( \frac{1}{2} ) teaspoon</td>
</tr>
<tr>
<td>brown sugar</td>
<td>( \frac{1}{2} \div 2 = \frac{1}{4} ) cup</td>
<td></td>
<td>( \frac{1}{4} ) cup</td>
</tr>
<tr>
<td>chocolate chips</td>
<td>( 2 \div 2 = 1 ) cup</td>
<td></td>
<td>1 cup</td>
</tr>
</tbody>
</table>
Teacher’s Notes

It is very important for students to have a solid understanding of fractions and how to do calculations with them. Their ability to work with fractions tends to diminish once they are given a calculator to work with, and this can be problematic in later mathematics classes.

A calculator is a great tool for computing results given numbers. However, when students are working with algebraic expressions that involve fractions, a calculator may not be able to help. For example, a calculator cannot help you simply an expression like:

\[
\frac{(x + 1)}{3} + \frac{(x - 5)}{2}
\]

Understanding and practise adding, subtracting, multiplying and dividing fractions early will make a difference for students in high school and beyond.
Problem of the Week
Problem A
Biking Around the Lake

Jessie lives on Island Lake. There is a road all the way around the lake called Ring Road. There is one road from town to the lake, called Town Road (not shown in the picture). The distance around Island Lake is 10 km.

Measuring the distance clockwise, the distance from Jessie’s house to the intersection of Ring Road and Town Road is 3 km, and the distance from the beach to that intersection is 9 km.

Jessie is biking to the beach, to meet her friend Brody who is already there. She wants to take the shortest route to the beach. Jessie rides 2 km, then remembers she promised to bring a water bottle for Brody. She cycles back to her house, picks up the water bottle, then bikes to the beach to meet her friend.

How many kilometres did Jessie travel to reach her final destination?

**Strands**  Number Sense and Numeration,  Measurement
Problem of the Week
Problem A and Solution
Biking Around the Lake

Problem
Jessie lives on Island Lake. There is a road all the way around the lake called Ring Road. There is one road from town to the lake, called Town Road (not shown in the picture). The distance around Island Lake is 10 km.

Measuring the distance clockwise, the distance from Jessie’s house to the intersection of Ring Road and Town Road is 3 km, and the distance from the beach to that intersection is 9 km.

Jessie is biking to the beach, to meet her friend Brody who is already there. She wants to take the shortest route to the beach. Jessie rides 2 km, then remembers she promised to bring a water bottle for Brody. She cycles back to her house, picks up the water bottle, then bikes to the beach to meet her friend.

How many kilometres did Jessie travel to reach her final destination?
Solution

The first thing we need to do is determine the shortest distance from Jessie’s house to the beach. It would be helpful to identify exactly where Town Road is relative to the location of the house and the beach. We could do this by creating a diagram that includes Town Road, and shows 10 equidistant markers on Ring Road. Since the distance around Ring Road is 10 km, the distance between adjacent markers is 1 km. Now, we can determine distances between points on Ring Road by counting markers. After deciding where the intersection for Ring Road and Town Road is on the diagram, add Jessie’s house and the beach. We can measure 3 km counter-clockwise from the intersection to identify the location of the house, and measure 9 km counter-clockwise from the intersection to the location of the beach.

![Diagram of Ring Road and Town Road with markers identifying distances from the intersection to Jessie's house and the beach.]

From this diagram, it is clear that it would be shorter for Jessie to ride clockwise on Ring Road to get from her house to the beach. Counting the markers, it is 4 km from her house to the beach if she rides clockwise on Ring Road, and it is 6 km if she rides counter-clockwise to the beach. Now we can make a number line to track how far Jessie rode.

![Number line showing distances ridden by Jessie: ride 2 km away from the house, ride 2 km back to the house, ride 4 km to the beach.]

In total, Jessie has ridden $2 + 2 + 4 = 8$ km.
Teacher’s Notes

We used a diagram in the solution of this problem. A diagram does not need to be an exact representation of the problem, nor does it even need to be a scaled representation. A diagram needs to include all known, and important information in an organized way that will help us find a solution. In this case, the important information is the relative location of the house to Town Road, the relative location of the beach to Town Road, and the length of Ring Road. Details such as the fact that Ring Road surrounds a lake and the exact shape of the road are irrelevant. A diagram is an abstraction of the actual problem. The process of creating a mathematical model from a real-world situation is an essential part of advanced mathematics and computer science.

Once we have created a good diagram, we can use deductive reasoning to determine the distance between Jessie’s house and the beach. As students continue to study mathematics, they will be introduced to Euclidean geometry. They will be asked to solve problems where they are required to determine an unknown value given some starting information about angles, lines, triangles, circles, and other polygons. Creating a useful diagram is often the first step to solving these types of problems. Students will practice using deductive reasoning to find the answers. Both of these skills can be applied to solving many other problems.
James saves all of the coins he can find. He is very organized, so he has a separate container for each type of coin. In Canada a toonie is worth $2 (2 dollars), a loonie is worth $1, a quarter is worth 25¢ (25 cents), a dime is worth 10¢, a nickel is worth 5¢, and $1 is equal to 100¢.

Each container James has holds 80 coins. Here is a picture representing the containers. The shaded part represents how many coins are in each container.

A) Which container holds the most money? Justify your answer.

B) Approximately what is the difference between the amount James has saved in quarters and the amount he has saved in dimes?

C) Approximately what is the total amount of money James has saved so far?

**STRANDS**  Data Management and Probability, Number Sense and Numeration
Problem of the Week
Problem A and Solution
Smart Saver

Problem

James saves all of the coins he can find. He is very organized, so he has a separate container for each type of coin. In Canada a toonie is worth $2 (2 dollars), a loonie is worth $1, a quarter is worth 25¢ (25 cents), a dime is worth 10¢, a nickel is worth 5¢, and $1 is equal to 100¢.

Each container James has holds 80 coins. Here is a picture representing the containers. The shaded part represents how many coins are in each container.

![Containers with coins](image)

A) Which container holds the most money? Justify your answer.

B) Approximately what is the difference between the amount James has saved in quarters and the amount he has saved in dimes?

C) Approximately what is the total amount of money James has saved so far?

Solution

We need to estimate what fraction of each container is filled with coins.

Container A appears to be approximately \( \frac{1}{4} \) full.
Container B appears to be approximately \( \frac{3}{4} \) full.
Container C appears to be approximately \( \frac{1}{2} \) full.
Container D appears to be approximately \( \frac{1}{2} \) full.
Container E appears to be completely full.
Half of 80 is 40, since $40 + 40 = 80$.

Half of 40 is 20, since $20 + 20 = 40$.

Since half of $\frac{1}{2}$ is $\frac{1}{4}$, then $\frac{1}{4}$ of 80 is 20.

Since $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$, then $\frac{3}{4}$ of 80 is $20 + 20 + 20 = 60$.

With this information we can estimate how many coins are in each container, as well as the value of the coins that have been collected. To determine the amount of money in each container, we can use skip counting, or multiplication.

<table>
<thead>
<tr>
<th>Container</th>
<th>Number of Coins</th>
<th>Value of Coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20</td>
<td>$20 \times 2 = $40$</td>
</tr>
<tr>
<td>B</td>
<td>60</td>
<td>$60 \times 1 = $60$</td>
</tr>
<tr>
<td>C</td>
<td>40</td>
<td>$40 \times 25 = 1000\text{¢} = $10$</td>
</tr>
<tr>
<td>D</td>
<td>40</td>
<td>$40 \times 10 = 400\text{¢} = $4$</td>
</tr>
<tr>
<td>E</td>
<td>80</td>
<td>$80 \times 5 = 400\text{¢} = $4$</td>
</tr>
</tbody>
</table>

It may be easier to calculate the values of the quarters, dimes, and nickels by noting how many of each coin we need to make $1$. It takes 4 quarters, 10 dimes, or 20 nickels to make $1$. To calculate the total value of the quarters, we can skip count by 4: $4, 8, 12, 16, 20, 24, 28, 32, 36, 40$. This means that 40 is equal to 10 groups of 4. Since each of these groups of 4 is worth $1$, then the 40 quarters are worth $10$. Similarly, if we skip count by 10: $10, 20, 30, 40$, we see that the 4 groups of 10 dimes James has saved are worth $4$, and if we skip count by 20: $20, 40, 60, 80$, we see that the 4 groups of 20 nickels James has saved are worth $4$.

A) Based on the calculations shown in the table, it appears that the container holding the loonies has the most money.

B) Based on the calculations shown in the table, it appears that James has saved $10 - 4 = \$6$ more in quarters than he has in dimes.

C) James has saved approximately, $40 + 60 + 10 + 4 + 4 = \$118$, in his containers.
Teacher’s Notes

Here is a question that you might want to consider. Which answers would change if the size of the containers changed? For example, suppose the containers hold 200 coins instead of 80 coins. The answer to part A) would not change, but the answers to part B) and part C) would change.

We could describe the amount of money each container holds algebraically. Let \( x \) represent the maximum number of coins each container can hold. Then,

\[
\begin{align*}
\text{number of cents in Toonies} &= \frac{200 \cdot x}{4} = 50x \\
\text{number of cents in Loonies} &= \frac{100 \cdot 3x}{4} = 75x \\
\text{number of cents in Quarters} &= \frac{25 \cdot x}{2} = 12.5x \\
\text{number of cents in Dimes} &= \frac{10 \cdot x}{2} = 5x \\
\text{number of cents in Nickels} &= 5x
\end{align*}
\]

So, no matter what the size of the container is, the most amount of money saved is in the Loonies container. Interestingly, there is an equal amount of money saved in the container with dimes when compared to the container with nickels.

The difference between the amount of money saved in quarters and the amount of money saved in dimes would depend on the size of the container. We can describe that difference as: \( 12.5x - 5x = 7.5x \)

We can also generally describe the total amount of money saved so far, with an unknown container size, as an equation: \( 50x + 75x + 12.5x + 5x + 5x = 147.5x \)
Problem of the Week
Problem A
Ups and Downs

Graham rides his bike to school. He takes a different route home. On the way to school he rides up a hill for 500 metres, then he rides on a flat section for 3 kilometres, and then he rides downhill for 1 kilometre. On his way home he rides on a flat section for 250 metres, then he rides uphill for 750 metres, followed by another flat section for 2 kilometres, then downhill for 2 kilometres, and finally on a flat section for 500 metres.

A) How far does Graham ride to and from school every day?

B) What fraction of the total distance travelled in one day is Graham riding downhill?
Problem of the Week
Problem A and Solution
Ups and Downs

Problem
Graham rides his bike to school. He takes a different route home. On the way to school he rides up a hill for 500 metres, then he rides on a flat section for 3 kilometres, and then he rides downhill for 1 kilometre. On his way home he rides on a flat section for 250 metres, then he rides uphill for 750 metres, followed by another flat section for 2 kilometres, then downhill for 2 kilometres, and finally on a flat section for 500 metres.

A) How far does Graham ride to and from school every day?
B) What fraction of the total distance travelled in one day is Graham riding downhill?

Solution
A) We can use a number line to calculate the total distance Graham travelled.
Each interval on this line represents 250 m.

From the number line we can see that Graham travels a total of 10 km.
We could also add the individual distances together to determine the total. It is probably easier to do this if all of the distances were measured with the same unit. So, converting all of the distances to metres, we get:

\[ 500 + 3000 + 1000 + 250 + 750 + 2000 + 2000 + 2000 + 500 = 10000 \]

and 10000 m is equal to 10 km.

B) Graham travels 1 km downhill on the way to school and 2 km downhill on the way home. This is a total of \(1 + 2 = 3\) km.

Since he travels a total of 10 km in one day, the downhill portion of the distance he travels is: \(\frac{3}{10}\).
Teacher’s Notes

In this solution we use a number line to accumulate the distance Graham travels. To make the number line a useful tool, we must make a good choice for the size of its intervals. A simple number line might have each interval be equal to one unit, however that scale does not work in this case. If we had each interval represent 1 km, then it would be hard to accurately accumulate fractional values like 250 m or 500 m. If we had each interval represent 1 m, then the number line would either be extremely long or the spaces between intervals would be extremely small. Even choosing 100 m as the distance between intervals would not be the best choice, since some of the accumulated values would land between the tick marks on the number line.

One way to pick the interval size is to consider the greatest common divisor or \( GCD \) of the numbers that we are accumulating. The \( GCD \) of a set of numbers is the largest integer that divides evenly into each of the numbers in the set. In this case, we have the numbers 500, 3000, 1000, 250, 750, and 2000. For all of these numbers, we get an integer result when we divide them by 250. By choosing an interval size that is a divisor of each of the numbers we are accumulating, we guarantee that accumulated values will always land on one of the tick marks of our number line. Choosing the interval size to be the greatest common divisor, means we have the fewest intervals necessary to guarantee accumulated values will always end up landing on a tick mark.

In this case, the \( GCD \) also happens to be the smallest number in our set. However, that is not always the case. For example, the \( GCD \) of the numbers 400, 2000, and 900 is 100. The \( GCD \) will always be less than or equal to the smallest number in the set and greater than or equal to 1.
Patterning & Algebra
Problem of the Week
Problem A
Following Directions

Yeni is playing a game that uses a $12 \times 12$ grid like the one shown below. She moves pieces on the grid by giving a sequence of steps. Each step is a direction indicated by an arrow: ↑ (up one block), ↓ (down one block), ← (left one block), and → (right one block).

Yeni has a special move which combines six steps: → → ↓ ↓ ← ↑

A) Yeni starts by putting a marker (X) at position A1.

What is the position of the marker after making the special move three times in a row?

B) How many more times can she repeat the special move before the marker will move off the grid?

**Strands**  PATTERNING AND ALGEBRA, GEOMETRY AND SPATIAL SENSE
Problem of the Week
Problem A and Solution
Following Directions

Problem
Yeni is playing a game that uses a $12 \times 12$ grid like the one shown below. She moves pieces on the grid by giving a sequence of steps. Each step is a direction indicated by an arrow: ↑ (up one block), ↓ (down one block), ← (left one block), and → (right one block).

Yeni has a special move which combines six steps: → → ↓ ↓ ← ↑

A) Yeni starts by putting a marker (X) at position A1. What is the position of the marker after making the special move three times in a row?

B) How many more times can she repeat the special move before the marker will move off the grid?

Solution
A) Starting at position A1, after making the special move once, the marker ends up at position B2.

```
<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>→</td>
<td>→</td>
<td>↓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>X</td>
<td>↓</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>↑</td>
<td>←</td>
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<tr>
<td>4</td>
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<td></td>
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<tr>
<td>5</td>
<td></td>
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<td>6</td>
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<td></td>
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<tr>
<td>7</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>8</td>
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</tr>
</tbody>
</table>
```

Each time Yeni uses the special move, the marker ends up one row below and one column to the right of its starting position. So after the second time she executes the move, the marker moves to position C3, and after the third time she executes the move, the marker moves to position D4.

B) If we continue repeating this pattern, after the 10th time she repeats the move, the marker moves to position K11. From that position, if Yeni tries the special move she can move to the right once safely. However if she tries to move to the right a second time that would mean the marker moves off the edge of the grid. This means that starting from position A1 Yeni can repeat the special move 10 times safely, or 7 more times after using the special move 3 times.
Teacher’s Notes

This question was inspired by *turtle graphics* and *Karel the Robot*. In the 1960s, the programming language Logo allowed coders to use a turtle robot to draw patterns on the screen. The turtle normally appeared as a simple triangle, which indicated a direction it would move next. Programmers could easily write programs that would make the turtle move forward, turn, and draw lines. With a small set of instructions, people could create very interesting patterns and geometric images on the screen. Karel the Robot was designed to teach students how to code by having them control a simple, graphical robot that could move around a grid on the screen. Karel only responded to a small set of instructions, but this was enough to teach and learn the essential concepts of computer programming. Both the Logo turtle and Karel the Robot have been replicated in more modern programming languages.

This problem demonstrates two of those essential programming concepts: *modularization* and *repetition*. When writing programs of any size, it is important to be able to break up the problem into smaller subproblems. Programmers will bundle instructions together into subprograms, that are often referred to as functions, procedures, subroutines, or methods. Identifying these smaller pieces of the problem makes them easier to solve, and the code used to solve them more flexible.

Having the ability to repeat instructions in code, leads to more and better solutions to the problems that can be solved by computers. It can be tricky for new programmers to use repetition properly. Knowing when and how to stop the repetition is essential to success in coding.
Problem of the Week
Problem A
Stella’s Vegetable Garden

Stella wants to plant some vegetables in her garden so her family can enjoy them. She has five types of plants in the garden: beans, peas, carrots, lettuce, and tomatoes. She has four more bean plants than tomato plants. She has twice as many pea plants as lettuce plants. She has 10 fewer lettuce plants than carrot plants. She has the same number of bean plants as pea plants. Stella planted 16 carrot plants. How many of each type of plant does Stella have in her garden?

Strands
Number Sense and Numeration, Patterning and Algebra
Problem of the Week
Problem A and Solution
Stella’s Vegetable Garden

Problem
Stella wants to plant some vegetables in her garden so her family can enjoy them. She has five types of plants in the garden: beans, peas, carrots, lettuce, and tomatoes. She has four more bean plants than tomato plants. She has twice as many pea plants as lettuce plants. She has 10 fewer lettuce plants than carrot plants. She has the same number of bean plants as pea plants. Stella planted 16 carrot plants. How many of each type of plant does Stella have in her garden?

Solution
We know the number of carrot plants without knowing any other information about the garden. All other numbers of plants are related to other the information about other plants. The number of lettuce plants can be determined given the number of carrots, so we can calculate it relatively easily. There are $16 - 10 = 6$ lettuce plants.

Knowing the number of lettuce plants, we can calculate the number of pea plants. There are $6 \times 2 = 12$ pea plants. Since she has an equal number of pea plants and bean plants, she also has 12 bean plants.

If Stella has four more bean plants than tomato plants, that is the same as saying she has four fewer tomato plants than bean plants. Knowing that she has 12 bean plants, she must have $12 - 4 = 8$ tomato plants.

In summary, Stella’s garden contains:

- 12 bean plants
- 12 pea plants
- 16 carrot plants
- 6 lettuce plants
- 8 tomato plants
Teacher’s Notes

It is possible to solve this problem algebraically. We could assign variables to each of the types of plants and then set up a system of equations that reflects all of the relationships between the numbers of each of the plants. Then we could solve the equations to determine how many of each plant are in the garden. For example if we use variables $b, p, c, l, t$ for the number of beans, peas, carrots, lettuce, and tomatoes, we can describe the relationships as follows:

\[
\begin{align*}
  t + 4 &= b \\
  2 \times l &= p \\
  c - 10 &= l \\
  b &= p \\
  c &= 16
\end{align*}
\]

There are many algebraic techniques we could use to solve the equations and determine the values for each of the variables. We could also describe the relationships in such a way so that each different variable appears by itself on one side of the equation as follows:

\[
\begin{align*}
  t &= b - 4 \\
  p &= 2 \times l \\
  l &= c - 10 \\
  b &= p \\
  c &= 16
\end{align*}
\]

Using these relationships we can set up formulae in a spreadsheet like this:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Type</td>
</tr>
<tr>
<td>2</td>
<td>Tomatoes</td>
</tr>
<tr>
<td>3</td>
<td>Peas</td>
</tr>
<tr>
<td>4</td>
<td>Lettuce</td>
</tr>
<tr>
<td>5</td>
<td>Beans</td>
</tr>
<tr>
<td>6</td>
<td>Carrots</td>
</tr>
<tr>
<td>2</td>
<td>Number</td>
</tr>
<tr>
<td>3</td>
<td>$=B5 - 4$</td>
</tr>
<tr>
<td>4</td>
<td>$=2 * B4$</td>
</tr>
<tr>
<td>5</td>
<td>$=B6 - 10$</td>
</tr>
<tr>
<td>6</td>
<td>$=B3$</td>
</tr>
<tr>
<td></td>
<td>16</td>
</tr>
</tbody>
</table>

Using this setup, a spreadsheet will show you the same results for each type of plant that we calculated in our solution.
Problem of the Week
Problem A
Dining Dilemma

We want to start a new restaurant. We have square tables that allow one chair on each side. Therefore, we can arrange four chairs around each table.

A) If the restaurant has 32 tables, how many chairs do we need to buy?

B) As we set up the restaurant, we put out one table at a time with its full set of chairs surrounding it. If we have put out 36 chairs, how many tables have been set up so far?

C) When we have banquets we sometimes need to push the tables together. This changes the amount of chairs we can put around the table grouping, as shown in the following picture.

How many chairs are required if we set up the 32 tables in pairs?

D) How would the answer to part (C) change if we group 8 tables end to end, and still use all 32 tables?

Strands Patterning and Algebra, Number Sense and Numeration
Problem of the Week
Problem A and Solution
Dining Dilemma

Problem
We want to start a new restaurant. We have square tables that allow one chair on each side. Therefore, we can arrange four chairs around each table.

A) If the restaurant has 32 tables, how many chairs do we need to buy?

B) As we set up the restaurant, we put out one table at a time with its full set of chairs surrounding it. If we have put out 36 chairs, how many tables have been set up so far?

C) When we have banquets we sometimes need to push the tables together. This changes the amount of chairs we can put around the table grouping, as shown in the following picture.

How many chairs are required if we set up the 32 tables in pairs?

D) How would the answer to part (C) change if we group 8 tables end to end, and still use all 32 tables?
Solution

A) We can use multiplication to calculate the number of chairs: \(32 \times 4 = 128\).

We might also notice that multiplying by 4 is the same as doubling a number and then doubling the answer. For example, we can find the answer to \(32 \times 4\) by calculating \(32 \times 2 = 64\) and then calculating \(64 \times 2 = 128\).

B) We can use skip counting to find out how many tables are out.

We skip count by 4 to count the chairs: 4, 8, 12, 16, 20, 24, 28, 32, 36. This means there are 9 tables set up so far. We could have also used division to calculate this answer: \(36 \div 4 = 9\).

C) We can make a table showing the relationship between the number of tables and the number of chairs using this configuration. Each cluster of 2 tables is surrounded by 6 chairs. So we add 2 to the number of tables from one row to the next, and we add 6 to the number of chairs from one row to the next.

<table>
<thead>
<tr>
<th>Number of Tables</th>
<th>Number of Chairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>12</td>
<td>36</td>
</tr>
<tr>
<td>14</td>
<td>42</td>
</tr>
<tr>
<td>16</td>
<td>48</td>
</tr>
<tr>
<td>18</td>
<td>54</td>
</tr>
<tr>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>22</td>
<td>66</td>
</tr>
<tr>
<td>24</td>
<td>72</td>
</tr>
<tr>
<td>26</td>
<td>78</td>
</tr>
<tr>
<td>28</td>
<td>84</td>
</tr>
<tr>
<td>30</td>
<td>90</td>
</tr>
<tr>
<td>32</td>
<td>96</td>
</tr>
</tbody>
</table>

Another way to calculate this result is as follows:

Since there are 32 tables in total, there are \(32 \div 2 = 16\) pairs of tables. In this configuration, there 6 chairs around each pair of tables. Therefore, there are a total of \(16 \times 6 = 96\) chairs required for this set up.
D) With 8 tables arranged end to end, there will be one chair on each end, and eight chairs on each side. This is a total of $1 + 1 + 8 + 8 = 18$ chairs around the tables.

We can make a new table showing the relationship between the number of tables and the number of chairs using this configuration. In this case we add 8 to the number of tables from one row to the next, and we add 18 to the number of chairs from one row to the next.

<table>
<thead>
<tr>
<th>Number of Tables</th>
<th>Number of Chairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>18</td>
</tr>
<tr>
<td>16</td>
<td>36</td>
</tr>
<tr>
<td>24</td>
<td>54</td>
</tr>
<tr>
<td>32</td>
<td>72</td>
</tr>
</tbody>
</table>

Another way to calculate this result is as follows:
Since there are 32 tables in total, there are $32 \div 8 = 4$ sets of 8 tables. In this configuration, there 18 chairs around each pair of tables. Therefore, there are a total of $18 \times 4 = 72$ chairs required for this set up.
Teacher’s Notes

Part of the solutions for parts C and D can be described as a table of values (no pun intended). A table of values can be used to list specific values that are described by a function. We write functions in a variety of ways. For example, we can write a function describing the values in part D in the form of an equation that uses a variable \( c \) to represent the number of chairs and a variable \( t \) that represents the number of tables. This equation would be:

\[
c = \frac{t}{8} \times 18
\]

Another way of describing this function uses \( x \) to represent the number of tables and \( f(x) \) to represent the number of chairs. The function that uses this notation would be:

\[
f(x) = \frac{x}{8} \times 18
\]

Normally when we use these formats to describe functions, we expect that the variables in the equations represent many possible numbers - sometimes infinitely many numbers. However, for this problem there is a finite set of possible values for the number of tables we arrange in the restaurant. In particular, we only consider groupings of 8, 16, 24, or 32 tables. So numbers like 1, 2, 3, ..., 7, 9, 10 and so on are not actual values that would be used for \( t \) or \( x \), in the functions we have written.

Another way we describe functions is called a mapping. This is a visual representation of the relationship between two sets of numbers. For example, we use a mapping like this to describe the values in part D of this problem:

In a mapping, an oval represents a set of values. In this example, the oval on the left represents the set of values representing the number of tables we may group together in this problem. The oval on the right represents the set of values that are the related number of chairs. Values in the set on the left are connected, using an arrow, to a particular value in the set on the right. Altogether, this picture represents a function that describes the relationship between 8, 16, 24, or 32 tables and the number of chairs required in each case. A mapping is a nice way to represent a function with a finite set of values.
Mrs. Zhang needs to buy pencil crayons for her classroom. A local store is running a sale where the shopper will receive a free box of pencil crayons for every 4 boxes purchased. Her teaching partner, Mr. Holland also mentioned that he needs 5 more boxes of pencil crayons for his classroom. How many boxes of pencil crayons must Mrs. Zhang purchase in order to get 5 free boxes to give to Mr. Holland?
Problem of the Week
Problem A and Solution
Purchasing Pencil Crayons

Problem
Mrs. Zhang needs to buy pencil crayons for her classroom. A local store is running a sale where the shopper will receive a free box of pencil crayons for every 4 boxes purchased. Her teaching partner, Mr. Holland also mentioned that he needs 5 more boxes of pencil crayons for his classroom. How many boxes of pencil crayons must Mrs. Zhang purchase in order to get 5 free boxes to give to Mr. Holland?

Solution
We can use a table to show how many free boxes Mrs. Zhang receives as she buys her pencil crayons. On each row, we increase the number of boxes purchased by 4 and the number of free boxes by 1.

<table>
<thead>
<tr>
<th>Boxes Purchased</th>
<th>Free Boxes</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
</tr>
</tbody>
</table>

From the table we see that Mrs. Zhang would need to buy 20 boxes of pencil crayons in order to get 5 free boxes.

Alternatively we can determine the result this way. In order to get the pencil crayons that Mr. Holland needs, Mrs. Zhang needs to buy 5 sets of 4 boxes. This means that she needs to buy a total of $5 \times 4 = 20$ boxes of pencil crayons.
Teacher’s Notes

We can describe the relationship between the number of boxes Mrs. Zhang bought and the number of boxes she got for free in a number of ways.

The **ratios** 4 : 1 and 20 : 5 show the relative number of paid boxes to the number of free boxes. Since these two ratios show the same relationship, they are said to be **proportional**.

The fraction of free boxes in the total that Mrs. Zhang acquired is $\frac{5}{25}$ or $\frac{1}{5}$. These are examples of equivalent fractions.

The **percentage** of free boxes is 20% which can also be written as the decimal number 0.2 or the fraction $\frac{20}{100}$. The term percent, literally means *per 100*.

The less well known term *per mille* is used to describe a fraction out of 1000. This is written using the symbol $\%$. So, $2\%$ of the boxes acquired were free.
Problem of the Week
Problem A
Flowers for Volunteers

To thank the volunteers at school, the school council would like to give 5 flowers to each volunteer. There are 15 volunteers to thank. You cannot buy individual flowers; they are only sold by the dozen.

A) How many dozens will the school council need to buy?

B) If one dozen flowers costs $10.00, how much will the school council need to spend on flowers?

Strands  Number Sense and Numeration, Patternning and Algebra
Problem of the Week
Problem A and Solution
Flowers for Volunteers

Problem
To thank the volunteers at school, the school council would like to give 5 flowers to each volunteer. There are 15 volunteers to thank. You cannot buy individual flowers; they are only sold by the dozen.

A) How many dozens will the school council need to buy?

B) If one dozen flowers costs $10.00, how much will the school council need to spend on flowers?

Solution
A) Since the school council wants to give each volunteer 5 flowers, then they need a total of $15 \times 5 = 75$ flowers. Alternatively, we can skip count by 5: $5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75$ to calculate this total.

To calculate how many dozen flowers we need, we could use repeated subtraction of 12, starting with 75:

\[
75 - 12 = 63 \\
63 - 12 = 51 \\
51 - 12 = 39 \\
39 - 12 = 27 \\
27 - 12 = 15 \\
15 - 12 = 3
\]

So after subtracting 12, six times, we still have 3 flowers to buy. Since the smallest number of flowers we can buy is 12, we need to buy a total of 7 dozen flowers.

Alternatively, we can skip count by 12 until we get a number greater than or equal to 75: $12, 24, 36, 48, 60, 72, 84$. This also shows us that we need at least 7 dozen flowers to have enough for the volunteers.

Also we can use division to see that $75 \div 12 = 6$ remainder 3. Again, this means that we need 7 dozen flowers.

B) Since flowers cost $10.00 per dozen, then the school council must spend $7 \times 10.00 = $70.00 to buy enough flowers for the volunteers.
Teacher’s Notes

Dividing integers results in a two part answer: the quotient and the remainder. For example $38 \div 11 = 3$ with a remainder of 5. This is an exact result. If we use a calculator to do division, we may end up with an inexact result. For example, the result of dividing 38 by 11 appears as 3.45454545 on a calculator. This is an approximation. The exact answer has the digits 45 repeating forever.

When computers calculate the result of division with integers, the quotient and remainder are stored separately. This means that it is possible to access those two parts separately. Most computer languages have commands that allow you to access each of the parts. They also have commands that allow you to generate integer values from non-integer results. For example, you may be able to round off a number (to the nearest integer), round down a number (often referred to as finding the floor) or round up a number (often referred to as finding the ceiling).

If we want to see single result of a division where the decimal representation has infinitely repeating decimals, a computer can only display a finite number of digits. These types of results are converted into a different format, usually known as floating point numbers. Since this value is an inexact result of a calculation, small errors can appear when we work with this kind of number. You might see the result of working with inexact data when you expect to see a result like 4 after doing some calculations on your computer, but you see the number 3.9999999999 or 4.0000000001 instead.
Problem of the Week
Problem A
Saving to Surf

Henri is saving money to purchase a new tablet. The tablet he wishes to buy costs $260.00. Henri started with $65.00 in his bank account. Last week, he received $100.00 from his grandmother for his birthday and an additional $25.00 from his uncle. He deposited his birthday money into his bank account.

A) After depositing his birthday gifts, how much money does Henri have in his bank account?

B) Henri earns a weekly allowance of $8.00. How many weeks will it take Henri to save enough money from his allowance to be able to purchase the tablet?

Show your thinking.
Problem of the Week
Problem A and Solution
Saving to Surf

Problem
Henri is saving money to purchase a new tablet. The tablet he wishes to buy costs $260.00. Henri started with $65.00 in his bank account. Last week, he received $100.00 from his grandmother for his birthday and an additional $25.00 from his uncle. He deposited his birthday money into his bank account.

A) After depositing his birthday gifts, how much money does Henri have in his bank account?

B) Henri earns a weekly allowance of $8.00. How many weeks will it take Henri to save enough money from his allowance to be able to purchase the tablet?

Show your thinking.

Solution

A) Henri received a total of $100.00 + $25.00 = $125.00 for his birthday. After he deposited his birthday money, Henri had $65.00 + $125.00 = $190.00 in the bank.

B) We can make a table to determine how much money Henri saves:

<table>
<thead>
<tr>
<th>Week</th>
<th>Total Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$190.00 + $8.00 = $198.00</td>
</tr>
<tr>
<td>2</td>
<td>$198.00 + $8.00 = $206.00</td>
</tr>
<tr>
<td>3</td>
<td>$206.00 + $8.00 = $214.00</td>
</tr>
<tr>
<td>4</td>
<td>$214.00 + $8.00 = $222.00</td>
</tr>
<tr>
<td>5</td>
<td>$222.00 + $8.00 = $230.00</td>
</tr>
<tr>
<td>6</td>
<td>$230.00 + $8.00 = $238.00</td>
</tr>
<tr>
<td>7</td>
<td>$238.00 + $8.00 = $246.00</td>
</tr>
<tr>
<td>8</td>
<td>$246.00 + $8.00 = $254.00</td>
</tr>
<tr>
<td>9</td>
<td>$254.00 + $8.00 = $262.00</td>
</tr>
</tbody>
</table>

After nine weeks, Henri would have saved enough of his allowance in combination with the money in his bank account so that he can buy the tablet.
Teacher’s Notes

This problem can be solved algebraically. We can write an equation that shows the relationship between the time that has passed and the money that has been saved. Let \( w \) represent the number of weeks Henri has been saving money since his birthday. Let \( m \) represent the total amount of money that has been saved. We have calculated that there is $190.00 in the bank after depositing his birthday gifts. This is the equation that describes the relationship between weeks passed and money saved:

\[
m = 8w + 190
\]

We know that Henri needs $260.00 to buy the tablet, so we can substitute 260 in place of \( m \) in the equation.

\[
260 = 8w + 190
\]

Now we need to find out the value of \( w \). We subtract 190 from both sides of the equation.

\[
260 - 190 = 8w + 190 - 190
\]
\[
70 = 8w
\]

Then we divide both sides by 8.

\[
\frac{70}{8} = \frac{8w}{8}
\]
\[
8.75 = \frac{8w}{8}
\]
\[
8.75 = w
\]

This means Henri needs more than 8 weeks to save enough money for the tablet. He will have what he needs after 9 weeks of saving.
Problem of the Week
Problem A
Swimming Practice

Kunik takes swimming lessons at the community pool. A lap is the length of the pool, which is 25 m. During the lessons Kunik swims laps, from one end of the pool to the other end and back to the starting point. Kunik can swim 10 laps of the pool in 5 minutes if she is doing the front crawl, and she can swim 6 laps of the pool in 4 minutes if she is doing the backstroke.

A) How many metres can Kunik swim in 10 minutes, using the front crawl?

B) Approximately how long will it take Kunik to swim 60 laps of the pool if she uses the front crawl for half the laps and the backstroke for half the laps?

C) If Kunik wants to swim faster when she is in the pool, should she do the front crawl or the backstroke? Justify your answer.

**STRANDS**  Number Sense and Numeration, Patterning and Algebra, Measurement
Kunik takes swimming lessons at the community pool. A lap is the length of the pool, which is 25 m. During the lessons Kunik swims laps, from one end of the pool to the other end and back to the starting point. Kunik can swim 10 laps of the pool in 5 minutes if she is doing the front crawl, and she can swim 6 laps of the pool in 4 minutes if she is doing the backstroke.

A) How many metres can Kunik swim in 10 minutes, using the front crawl?

B) Approximately how long will it take Kunik to swim 60 laps of the pool if she uses the front crawl for half the laps and the backstroke for half the laps?

C) If Kunik wants to swim faster when she is in the pool, should she do the front crawl or the backstroke? Justify your answer.

Solution

A) Notice that 10 minutes is twice as much as 5 minutes. Since Kunik can swim 10 laps in 5 minutes using the front crawl, if we double the amount of time in the pool, we expect her to swim double the number of laps. Since double the time is $2 \times 5 = 10$ minutes, then we expect her to swim $2 \times 10 = 20$ laps in that time. Since each lap is 25 m, Kunik can swim $25 \times 20 = 500$ m in 10 minutes.
B) Half of 60 is 30 since $30 + 30 = 60$, so Kunik will be swimming 30 laps using the front crawl and 30 laps using the backstroke. For each stroke, we can use a table to determine how long it takes Kunik to swim 30 laps.

<table>
<thead>
<tr>
<th>Minutes</th>
<th>Laps</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>15</td>
<td>30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Minutes</th>
<th>Laps</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>16</td>
<td>24</td>
</tr>
<tr>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

From the tables, we expect it will take Kunik $15 + 20 = 35$ minutes to swim 60 laps if she uses each stroke half of the time.

C) From the tables we notice that it takes Kunik 15 minutes to swim 30 laps using the front crawl, and it takes her 20 minutes to swim the 30 laps using the backstroke. Since it takes longer for her to swim the same number of laps using the backstroke, she should do the front crawl if she wants to swim faster.
Teacher’s Notes

This problem could be solved using the rates of speed. The formula for calculating speed is:

\[ \text{speed} = \frac{\text{distance}}{\text{time}} \]

Since Kunik swims 10 laps of the pool in 5 minutes, then her front crawl speed is:

\[ \frac{250}{5} = 50 \text{ metres/min} \]

Since she swims 6 laps of the pool in 4 minutes, then her backstroke speed is:

\[ \frac{150}{4} = 37.5 \text{ metres/min} \]

Based on these calculations, we can see that Kunik’s rate of speed is faster doing the front crawl.

If we want to find out the time of an activity given the speed and distance, we can rearrange the formula this way:

\[ \text{time} = \frac{\text{distance}}{\text{speed}} \]

To calculate her time in part B) of the question we can substitute the distances and speeds into our formula for finding the time. The distance she swims using each stroke is 750 m.

\[ \text{time} = \frac{750 \text{ metres}}{50 \text{ metres/min}} + \frac{750 \text{ metres}}{37.5 \text{ metres/min}} = 15 \text{ minutes} + 20 \text{ minutes} = 35 \text{ minutes} \]

Notice the units in the calculation of the time. The distance unit (metres) appears in the numerator and denominator of the fraction. These units cancel each other. The time unit (min) appears in the denominator of the unit of speed in the denominator of the fraction. This means that the units we are calculating in the end are minutes. Keeping track of the units in your calculations that involve speed, time, and distance is a good way to check that you are working with the proper formula.
Problem of the Week
Problem A
Weighing Brown Bats

“Brown bats, common in Canada, weigh as much as two nickels and a dime.”

Riya, who has always been curious about bats, decided to count the number of
bats that flew through her yard. Riya counted 15 bats in one hour. She wondered
what the total weight of these bats would be using the information from the
quotation above. She made a table to figure out how much 15 bats would weigh,
if we actually measured weight in terms of nickels and dimes.

A) Finish her table.

<table>
<thead>
<tr>
<th>Number of Bats</th>
<th>Number of Nickels</th>
<th>Number of Dimes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
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<tr>
<td>4</td>
<td></td>
<td></td>
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<tr>
<td>5</td>
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<td>9</td>
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<td>10</td>
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<td>12</td>
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<tr>
<td>13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

B) In Canada, a nickel is worth 5¢ (5 cents), a dime is worth 10¢, and
$1 (one dollar) is equal to 100¢. What would the total value of 15 bats be if
each bat was actually worth “two nickels and a dime”?

Strands  Patterning and Algebra, Measurement
Problem

“Brown bats, common in Canada, weigh as much as two nickels and a dime.”

Riya, who has always been curious about bats, decided to count the number of bats that flew through her yard. Riya counted 15 bats in one hour. She wondered what the total weight of these bats would be using the information from the quotation above. She made a table to figure out how much 15 bats would weigh, if we actually measured weight in terms of nickels and dimes.

A) Finish her table.

B) In Canada, a nickel is worth 5¢ (5 cents), a dime is worth 10¢, and $1 (one dollar) is equal to 100¢. What would the total value of 15 bats be if each bat was actually worth “two nickels and a dime”?

Solution

A) Here is the completed table:

<table>
<thead>
<tr>
<th>Number of Bats</th>
<th>Number of Nickels</th>
<th>Number of Dimes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>18</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>22</td>
<td>11</td>
</tr>
<tr>
<td>12</td>
<td>24</td>
<td>12</td>
</tr>
<tr>
<td>13</td>
<td>26</td>
<td>13</td>
</tr>
<tr>
<td>14</td>
<td>28</td>
<td>14</td>
</tr>
<tr>
<td>15</td>
<td>30</td>
<td>15</td>
</tr>
</tbody>
</table>
B) According to the table, 15 bats are equal to 30 nickels and 15 dimes.

- The value of the nickels is: \(30 \times 5 = 150\)¢.
- The value of the dimes is: \(15 \times 10 = 150\)¢.
- Total value of 15 bats would be: \(150 + 150 = 300\)¢ or $3.

Another way to calculate the total monetary value of the bats would be to add a column to the table. The extra column in the table can be used to keep track of how much the bats are worth.

Since 2 nickels are equal to: \(5 + 5 = 2 \times 5 = 10\)¢, then the value of 2 nickels and a dime would be: \(10 + 10 = 20\)¢.

<table>
<thead>
<tr>
<th>Number of Bats</th>
<th>Number of Nickels</th>
<th>Number of Dimes</th>
<th>Value in ¢</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>3</td>
<td>60</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>5</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>6</td>
<td>120</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
<td>7</td>
<td>140</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
<td>8</td>
<td>160</td>
</tr>
<tr>
<td>9</td>
<td>18</td>
<td>9</td>
<td>180</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>10</td>
<td>200</td>
</tr>
<tr>
<td>11</td>
<td>22</td>
<td>11</td>
<td>220</td>
</tr>
<tr>
<td>12</td>
<td>24</td>
<td>12</td>
<td>240</td>
</tr>
<tr>
<td>13</td>
<td>26</td>
<td>13</td>
<td>260</td>
</tr>
<tr>
<td>14</td>
<td>28</td>
<td>14</td>
<td>280</td>
</tr>
<tr>
<td>15</td>
<td>30</td>
<td>15</td>
<td>300</td>
</tr>
</tbody>
</table>
Teacher’s Notes

The solution for part B) of this problem can be determined algebraically. If we let $x$ represent the number of bats, and let $y$ represent their monetary worth in cents, we can write the following equation: $y = ((2 \times 5) + 10) \times x$.

This equation can be simplified to: $y = 20x$

This equation shows a linear relationship between the number of bats and their monetary value. We can see that it is a linear relationship in the equation because it reflects one of the standard equations for a line: $y = mx + b$

This standard equation shows a relationship between variables $x$ and $y$, given the constants $m$ and $b$. The constants in our equation $y = 20x$ are:

- 20 for the value of $m$
- 0 for the value of $b$

The constant $m$ in this form of the equation of a line represents the slope of the line. The slope is defined as the rise (the amount $y$ changes) over the run (the amount $x$ changes) between two points on the line. The slope of a line is the same for every pair of points on the line. Using our table, we can check the slope using two random pairs of values. For example, according to the table, when we have 12 bats, the monetary value is 240¢, and when we have 7 bats, the monetary value is 140¢. So the rise over run is:

$$\frac{(240 - 140)}{(12 - 7)} = \frac{100}{5} = 20$$

The constant $b$ in this form represents the $y$-intercept of the line. The $y$-intercept is the place where the line crosses the Y-axis if we draw a graph the equation. It is also the value of $y$ when $x$ is 0. In our equation, when the number of bats is 0 then the monetary value is 0.
Problem of the Week

Problem A

Fuel Frenzy

Different vehicles are more fuel efficient than others, meaning some vehicles can go further on a tank of gas than others. Our car uses 6 litres for every 100 km of driving.

A) The car has a 42 litre tank. How many kilometres can our car drive on one tank of gas?

B) We are going on a trip to visit our family, 900 kilometres away. How much fuel will we use to get to our destination?

C) Fuel costs $2.00 per litre. How much will the fuel cost for the trip?

**Strands** Patterning and Algebra, Measurement
Problem of the Week
Problem A and Solution
Fuel Frenzy

Problem
Different vehicles are more fuel efficient than others, meaning some vehicles can go further on a tank of gas than others. Our car uses 6 litres for every 100 km of driving.

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B) We are going on a trip to visit our family, 900 kilometres away. How much fuel will we use to get to our destination?

C) Fuel costs $2.00 per litre. How much will the fuel cost for the trip?

Solution

A) We can use a table to see the relationship between the number of litres of gas used and how far the car can travel. On each row we increase the litres used by 6 and the km travelled by 100.

<table>
<thead>
<tr>
<th>Litres Used</th>
<th>km Travelled</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>100</td>
</tr>
<tr>
<td>12</td>
<td>200</td>
</tr>
<tr>
<td>18</td>
<td>300</td>
</tr>
<tr>
<td>24</td>
<td>400</td>
</tr>
<tr>
<td>30</td>
<td>500</td>
</tr>
<tr>
<td>46</td>
<td>600</td>
</tr>
<tr>
<td>42</td>
<td>700</td>
</tr>
<tr>
<td>48</td>
<td>800</td>
</tr>
<tr>
<td>54</td>
<td>900</td>
</tr>
</tbody>
</table>

From this table we see that you can travel 700 km on 42 litres of fuel.

B) From the table in part A) we can also see that we use 54 litres of fuel to travel 900 km.

Alternatively, we can calculate how much fuel is required by using division and multiplication. We see that $900 \div 100 = 9$. This means we require 9 times as much fuel as it would take to travel 100 km. Since we know it takes 6 litres to travel 100 km, then it will take $9 \times 6 = 54$ litres of fuel to travel 900 km.

C) It will cost $2.00 \times 54 = $108.00 to buy the fuel necessary for the trip.
Teacher’s Notes

This problem is analogous to questions that deal with the relationship between velocity, distance and time. We could use an equation to describe the relationship between the rate of fuel consumption \((R)\), the volume of fuel \((V)\), and the distance travelled \((d)\):

\[ R = \frac{V}{d} \]

Given any two of the values for \(R\), \(V\), and \(d\) we can calculate the third value. Rearranging the equation we get:

\[ d = \frac{V}{R} \]

or

\[ V = R \cdot d \]

In this problem, we are given the rate \(\frac{6}{100} = 0.06\).

In part A) we are given the volume and are asked to find the distance travelled. We can calculate that by substituting the known values into our equation for \(d\):

\[ d = \frac{42}{0.06} = 700 \]

In part B) we are given the distance travelled and asked to find the volume of fuel consumed. We can calculate that by substituting the known values into our equation for \(V\):

\[ V = 0.06 \cdot 900 = 54 \]

When we are able to describe these kinds of relationships with equations, we can calculate values quickly. Equations also allows us to use tools such as spreadsheets for quick calculations. However, understanding the relationship between these values is important. Creating a table can help solidify understanding this kind of relationship.