The problems in this booklet are organized into strands. A problem often appears in multiple strands. The problems are suitable for most students in Grade 5 or higher.
Data Management

&

Probability

TAKE ME TO THE COVER
Suppose that 100 ten to twelve year-old adolescents are asked to pick their favourite among four popular singers, with the following outcomes:

- Pasty Kerry: favourite of 20 out of each 50 students
- Tustin Jimberlake: favourite of 30% of the 100 students
- Bustin Jeiber: favourite of 10% of the 100 students
- Tregan Maynor: favourite of the remaining students

a) How many adolescents voted for each artist? Use a table to show your answers.

b) What fraction of the students voted for Tregan Maynor?

c) Which types of graphs are best suited to comparing this data?

d) Choose one type, and construct a graph to display the information in your table from part a).

e) Based on the given data, how many votes as favourite would each artist get if 250 adolescents were surveyed?
Problem of the Week
Problem B
Stop Pars!

Problem
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e) Based on the given data, how many votes as favourite would each artist get if 250 adolescents were surveyed?

Solution

<table>
<thead>
<tr>
<th>Artist</th>
<th>Given Data</th>
<th>No.of Votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pasty Kerry</td>
<td>20 out of 50</td>
<td>[\frac{20}{50} = \frac{40}{100}, \text{ so } 40 \text{ votes}]</td>
</tr>
<tr>
<td>Tustin Jimberlake</td>
<td>30% of 100</td>
<td>[\frac{30%}{100} = \frac{30}{100}, \text{ so } 30 \text{ votes}]</td>
</tr>
<tr>
<td>Bustin Jeiber</td>
<td>10% of 100</td>
<td>[10% = \frac{10}{100}, \text{ so } 10 \text{ votes}]</td>
</tr>
<tr>
<td>Tregan Maynor</td>
<td>remainder</td>
<td>[100 - 40 - 30 - 10 = 20, \text{ so } 20 \text{ votes}]</td>
</tr>
</tbody>
</table>
b) The number of students who voted for Tregan Maynor is 20 out of 100, or \( \frac{1}{5} \) of the students.

c) Good types of graphs might be

- a Bar Graph, which is good for comparing data;
- a Circle Graph, which is good for showing parts of a whole;
- a Pictograph, which is also good for comparing data.

d) Here are samples of a bar graph and a circle graph displaying the given data.

e) Based on the given data, if 250 adolescents were surveyed, the results are:

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</tr>
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</tr>
<tr>
<td>Tregan Maynor</td>
<td>remainder (( \frac{1}{5} ))</td>
<td>( \frac{1}{5} \times 250 = 50 ), so 50 votes</td>
</tr>
</tbody>
</table>
Sarah has three number discs in her pocket, with the digit 5 on one, 8 on another, and 9 on the third.

Miriam also has three number discs in her pocket, but hers have the numbers 2, 4, and 7 on them.

Sarah claims that if they each pull two discs out of their own pocket, without looking, then the sum of the digits on her two discs will most likely be greater than the product of the two digits from Miriam’s pocket.

a) The tree below will help to show all the possible combinations of sums and products. One of Sarah’s sums and one of Miriam’s products are filled in. Complete the tree by filling in the rest of the sums and products. Then determine the outcomes, using \( S \) if Sarah’s sum is greater than Miriam’s product and \( M \) if not. Hence find the theoretical probability that Sarah’s discs will have a higher sum than Miriam’s product.

![Tree diagram with filled in numbers]

b) Suppose instead that Miriam’s discs are numbered 2, 3, and 7. Do you think this would improve Sarah’s chances of being right? Explain your reasoning, and then check your conclusion by determining the theoretical probability as in a).

c) Is Sarah’s claim correct if Miriam’s discs are numbered 2, 4, and 6? Explain.

**Strand** Data Management and Probability
Problem of the Week
Problem B
Disc-Us This

Problem
Sarah has three number discs in her pocket, with the digit 5 on one, 8 on another, and 9 on the third.
Miriam also has three number discs in her pocket, but hers have the numbers 2, 4, and 7 on them.
Sarah claims that if they each pull two discs out of their own pocket, without looking, then the sum of the digits on her two discs will most likely be greater than the product of the two digits from Miriam’s pocket.

a) The tree below will help to show all the possible combinations of sums and products. One of Sarah’s sums and one of Miriam’s products are filled in. Complete the tree by filling in the rest of the sums and products. Then determine the outcomes, using S if Sarah’s sum is greater than Miriam’s product and M if not. Hence find the theoretical probability that Sarah’s discs will have a higher sum than Miriam’s product.

b) Suppose instead that Miriam’s discs are numbered 2, 3, and 7. Do you think this would improve Sarah’s chances of being right? Explain your reasoning, and then check your conclusion by determining the theoretical probability as in a).

c) Is Sarah’s claim correct if Miriam’s discs are numbered 2, 4, and 6? Explain.

Solution

a) Sarah’s possible sums are 5 + 8 = 13, 5 + 9 = 14, and 8 + 9 = 17; Miriam’s products are 2 × 4 = 8, 2 × 7 = 14, and 4 × 7 = 28. The completed tree is:

Since Sarah’s sums are greater in only 4 of the 9 outcomes, the theoretical probability is \( \frac{4}{9} \), and her claim is incorrect.
b) Sarah’s sums are the same; Miriam’s products are now $2 \times 3 = 6$, $2 \times 7 = 14$, and $3 \times 7 = 21$. The completed tree reveals that the outcomes do not change, so Sarah’s chances do not improve.

<table>
<thead>
<tr>
<th>Sarah’s Sums</th>
<th>13</th>
<th>14</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miriam’s Products</td>
<td>6</td>
<td>14</td>
<td>21</td>
</tr>
<tr>
<td>Outcomes</td>
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Note that decreasing the greater number 7 (to 6) has a more dramatic effect on the outcome. Perhaps a class discussion on why this is so would be worthwhile.

c) Sarah’s sums are still the same; Miriam’s possible products are now $2 \times 4 = 8$, $2 \times 6 = 12$, and $4 \times 6 = 24$. The completed tree this time reveals that the chance that Sarah’s sum is greater is now $\frac{6}{9} = \frac{2}{3}$, so her claim would be correct.

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Problem of the Week
Problem B
Pumpkins on Parade

Arrange with your teacher to bring several pumpkins to class, enough for each group of two or three students.

a) Discuss how many different ways could you measure a pumpkin, and how such measurements could be made.

b) Pick several of the possible measures to be done. Then team up in groups of two or three, make and record these measurements for your pumpkin. Compare your results with those of your classmates’ teams. Answer the following questions:

(i) What is the average (mean) of each of the specific measurements (e.g., the average of the diameters measured by each group of students)?
(ii) What is the range of each measurement?
(iii) What is the median of each measurement?
(iv) Is there a mode for each measurement?

Strands Measurement, Data Management and Probability
Problem of the Week
Problem B
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Solution

a) Possible measurements include circumference, diameter, height, length of stem, volume, mass (before and after scraping out the interior), number of seeds, number of grooves, temperature in degrees C (of the interior of the pumpkin), and capacity of the empty shell. Students may suggest others.

b) Answers will vary. Consider a sample set of diameter measurements

{22, 21, 22, 20, 23, 20, 23}, measured in cm.

(i) The mean of these 7 diameters is their sum divided by 7, i.e.,

\[ \frac{151}{7} \approx 21.6 \text{ cm}. \]

(ii) The range of this data set is \(23 - 20 = 3 \text{ cm}.\)

(iii) The median, or ‘middle’ number of this set is 22 cm.

(iv) Each of 20, 22, and 23 occurs twice. If there are two numbers occurring most often, we would say the data is bimodal. In this case, we might say it is multimodal.
Problem of the Week
Problem B
Mighty Math Means

Mary has had five unit tests in math so far this semester.

She has achieved scores of 86, 82, 93, 88, and 89 out of 100.

She knows there are only two unit tests left, and would like to achieve an overall average of 90 on the seven tests.

a) Can she achieve her desired average? If so, what average would she need to achieve on the last two tests?

b) What is the minimum score she could get on either of the last two tests, and still achieve an overall average of 90?
Problem of the Week
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Solution

a) To achieve an overall average of 90 on 7 tests, Mary needs $90 \times 7 = 630$ marks in total.
She currently has $86 + 82 + 93 + 88 + 89 = 438$ marks.
So she needs $630 - 438 = 192$ marks on the remaining two tests.
Thus her average on these two tests must be $192 \div 2 = 96$.

b) If Mary achieved the maximum possible grade of 100 on one of the last two tests, the minimum she could score on the other is 92, in order to obtain the 192 marks she needs.
Jon, Laura, Geoff, and Riley are seated around a circular card table ready to play a game.

a) How many different seating arrangements are possible?

b) If Katia joins the group, how many seating arrangements are now possible?
Problem of the Week  
Problem B  
Cards, Anyone?

Problem  
Jon, Laura, Geoff, and Riley are seated around a circular card table ready to play a game.

a) How many different seating arrangements are possible?

b) If Katia joins the group, how many seating arrangements are now possible?

Solution

a) For the four people Laura (L), Jon (J), Geoff (G), and Riley (R), there are six possible seating arrangements, as determined by the tree below.

```
  L
 /   \
J     G
 /   / \  \
R   R   J  J
 / |   |   |   |
G | R | J   R  G
 /     |   |   |   |
R   G  J  R  G  J
```

This result can also be obtained as follows: Assign Seat 1 to one player, say Laura; then there are three choices of who can sit to her left, in Seat 2. Once that player is chosen, say Jon, there are two players for Seat 3. Say Geoff is chosen; then only one possible player, Riley, remains for the Seat 4. Thus the number of possible arrangements is \( 3 \times 2 \times 1 = 6 \).
b) When a fifth player, Katia (K) is added to the game, the number of possible arrangements increases dramatically, from 6 to 24, as shown in the table below.

Using a similar alternative argument to the above, we can also arrive at 24 possible arrangements by choosing a player for Seat 1, which leaves four possible choices for Seat 2, three for Seat 3, two for seat 4, and only one for Seat 5. Thus the number of possible arrangements is \(4 \times 3 \times 2 \times 1 = 24\).

In the tree structure, these factors are portrayed by the number of lines connecting each level of the tree, 4 from Seat 1 to Seat 2, 3 from Seat 2 to Seat 3, 2 from Seat 3 to Seat 4, and 1 from Seat 4 to Seat 5.

Note that in either case, rotating the players around the table is not regarded as a new arrangement since they would still play the card game in the same order.
Problem of the Week
Problem B
Cell Phone Dilemma

You’re excited: your parents are permitting you to get your first cell phone! They don’t want to spend too much money on your plan, but they don’t want to do the research about different cell phone plans. It’s your phone; they expect you to make sure they pay the least amount possible.

Having done some digging, you’ve discovered you can get the $\pi$Phone 3.14, or an equivalent cool phone, with a two-year contract on any of the following plans from Telem, Smell, or Doggers. However, the cost of the phone varies.

<table>
<thead>
<tr>
<th>Phone</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Telem</td>
<td>$200</td>
</tr>
<tr>
<td>Smell</td>
<td>$100</td>
</tr>
<tr>
<td>Doggers</td>
<td>Free</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Plan</th>
<th>Cost/Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>500 MB</td>
<td>$45</td>
</tr>
<tr>
<td>1 GB</td>
<td>$70</td>
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</table>

A fourth possibility is the Hurricane Network, with which you can get a phone for $648, and pay the monthly fees shown at the right, but without a contract.

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Which of these four plans should you choose in order to minimize your parents' costs over two years, assuming you wish 1 GB of data per month?

Use the following steps to guide your reasoning:

1. Make a table for each plan showing the costs for one, two, and three months.
2. Write a rule which gives the cost for the phone after any number of months, for each plan. Describe clearly the meaning of each symbol in your rule.
Problem of the Week
Problem B
Cell Phone Dilemma

Problem
You’re excited: your parents are permitting you to get your first cell phone! They don’t want to spend too much money on your plan, but they don’t want to do the research about different cell phone plans. It’s your phone; they expect you to make sure they pay the least amount possible.

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Telem
Phone: $200

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Smell
Phone: $100

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Doggers
Phone: Free

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Which of these four plans should you choose in order to minimize your parents’ costs over two years, \textbf{assuming you wish 1 GB of data per month}?

Use the following steps to guide your reasoning:

1. Make a table for each plan showing the costs for one, two, and three months.

2. Write a rule which gives the cost for the phone after any number of months, for each plan. Describe clearly the meaning of each symbol in your rule.

\textbf{Solution}

Assuming a 1 GB data plan (using ‘unlimited’ where necessary), the costs for 1, 2, and 3 months are shown in the table, plus the cost of the phone.

\begin{center}
\begin{tabular}{|l|c|c|c|c|}
\hline
Vendor & Phone Cost & Cost for 1 month & Cost for 2 months & Cost for 3 months \\
\hline
Telem & $200 & $70 & $140 & $210 \\
Smell & $100 & $80 & $160 & $320 \\
Doggers & $0 & $85 & $170 & $255 \\
Hurricane & $648 & $50 & $100 & $150 \\
\hline
\end{tabular}
\end{center}

General rules for the cost $C$ for $N$ months, and for 24 months, for 1 GB of data are

- Telem: $C = 200 + N \times 70$, so $N = 24$ gives $C = 200 + 24 \times 70 = 1880$.
- Smell: $C = 100 + N \times 80$, so $N = 24$ gives $C = 100 + 24 \times 80 = 2020$.
- Doggers: $C = 0 + N \times 85$, so $N = 24$ gives $C = 0 + 24 \times 85 = 2040$.
- Hurricane: $C = 648 + N \times 50$, so $N = 24$ gives $C = 648 + 24 \times 50 = 1848$.

Thus the plan of least cost is with Hurricane, due to the fact that over a long time period, the much lower monthly cost offsets the initially large cost for the phone.

Telem is a close second, only $32$ more, so it would also be a sensible choice. This second choice would require a much smaller initial outlay of money towards the plan.

A discussion could follow about the merits of each plan and why someone might still choose the Dogger’s plan.
Brave Bart rides his bike from his home to his Uncle Sam’s house. On the way, he stops at his friend Pietro’s house, and later at the grocery store to buy apples for his Uncle.

The distance and time for each leg of his trip are shown on the diagram below, not including the stops.

a) What is Bart’s speed (M1) in km per minute on the first leg of his trip? On the second leg (M2)? On the third leg (M3)?

b) Convert Bart’s speeds M1, M2, M3 to km per hour.

c) What is Bart’s average (mean) speed (A) in km per minute for the whole bike trip from home to Uncle Sam’s? In km per hour?

d) Is your answer to b) the same as the average (mean) of M1, M2, and M3 (i.e., does $A=\frac{1}{3}(M1 + M2 + M3)$)? Think carefully about why it is, or is not.

**STRANDS** Data Management and Probability, Measurement
Problem of the Week
Problem B and Solution
Bart’s Likin’ Bikin’

Problem
Brave Bart rides his bike from his home to his Uncle Sam’s house. On the way, he stops at his friend Pietro’s house, and later at the grocery store to buy apples for his Uncle.

The distance and time for each leg of his trip are shown on the diagram below, not including the stops.

![Diagram of Bart's bike trip]

a) What is Bart’s speed (M1) in km per minute on the first leg of his trip? On the second leg (M2)? On the third leg (M3)?

b) Convert Bart’s speeds M1, M2, M3 to km per hour.

c) What is Bart’s average (mean) speed (A) in km per minute for the whole bike trip from home to Uncle Sam’s? In km per hour?

d) Is your answer to b) the same as the average (mean) of M1, M2, and M3 (i.e., does \[ A = \frac{1}{3} (M1 + M2 + M3) \])? Think carefully about why it is, or is not.

Solution

a) Dividing distance by time on each leg of his trip gives Bart’s speeds as
\[ M1 = \frac{5}{20} = \frac{1}{4} \text{ km/min}, \quad M2 = \frac{10}{30} = \frac{1}{3} \text{ km/min}, \quad \text{and} \quad M3 = \frac{8}{30} = \frac{4}{15} \text{ km/min}. \]

b) Multiplying by 60 min/h gives these speeds in km/h as M1 = 15 km/h, M2 = 20 km/h, and M3 = 16 km/h.

c) Bart’s average (mean) speed for the whole trip is
\[ A = \frac{\text{total distance}}{\text{total time}} = \frac{5 + 10 + 8}{20 + 30 + 30} = \frac{23}{80} \text{ km/min}, \]
which gives A = 60 × \(\frac{23}{80}\) = 17\(\frac{1}{4}\) km/h.

d) The value of \(\frac{1}{3}(M1 + M2 + M3)\) is \(\frac{1}{3}(15 + 20 + 16) = \frac{51}{3} = 17 \text{ km/h}\), which is NOT the same as A. This is because the times for the three legs are not the same; if they were, the two different averages would be the same.
Geometry & Spatial Sense
Problem of the Week
Problem B
Gimme Five!

Sports teams often celebrate a good play or a winning goal by giving one another ‘high fives’.

a) Emelia’s basketball team won their game. Everyone gave everyone else on the team (5 players in total) a single high five. How many high fives were exchanged?

b) If the ‘spare’ player on Emelia’s team were included in the high fives, how many exchanges would occur?

c) Yousef scored the winning goal for his soccer team. If all 11 players gave high fives to each other, how many high fives were exchanged?

Strands  Patterning and Algebra, Geometry and Spatial Sense
Problem of the Week
Problem B
Gimme Five!

Problem
Sports teams often celebrate a good play or a winning goal by giving one another ‘high fives’.

a) Emelia’s basketball team won their game. Everyone gave everyone else on the team (5 players in total) a single high five. How many high fives were exchanged?

b) If the ‘spare’ player on Emelia’s team were included in the high fives, how many exchanges would occur?

c) Yousef scored the winning goal for his soccer team. If all 11 players gave high fives to each other, how many high fives were exchanged?

Solution

a) This is a great example of a problem that students should “act out” to understand. Let the five players (students) be represented by A, B, C, D, and E. Then, thinking of the high fives as being done in order, proceed as follows.
• A gives 4 high fives, one to each of B, C, D, and E;

• B gives 3 high fives, one to each of C, D, and E (she’s already done A);

• C gives 2 high fives, one to each of D, and E (she’s already done A and B);

• D gives 1 high five to E (she’s already done the others).

Thus a total of $4 + 3 + 2 + 1 = 10$ high fives were exchanged.

This is illustrated by the diagram at the right. Each dashed line represents one high five.

b) Let the ‘spare’ be represented by F. Then, following the same reasoning as in part a), we see from the diagram that A would give 5 high fives (to B, C, D, E, and F), B would give 4 high fives (to C, D, E, and F), C would give 3, D would give 2, and E would give 1.

Thus a total of $5 + 4 + 3 + 2 + 1 = 15$ high fives would be exchanged.

c) The reasoning in a) and b) generalizes to the high fives among the 11 players on Yousef’s soccer team, giving a total exchange of

$$10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 55$$

high fives.

Note that each of these can be visualized as a geometry problem: the number of high fives is equal to the number of sides plus the number of diagonals of a regular polygon.
Problem of the Week

Problem B

Colour Me Happy!

a) Below is a map of Canada and its provinces and territories. What is the fewest number of colours required to colour each province and territory so that no two adjacent (side-by-side) regions are the same colour?

Regions that meet only at a corner could be the same colour. You may assume that regions separated by a body of water are not adjacent.

b) Now consider a map of Europe. What is the fewest number of colours needed so that no two adjacent countries are the same colour?

HINT: Start with the smallest countries (i.e., San Marino, Vatican City, Liechtenstein, Luxembourg).
Strand  Geometry and Spatial Sense
Problem of the Week
Problem B and Solution
Colour Me Happy!

Problem

a) Below is a map of Canada and its provinces and territories. What is the fewest number of colours required to colour each province and territory so that no two adjacent (side-by-side) regions are the same colour? Regions that meet only at a corner could be the same colour. You may assume that regions separated by a body of water are not adjacent.

b) Now consider a map of Europe. What is the fewest number of colours needed so that no two adjacent countries are the same colour?

Hint: Start with the smallest countries (i.e., San Marino, Vatican City, Liechtenstein, Luxembourg).

Solution

a) The map to the right reveals that only three colours are required to colour a map of Canada such that no two adjacent provinces or territories are the same colour.

b) The second map reveals that four colours are required to colour a map of Europe such that no two adjacent countries are the same colour. Colours are represented by the numbers 1 to 4 inside the boundaries of the countries.

Note: Greenland (not shown) is part of the Kingdom of Denmark, hence also technically in Europe.

See the next page for a fuller explanation of the need for four colours.
Part of the map of Europe has been magnified and is shown below. Austria (1) is labelled on the map below to the left. If you have a colour printer Austria is coloured green. (Otherwise, Austria will appear to be shaded.)

It may not be obvious that you must use four colours. Austria is bordered by 7 distinct countries. A diagram showing Austria and the seven border countries is shown above to the right. Use colour (1) (green if you have a colour printer) for Austria. Starting with Hungary use purple (3). Going clockwise, use pink (2) for Slovenia. Then use purple (3) for Italy, pink (2) for Switzerland, purple (3) for Germany and pink (2) for the Czech Republic. So far only three colours have been used. So what colour do we use for Slovakia? It cannot be purple (3) or it would be the same as Hungary. It cannot be green (1) or it would be the same as Austria. It cannot be pink (2) or it would be the same as the Czech Republic. No matter how we attempt to colour, a fourth colour is required.

**Comment:**

A famous mathematical theorem, the Four Colour Theorem, proves that if you divide up the plane into contiguous (adjoining) regions, no more than four colours are needed to colour any such map so that no two adjacent regions are the same colour. (Adjacent regions share a common boundary that is not just a point like the corner where Nunavut, Manitoba, Saskatchewan, and the Northwest Territories meet.)
Problem of the Week
Problem B
Chipping Away

In the popular game video game Minecraft™, players have to get materials to build the tools they need. Marsha is playing and wants to make sure she has a good stock of the following tools:

- wood pick axes, each requiring 5 sticks;
- swords, each requiring 1 stick and 2 cobblestone blocks;
- shovels, each requiring 2 sticks and 1 cobblestone block;
- hoes, each requiring 2 sticks and 2 cobblestone blocks.

a) If she wants to have 6 of each tool, how many cobblestone blocks and how many sticks will she need?

b) Marsha has decided to build a house with a square base 8 blocks long by 8 blocks wide, and 4 blocks high. How many blocks will she need to build all the walls of her house? Be careful with the corners!

c) Once the walls are constructed, Marsha decides to build a pyramid-shaped roof on her house. For example, the first layer will have dimensions of 7 by 7 blocks; think about why this layer will require 24 blocks. The second layer will be 6 by 6 blocks, etcetera, until there is only 1 block on top. How many blocks will she need for the roof?

d) If each pick axe can mine 60 blocks, how many pick axes are needed to build the walls and roof of Marsha’s house?
Problem of the Week
Problem B and Solution
Chipping Away

Problem
In the popular game video game Minecraft\textsuperscript{TM}, players have to get materials to build the tools they need. Marsha is playing and wants to make sure she has a good stock of the following tools:

- wood pick axes, each requiring 5 sticks;
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c) Once the walls are constructed, Marsha decides to build a pyramid-shaped roof on her house. For example, the first layer will have dimensions of 7 by 7 blocks; think about why this layer will require 24 blocks. The second layer will be 6 by 6 blocks, etcetera, until there is only 1 block on top. How many blocks will she need for the roof?

d) If each pick axe can mine 60 blocks, how many pick axes are needed to build the walls and roof of Marsha’s house?

Solution

a) Examining the given information, we see that one of each tool would require $5 + 1 + 2 + 2 = 10$ sticks, and $0 + 2 + 1 + 2 = 5$ cobblestone blocks. Thus, if Marsha wants to have 6 of each tool, she will need $10 \times 6 = 60$ sticks, and $5 \times 6 = 30$ cobblestone blocks.

b) From the diagram, one layer of Marsha’s house will require $8 + 8 + 6 + 6 = 28$ blocks. (Note that you can think of the square base as two ‘lengths’ of 8 blocks, with two ‘widths’ of 6 blocks tucked between them.) Thus if her house is to be 4 blocks high, all the walls will require $28 \times 4 = 112$ blocks.
c) Each layer of the pyramidal roof will be a square, similar to the base walls. The table below lists the dimensions of each layer in blocks (from the bottom up), and the total blocks required for each layer, and for the entire roof. The terms ‘width’ and ‘length’ are used here in the same manner as in the solution for part b).

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>‘Width’</th>
<th>‘Length’</th>
<th>Layer Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 × 7</td>
<td>5</td>
<td>7</td>
<td>24</td>
</tr>
<tr>
<td>6 × 6</td>
<td>4</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>5 × 5</td>
<td>3</td>
<td>5</td>
<td>16</td>
</tr>
<tr>
<td>4 × 4</td>
<td>2</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>3 × 3</td>
<td>1</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>2 × 2</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>1 × 1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>Total Blocks 85</strong></td>
</tr>
</tbody>
</table>

Here’s how the roof would look:

---

d) Since the total number of blocks for both walls and roof is $112 + 85 = 197$, and each pick axe can mine 60 blocks, she would need 4 pick axes. (Three is too few, since they would only mine $3 \times 60 = 180$ blocks.)
Mrs. Aviary’s class wanted to study the birds on the school’s property for their Biodiversity Study. They decided to make some bird houses for the different birds they wanted to attract. Here is a front view and an angled view of their bird house design:

\[ a) \text{ Using } 1 \text{ cm grid paper, carefully sketch a right-angled triangle with sides } 6 \text{ cm and } 7 \text{ cm. Then measure the length of the hypotenuse } R \text{ to determine the slanted roof length.} \]

\[ b) \text{ Make a net to show how the pieces of the bird house could be laid out before putting it together. Compare your net to those of your classmates. Are all of the nets the same?} \]

\[ c) \text{ Polly and her friend Sylvester have decided to make their birdhouse out of cedar, because of its pleasant aroma and durability. } \]

How many square centimetres of cedar wood will they need to construct their beautiful bird abode?

**Strands** Geometry and Spatial Sense, Measurement
Problem of the Week
Problem B
This Problem is for the Birds!

Problem
Mrs. Aviary’s class wanted to study the birds on the school’s property for their Biodiversity Study. They decided to make some bird houses for the different birds they wanted to attract. Here is a front view and an angled view of their bird house design:

![Bird House Diagram]

a) Using 1 cm grid paper, carefully sketch a right-angled triangle with sides 6 cm and 7 cm. Then measure the length of the hypotenuse $R$ to determine the slanted roof length.

b) Make a net to show how the pieces of the bird house could be laid out before putting it together. Compare your net to those of your classmates. Are all of the nets the same?

c) Polly and her friend Sylvester have decided to make their birdhouse out of cedar, because of its pleasant aroma and durability. How many square centimetres of cedar wood will they need to construct their beautiful bird abode?

Solution
a) Careful construction and measurement reveals that R is approximately 9.2 cm in length.
b) Here are two possible nets (you and your classmates may find others).

c) The area of the pieces of the left net above are calculated in the table below, revealing that the total area of cedar required is \(927.2 \text{ cm}^2\). While both nets have the same total area, the net on the right would be a more efficient way to use the wood. Using that net, the pieces could be sawn from a cedar board \(33 \text{ cm} \times 40 \text{ cm}\).

<table>
<thead>
<tr>
<th>Pieces</th>
<th>Area in cm(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 of A1</td>
<td>(2 \times (9.2 \times 8) = 147.2)</td>
</tr>
<tr>
<td>2 of A2</td>
<td>(2 \times (15 \times 8) = 240)</td>
</tr>
<tr>
<td>2 of A3</td>
<td>(2 \times \frac{1}{2}(12 \times 7) = 84)</td>
</tr>
<tr>
<td>2 of A4</td>
<td>(2 \times (12 \times 15) = 360)</td>
</tr>
<tr>
<td>1 of A5</td>
<td>(12 \times 8 = 96)</td>
</tr>
<tr>
<td>Total</td>
<td>(927.2)</td>
</tr>
</tbody>
</table>
Measurement
Problem of the Week
Problem B
Moe is no Stooge!

Moe helps out the seniors Larry and Curly on his block by mowing their lawns during the summer. He can mow a square lawn of side length 30 m in 45 minutes.

a) If he works at the same rate, how many minutes will it take Moe to mow a square lawn of side length 60 m?

b) What are the dimensions of possible rectangular lawns with the same area as the lawn of side 30 m, and dimensions which are whole numbers? Which of these are you likely to see in the city?
Problem of the Week
Problem B
Moe is no Stooge!

Problem
Moe helps out the seniors Larry and Curly on his block by mowing their lawns during the summer. He can mow a square lawn of side length 30 m in 45 minutes.

a) If he works at the same rate, how many minutes will it take Moe to mow a square lawn of side length 60 m?

b) What are the dimensions of possible rectangular lawns with the same area as the lawn of side 30 m, and dimensions which are whole numbers? Which of these are you likely to see in the city?

Solution

a) The area of a lawn 30 m by 30 m is $30 \times 30 = 900 \text{ m}^2$. Thus Moe can mow $900 \text{ m}^2$ in 45 minutes. We can see from the diagram that a lawn 60 m by 60 m is four times the area of the 30 m by 30 m lawn. Thus it will take Moe $4 \times 45 = 180$ minutes, or 3 hours to mow the larger lawn.

b) The dimensions of a lawn of area $900 \text{ m}^2$ must be factors of 900. The first eight possibilities (highly unlikely in a city) are $1 \times 900$, $2 \times 450$, $3 \times 300$, $4 \times 225$, $5 \times 180$, $6 \times 150$, $9 \times 100$, $10 \times 90$, and $12 \times 75$. The possibilities which are perhaps more likely include $12 \times 75$, $15 \times 60$, $18 \times 50$, $20 \times 45$, $25 \times 36$, and $30 \times 30$. 

\[
\begin{array}{c}
30 \\
\hline
30\ \text{m} \quad 30\ \text{m} \quad 30\ \text{m} \\
\hline
60\ \text{m} \quad 60\ \text{m} \\
\end{array}
\]
Problem of the Week
Problem B
Pumpkins on Parade

Arrange with your teacher to bring several pumpkins to class, enough for each group of two or three students.

a) Discuss how many different ways could you measure a pumpkin, and how such measurements could be made.

b) Pick several of the possible measures to be done. Then team up in groups of two or three, make and record these measurements for your pumpkin. Compare your results with those of your classmates’ teams. Answer the following questions:

(i) What is the average (mean) of each of the specific measurements (e.g., the average of the diameters measured by each group of students)?
(ii) What is the range of each measurement?
(iii) What is the median of each measurement?
(iv) Is there a mode for each measurement?

Strands Measurement, Data Management and Probability
Problem of the Week
Problem B
Pumpkins on Parade

Problem
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(iv) Is there a mode for each measurement?

Solution

a) Possible measurements include circumference, diameter, height, length of stem, volume, mass (before and after scraping out the interior), number of seeds, number of grooves, temperature in degrees C (of the interior of the pumpkin), and capacity of the empty shell. Students may suggest others.

b) Answers will vary. Consider a sample set of diameter measurements \{22, 21, 22, 20, 23, 20, 23\}, measured in cm.

(i) The mean of these 7 diameters is their sum divided by 7, i.e.,\[
\frac{151}{7} \approx 21.6 \text{ cm}.
\]
(ii) The range of this data set is \(23 - 20 = 3\) cm.
(iii) The median, or ‘middle’ number of this set is 22 cm.
(iv) Each of 20, 22, and 23 occurs twice. If there are two numbers occurring most often, we would say the data is bimodal. In this case, we might say it is multimodal.
A good way to ensure that you do a complete job of washing your hands is to sing the ‘Happy Birthday’ song while you wash.

a) For approximately how long will you wash your hands each time?

b) If you wash your hands 6 times per day, how many seconds will you spend washing in total each day?

c) Supposing you could reach the bathroom sink to wash your own hands by your 4th birthday, and that you have done so ever since as in parts a) and b), how many hours and minutes have you spent washing your hands so far?

d) If you live to be 75 years old, how many days of handwashing will you have done by then?
Problem of the Week
Problem B
Grubby Paws

Problem
A good way to ensure that you do a complete job of washing your hands is to sing the ‘Happy Birthday’ song while you wash.

a) For approximately how long will you wash your hands each time?

b) If you wash your hands 6 times per day, how many seconds will you spend washing in total each day?

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d) If you live to be 75 years old, how many days of handwashing will you have done by then?

Solution
Note that all the answers in the solution to this problem will depend on your answer to part a), which may vary.

a) Hand washing to the tune ‘Happy Birthday’ takes about 10 seconds each time.

b) Thus, if you wash your hands 6 times per day, you will spend $6 \times 10 = 60$ seconds total per day washing your hands.

c) Answers here will depend on your age. Suppose you are 10 years old; then you will have been able to wash your own hands for the past 6 years.
Since there are 365 days in each year, 6 years equals $365 \times 6 = 2190$ days.
At 60 seconds, or 1 minute per day, you will have spent 2190 minutes washing your hands since you were 4 years old.
There are 60 minutes in an hour, so this is equivalent to $2190 \div 60 = 36.5$ hours, or 36 hours and 30 minutes.

d) If you live to be 75, you will have spent $(75 - 4) = 71$ years washing your hands. Since you spend 1 minute per day, this is $71 \times 365 \times 1 = 25915$ minutes, or $25915 \div 60 \approx 432$ hours, which is $432 \div 24 = 18$ days of hand washing.
Dr. Chapman has 4000 letters to mail to her clients. She has determined that she can

1. place a stamp, a sending label, and a return label on the envelope in 13 seconds,

2. sign, fold, and place the letter in the envelope in 15 seconds, and

3. seal and place the envelope in a box in 8 seconds.

a) How many hours will it take her to complete the task?

b) If Dr. Chapman hired two other people to help, with her doing step 2., and the others doing steps 1. and 3., how long will the task take?

c) Suppose she notices that she can sign and fold each letter in 10 seconds, but that it then takes the third person 13 seconds to place the letter in the envelope, seal it, and place it in a box. Would this be a more efficient way to do this task?
Problem of the Week
Problem B and Solution
Postal Processing

Problem
Dr. Chapman has 4 000 letters to mail to her clients. She has determined that she can

1. place a stamp, a sending label, and a return label on the envelope in 13 seconds,

2. sign, fold, and place the letter in the envelope in 15 seconds, and

3. seal and place the envelope in a box in 8 seconds.

a) How many hours will it take her to complete the task?

b) If Dr. Chapman hired two other people to help, with her doing step 2., and the others doing steps 1. and 3., how long will the task take?

c) Suppose she notices that she can sign and fold each letter in 10 seconds, but that it then takes the third person 13 seconds to place the letter in the envelope, seal it, and place it in a box. Would this be a more efficient way to do this task?

Solution
a) The three tasks will take the following times:

   Step 1. Stamping and labelling: \( 4 000 \times 13 = 52 000 \text{ s} \);

   Step 2. Signing, folding, and stuffing: \( 4 000 \times 15 = 60 000 \text{ s} \);

   Step 3. Sealing and boxing: \( 4 000 \times 8 = 32 000 \text{ s} \)

   Thus the total time is 144 000 s, which is \( 144 000 \div 3 600 = 40 \text{ hr} \), since there are \( 60 \times 60 = 3 600 \text{ s} \) in 1 hr.

b) Step 2. is the slowest: Dr. Chapman will have to wait 13 s for the first envelope from Step 1; and the person doing Step 3 will have to wait 28 s for the first letter to seal, but will make up that time quickly since Step 3 takes only 8 s. So the whole task will take 13 s, plus the 60 000 s of Step 2., plus 8 s for the final envelope to be sealed and placed in a box, giving a total time of 60 000 s = \( 16 \frac{2}{3} \text{ hr} \) (or 16 hr, 40 min), plus 21 s.

c) Since the longest task is now 13 s instead of 15 s, this would be a more efficient way to do this task. Further, the first two tasks are now independent and can be done simultaneously. Thus the total time would now be \( 4 000 \times 13 = 52 000 \text{ s} \), plus 13 s for the final envelope, or about \( 14 \frac{1}{2} \text{ hr} \).
Problem of the Week
Problem B
Ms Orlando “Takes the Cake”

Ms. Orlando’s Grade 5 class has baked her a birthday cake. They wish to divide the 12 cm by 18 cm cake evenly among the 29 students and the teacher.

a) If each person receives a piece of the same size, what would be its top surface area?

b) Assuming the pieces are rectangular, with length being a whole number of cm, what would be the width of piece which is closest to a square? (Measurements of widths need not be whole numbers.)

c) Could the cake be cut evenly into 30 pieces which are NOT rectangular? Illustrate your answer with diagram(s).

Strands  Number Sense and Numeration, Measurement
Problem of the Week
Problem B and Solution
Ms Orlando “Takes the Cake”

Problem
Ms. Orlando’s Grade 5 class has baked her a birthday cake. They wish to divide the 12 cm by 18 cm cake evenly among the 29 students and the teacher.

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c) Could the cake be cut evenly into 30 pieces which are NOT rectangular? Illustrate your answer with diagram(s).

Solution

a) The total surface area of the top of the cake is $18 \times 12 = 216 \text{ cm}^2$.
   Thus the top surface area of each equal piece would be $216 \div 30 = 7.2 \text{ cm}^2$.

b) With length a whole number and area 7.2 cm$^2$, possible dimensions of the pieces, in cm, are
   $1 \times 7.2, 2 \times 3.6, 3 \times 2.4, 4 \times 1.8, 5 \times 1.44, 6 \times 1.2, 8 \times 0.9, 12 \times 0.6$, and $18 \times 0.4$.
   Thus the one closest to a square would be 3 cm in length, with width 2.4 cm.

c) There are many possibilities for non-rectangular equal pieces. Here are three.
Problem of the Week
Problem B
Two’s Company

a) The dimensions $A\text{ cm} \times B\text{ cm}$, and $N\text{ cm} \times M\text{ cm}$, of the two rectangles shown below are four different single digits from 1, 2, \ldots, 9.

If the two rectangles have the same area, what digits CANNOT be $A$, $B$, $N$, or $M$?

b) Does your answer change if the rectangles are not congruent, but the digits can be repeated, i.e., $A$, $B$, $M$, and $N$ need not be distinct?
Problem of the Week
Problem B
Two’s Company

Problem

a) The dimensions \(A\) cm by \(B\) cm, and \(N\) cm by \(M\) cm, of the two rectangles shown below are four different single digits from 1, 2, \(\cdots\), 9.

If the two rectangles have the same area, what digits CANNOT be \(A\), \(B\), \(N\), or \(M\)?

\[\begin{array}{c}
A \\
B \\
M \\
N \\
\end{array}\]

b) Does your answer change if the rectangles are not congruent, but the digits can be repeated, i.e., \(A\), \(B\), \(M\), and \(N\) need not be distinct?

Solution

a) The table on the right displays all products of distinct digits; the shaded entries are repeated. For example, \(A \times B\) could be 1 \(\times\) 8 and \(M \times N\) could be 2 \(\times\) 4.

Those products involve digits 1, 2, 3, 4, 6, 8, 9. Thus we see that the only digits which cannot be \(A\), \(B\), \(M\), or \(N\) are 5 and 7.

\[
\begin{array}{cccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
1 & & & & & & & & & \\
2 & & & & & & & & & \\
3 & & & & & & & & & \\
4 & & & & & & & & & \\
5 & & & & & & & & & \\
6 & & & & & & & & & \\
7 & & & & & & & & & \\
8 & & & & & & & & & \\
\end{array}
\]

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
1 & 6 & 8 & 10 & 12 & 14 & 16 & 18 \\
3 & 12 & 15 & 18 & 21 & 24 & 27 & \\
4 & 20 & 24 & 28 & 32 & 36 & \\
5 & 30 & 35 & 40 & 45 & \\
6 & 42 & 48 & 54 & \\
7 & 56 & 63 & \\
8 & 72 & \\
\end{array}
\]

b) If the digits can be repeated, then we also have the possibilities \(1 \times 4 = 2 \times 2\), \(1 \times 9 = 3 \times 3\), \(2 \times 8 = 4 \times 4\), and \(4 \times 9 = 6 \times 6\), i.e., the perfect squares. However, neither 5 nor 7 appears, so the answer does not change.
Problem of the Week
Problem B
This Problem is for the Birds!

Mrs. Aviary’s class wanted to study the birds on the school’s property for their Biodiversity Study. They decided to make some bird houses for the different birds they wanted to attract.

Here is a front view and an angled view of their bird house design:

![Diagram of bird house]

a) Using 1 cm grid paper, carefully sketch a right-angled triangle with sides 6 cm and 7 cm. Then measure the length of the hypotenuse $R$ to determine the slanted roof length.

b) Make a net to show how the pieces of the bird house could be laid out before putting it together. Compare your net to those of your classmates. Are all of the nets the same?

c) Polly and her friend Sylvester have decided to make their birdhouse out of cedar, because of its pleasant aroma and durability.

How many square centimetres of cedar wood will they need to construct their beautiful bird abode?

**Strands**  Geometry and Spatial Sense, Measurement
Problem of the Week
Problem B
This Problem is for the Birds!

Problem
Mrs. Aviary’s class wanted to study the birds on the school’s property for their Biodiversity Study. They decided to make some bird houses for the different birds they wanted to attract. Here is a front view and an angled view of their bird house design:

![Diagram of bird house](image_url)

a) Using 1 cm grid paper, carefully sketch a right-angled triangle with sides 6 cm and 7 cm. Then measure the length of the hypotenuse \( R \) to determine the slanted roof length.

b) Make a net to show how the pieces of the bird house could be laid out before putting it together. Compare your net to those of your classmates. Are all of the nets the same?

c) Polly and her friend Sylvester have decided to make their birdhouse out of cedar, because of its pleasant aroma and durability. How many square centimetres of cedar wood will they need to construct their beautiful bird abode?

Solution
a) Careful construction and measurement reveals that \( R \) is approximately 9.2 cm in length.
b) Here are two possible nets (you and your classmates may find others).

![Diagram of nets]

While both nets have the same total area, the net on the right would be a more efficient way to use the wood. Using that net, the pieces could be sawn from a cedar board 33 cm by 40 cm.

c) The area of the pieces of the left net above are calculated in the table below, revealing that the total area of cedar required is 927.2 cm².

While both nets have the same total area, the net on the right would be a more efficient way to use the wood. Using that net, the pieces could be sawn from a cedar board 33 cm by 40 cm.

<table>
<thead>
<tr>
<th>Pieces</th>
<th>Area in cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 of A1</td>
<td>$2 \times (9.2 \times 8) = 147.2$</td>
</tr>
<tr>
<td>2 of A2</td>
<td>$2 \times (15 \times 8) = 240$</td>
</tr>
<tr>
<td>2 of A3</td>
<td>$2 \times \frac{1}{2}(12 \times 7) = 84$</td>
</tr>
<tr>
<td>2 of A4</td>
<td>$2 \times (12 \times 15) = 360$</td>
</tr>
<tr>
<td>1 of A5</td>
<td>$12 \times 8 = 96$</td>
</tr>
<tr>
<td>Total</td>
<td>927.2</td>
</tr>
</tbody>
</table>
Marianna has a 1 m long by 30 cm wide window box in which she is planting marigolds. Each plant must be at least 10 cm from the sides of the planter, and 10 cm from any other marigold.

a) What is the maximum number of marigolds Marianna can plant in her 1 m by 30 cm rectangular window box, with the desired 10 cm spacing?

b) Suppose another of Marianna’s window boxes has a trapezoidal shape, as shown below. (The diagram shows the surface of the soil, as viewed from above, so it tells you the shape within which her flowers must be planted.)

(i) Make a precise drawing of this cross-section, exactly to scale.
(ii) Use a protractor to measure the angle A on your diagram.
(iii) Keeping your scale in mind, use a ruler to see where Marianna could place her marigolds in this window box, assuming the same 10 cm constraints as above.

How many marigolds can she plant in this window box?
Problem of the Week
Problem B and Solution
Flower Power

Problem
Marianna has a 1 m long by 30 cm wide window box in which she is planting marigolds. Each plant must be at least 10 cm from the sides of the planter, and 10 cm from any other marigold.

a) What is the maximum number of marigolds Marianna can plant in her window box?

b) Suppose another of Marianna’s window boxes has a trapezoidal cross-section, as shown below at the right.

(i) Make a precise drawing of this cross-section, exactly to scale.

(ii) Use a protractor to measure the angle $A$ on your diagram.

(iii) Keeping your scale in mind, use a ruler to see where Marianna could place her marigolds in this window box, assuming the same 10 cm constraints as above.

How many marigolds can she plant in this window box?

Solution

a) The dashed lines on the diagram below are spaced at 10 cm, revealing that Marianna can plant a maximum of two rows of 9 marigolds in her rectangular window box, a total of 18 marigolds.
b) Using a protractor shows that the angle $A$ is about $56^\circ$.
Clearly two rows of plants will fit, since the width of this planter is 30 cm.
Careful measurement with a ruler reveals that the width of the planter
across the upper row is about 87 cm, and thus will fit 7 plants, while the
width across the lower row is about 73.5 cm, which will fit 6 plants.
Thus this box will hold a total of 13 marigolds.
Brave Bart rides his bike from his home to his Uncle Sam’s house. On the way, he stops at his friend Pietro’s house, and later at the grocery store to buy apples for his Uncle.

The distance and time for each leg of his trip are shown on the diagram below, not including the stops.

Strands: Data Management and Probability, Measurement
Problem of the Week
Problem B and Solution
Bart’s Likin’ Bikin’

Problem
Brave Bart rides his bike from his home to his Uncle Sam’s house. On the way, he stops at his friend Pietro’s house, and later at the grocery store to buy apples for his Uncle.

The distance and time for each leg of his trip are shown on the diagram below, not including the stops.

a) What is Bart’s speed (M1) in km per minute on the first leg of his trip? On the second leg (M2)? On the third leg (M3)?
b) Convert Bart’s speeds M1, M2, M3 to km per hour.
c) What is Bart’s average (mean) speed (A) in km per minute for the whole bike trip from home to Uncle Sam’s? In km per hour?
d) Is your answer to b) the same as the average (mean) of M1, M2, and M3 (i.e., does A=\frac{1}{3}(M1 + M2 + M3))? Think carefully about why it is, or is not.

Solution
a) Dividing distance by time on each leg of his trip gives Bart’s speeds as M1= \frac{5}{20} = \frac{1}{4} \text{ km/min}, M2= \frac{10}{30} = \frac{1}{3} \text{ km/min}, and M3= \frac{8}{30} = \frac{4}{15} \text{ km/min}.
b) Multiplying by 60 min/h gives these speeds in km/h as M1= 15 \text{ km/h}, M2= 20 \text{ km/h}, and M3= 16 \text{ km/h}.
c) Bart’s average (mean) speed for the whole trip is A= \frac{\text{total distance}}{\text{total time}} = \frac{5+10+8}{20+30+30} = \frac{23}{80} \text{ km/min}, which gives A= 60 \times \frac{23}{80} = 17 \frac{1}{4} \text{ km/h}.
d) The value of \frac{1}{3}(M1 + M2 + M3) is \frac{1}{3}(15 + 20 + 16) = \frac{51}{3} = 17 \text{ km/h}, which is NOT the same as A. This is because the times for the three legs are not the same; if they were, the two different averages would be the same.
Number Sense
&
Numeration
Problem of the Week
Problem B
For Your Amusement

Hassan’s family paid $80 for tickets to Wonderpark for 2 adults and 5 children.

a) If children’s tickets cost $10 each, how much will Bob’s family pay for 4 adults and 3 children?

b) Large groups get a 10% discount on the total cost. If your amazing teacher, and all the children in the class visit Wonderpark, what will be the cost?
Problem of the Week
Problem B
For Your Amusement

Problem
Hassan’s family paid $80 for tickets to Wonderpark for 2 adults and 5 children.

a) If children’s tickets cost $10 each, how much will Bob’s family pay for 4 adults and 3 children?

b) Large groups get a 10% discount on the total cost. If your amazing teacher, and all the children in the class visit Wonderpark, what will be the cost?

Solution

a) Since children’s tickets cost $10 each, the 5 children in Hassan’s family cost

\[5 \times \$10 = \$50.\]

Thus the 2 adults cost \(\$80 - \$50 = \$30\), and so the adults cost \(\$30 \div 2 = \$15\) each.

This means that Bob’s family of 4 adults and 3 children will cost

\[4 \times \$15 + 3 \times \$10 = \$60 + \$30 = \$90.\]

b) Answers will vary depending on class size. Let’s suppose there are 30 students plus the teacher. Then the full price cost would be

\[C = 30 \times \$10 + 1 \times \$15 = \$315.\]

The discount will then be 10% of \(C\), or \(0.10 \times \$315 = \$31.50\).

Thus the net cost will be \(\$315 - \$31.50 = \$283.50\), which is the same as

\[\$315 \times (1 - 0.10) = \$315 \times 0.9.\]

In general, the net cost will be

\[C = (((\text{the number of students} \times \$10) + \$15)) \times 0.9.\]
Problem of the Week

Problem B
Stop Pars!

Suppose that 100 ten to twelve year-old adolescents are asked to pick their favourite among four popular singers, with the following outcomes:

- Pasty Kerry: favourite of 20 out of each 50 students
- Tustin Jimberlake: favourite of 30% of the 100 students
- Bustin Jeiber: favourite of 10% of the 100 students
- Tregan Maynor: favourite of the remaining students

a) How may adolescents voted for each artist? Use a table to show your answers.

b) What fraction of the students voted for Tregan Maynor?

c) Which types of graphs are best suited to comparing this data?

d) Choose one type, and construct a graph to display the information in your table from part a).

e) Based on the given data, how many votes as favourite would each artist get if 250 adolescents were surveyed?

Strands
Data Management and Probability, Number Sense and Numeration
Problem of the Week
Problem B
Stop Pars!

Problem
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b) What fraction of the students voted for Tregan Maynor?
c) Which types of graphs are best suited to comparing this data?
d) Choose one type, and construct a graph to display the information in your table from part a).
e) Based on the given data, how many votes as favourite would each artist get if 250 adolescents were surveyed?

Solution
a)

<table>
<thead>
<tr>
<th>Artist</th>
<th>Given Data</th>
<th>No.of Votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pasty Kerry</td>
<td>20 out of 50</td>
<td>( \frac{20}{50} = \frac{40}{100} ), so 40 votes</td>
</tr>
<tr>
<td>Tustin Jimberlake</td>
<td>30% of 100</td>
<td>( \frac{30}{100} ), so 30 votes</td>
</tr>
<tr>
<td>Bustin Jeiber</td>
<td>10% of 100</td>
<td>( \frac{10}{100} ), so 10 votes</td>
</tr>
<tr>
<td>Tregan Maynor</td>
<td>remainder</td>
<td>( 100 - 40 - 30 - 10 = 20 ), so 20 votes</td>
</tr>
</tbody>
</table>
b) The number of students who voted for Tregan Maynor is 20 out of 100, or \( \frac{1}{5} \) of the students.

c) Good types of graphs might be

- a Bar Graph, which is good for comparing data;
- a Circle Graph, which is good for showing parts of a whole;
- a Pictograph, which is also good for comparing data.

d) Here are samples of a bar graph and a circle graph displaying the given data.

e) Based on the given data, if 250 adolescents were surveyed, the results are:

<table>
<thead>
<tr>
<th>Artist</th>
<th>Given Data</th>
<th>No.of Votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pasty Kerry</td>
<td>20 out of 50</td>
<td>( \frac{20}{50} = \frac{100}{250} ), so 100 votes</td>
</tr>
<tr>
<td>Tustin Jimberlake</td>
<td>30% of 100</td>
<td>( \frac{30}{100} = \frac{75}{250} ), so 75 votes</td>
</tr>
<tr>
<td>Bustin Jeiber</td>
<td>10% of 100</td>
<td>( \frac{10}{100} = \frac{25}{250} ), so 25 votes</td>
</tr>
<tr>
<td>Tregan Maynor</td>
<td>remainder (( \frac{1}{5} ))</td>
<td>( \frac{1}{5} \times 250 = 50 ), so 50 votes</td>
</tr>
</tbody>
</table>
Problem of the Week
Problem B
Plus Primes

The diagram below shows a set of ‘plus signs’ of different sizes, each formed from a set of unit squares. These unit squares can be rearranged to form rectangles of various sizes; for example, the unit squares in the first plus sign can form one rectangle with dimensions 1 unit by 5 units.

a) Rearrange the unit squares of each of the other plus signs to form as many rectangles as possible. Then construct the next three plus signs in this sequence and do the same for them.

b) Construct a table (or label your diagrams) showing, for each of the six ‘plus signs’, the number of unit squares and the dimensions of the different rectangles which can be formed.

c) What is special about the number of unit squares in those ‘plus signs’ which can only be rearranged to form ONE rectangle?

d) Some of the ‘plus signs’ permit rearrangements that are square, a special type of rectangle. What is special about the number of unit squares in those ‘plus signs’?

e) What kind of number of unit squares are in the ‘plus signs’ which can be rearranged into more than one rectangle?

f) After the sixth ‘plus sign’, what will be the next one with a perfect square number of unit squares?

Strands  Number Sense and Numeration, Patterning and Algebra
Problem of the Week

Problem B
Plus Primes

Problem
The diagram below shows a set of ‘plus signs’ of different sizes, each formed from a set of unit squares. These unit squares can be rearranged to form rectangles of various sizes; for example, the unit squares in the first plus sign can form one rectangle with dimensions 1 unit by 5 units.

a) Rearrange the unit squares of each of the other plus signs to form as many rectangles as possible. Then construct the next three plus signs in this sequence and do the same for them.

b) Construct a table (or label your diagrams) showing, for each of the six ‘plus signs’, the number of unit squares and the dimensions of the different rectangles which can be formed.

c) What is special about the number of unit squares in those ‘plus signs’ which can only be rearranged to form ONE rectangle?

d) Some of the ‘plus signs’ permit rearrangements that are square, a special type of rectangle. What is special about the number of unit squares in those ‘plus signs’?

e) What kind of number of unit squares are in the ‘plus signs’ which can be rearranged into more than one rectangle?

f) After the sixth ‘plus sign’, what will be the next one with a perfect square number of unit squares?

Solution
a),b) In the diagram below, the six ‘plus signs’ and their associated rectangles (including squares) are shown, numbered 1. to 6. The number of unit squares is shown at the bottom right of each ‘plus sign’, and the associated rectangles are labelled with their dimensions (e.g., for 5., there are two rectangles, one with dimensions 1 unit by 21 units, and one 3 units by 7 units.)
c) The ‘plus signs’ 1., 3., and 4. have only one possible rectangle. In this case, the number of unit squares is a prime number, 5, 13, and 17, respectively, each number having only two factors, 1 and itself.

d) The ‘plus signs’ 2. and 6., which permit squares (3 by 3 units and 5 by 5 units) have a perfect square, or square number of unit squares, 9 and 25, respectively.

e) The ‘plus signs’ 2., 5., and 6., which have two rectangles have 9, 21, and 25 unit squares respectively. These are composite numbers having more than two factors; 9 has factors 1, 3, 9, while 21 has factors 1, 3, 7, and 21, and 25 has factors 1, 5, and 25.

f) The next ‘plus signs’ with a perfect square number of unit squares will be the one with 49 (7 × 7) unit squares. Since it will have 4 ‘arms’ of length 12, it will be the 12th ‘plus sign’.

**Extension:**
What is the first ‘plus sign’ which can be rearranged to form three different rectangles?
Problem of the Week

Problem B

Say "Cheese"!

Mr. Peabody has to take a taxi home from the cheese shop. The fare is $0.90 for the first $\frac{1}{2}$ km, and $0.12$ for each additional $\frac{1}{8}$ km.

a) How far is his home from the shop if he spent $2.82$?

b) How much would this taxi cost for a $2$ km trip? A $4$ km trip? A $6$ km trip? An $8$ km trip? Is there a pattern to help you figure out the answer? Describe any patterns you see as a result of your calculations.

**Strand** Number Sense and Numeration
Problem of the Week
Problem B
Say "Cheese"!

Problem
Mr. Peabody has to take a taxi home from the cheese shop. The fare is $0.90 for the first \( \frac{1}{2} \) km, and $0.12 for each additional \( \frac{1}{8} \) km.

a) How far is his home from the shop if he spent $2.82?

b) How much would this taxi cost for a 2 km trip? A 4 km trip? A 6 km trip? An 8 km trip? Is there a pattern to help you figure out the answer? Describe any patterns you see as a result of your calculations.

Solution

a) The first \( \frac{1}{2} \) km costs $0.90. Since each additional \( \frac{1}{8} \) km costs $0.12, and there are four \( \frac{1}{8} \)'s in \( \frac{1}{2} \), we know that each additional \( \frac{1}{2} \) km will cost Mr. Peabody \( 4 \times 0.12 = 0.48 \). Here is a table of fares for the first 4 km.

<table>
<thead>
<tr>
<th>Distance</th>
<th>( \frac{1}{2} ) km</th>
<th>1 km</th>
<th>1 ( \frac{1}{2} ) km</th>
<th>2 km</th>
<th>2 ( \frac{1}{2} ) km</th>
<th>3 km</th>
<th>3 ( \frac{1}{2} ) km</th>
<th>4 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fare in $</td>
<td>0.90</td>
<td>1.38</td>
<td>1.86</td>
<td>2.34</td>
<td>2.82</td>
<td>3.30</td>
<td>3.78</td>
<td>4.26</td>
</tr>
</tbody>
</table>

Thus Mr. Peabody’s home is \( 2 \frac{1}{2} \) km from the cheese shop.

b) We see from the table that a 2 km trip costs $2.34; a 4 km trip costs $4.26. Note the pattern: each additional km adds $0.96 (i.e., \( 2 \times 0.48 = 0.96 \)) to the fare. Thus a 6 km trip will cost $4.26 + (2 \times 0.96) = $6.18, and an 8 km trip will cost $6.18 + (2 \times 0.96) = $8.10.

This solution can also be visualized on a number line, as shown below.
Problem of the Week
Problem B
Grubby Paws

A good way to ensure that you do a complete job of washing your hands is to sing the ‘Happy Birthday’ song while you wash.

a) For approximately how long will you wash your hands each time?

b) If you wash your hands 6 times per day, how many seconds will you spend washing in total each day?

c) Supposing you could reach the bathroom sink to wash your own hands by your 4th birthday, and that you have done so ever since as in parts a) and b), how many hours and minutes have you spent washing your hands so far?

d) If you live to be 75 years old, how many days of handwashing will you have done by then?

Strands  Number Sense and Numeration, Measurement
Problem of the Week

Problem B

Grubby Paws

Problem

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d) If you live to be 75 years old, how many days of handwashing will you have done by then?

Solution

Note that all the answers in the solution to this problem will depend on your answer to part a), which may vary.

a) Hand washing to the tune ‘Happy Birthday’ takes about 10 seconds each time.

b) Thus, if you wash your hands 6 times per day, you will spend $6 \times 10 = 60$ seconds total per day washing your hands.

c) Answers here will depend on your age. Suppose you are 10 years old; then you will have been able to wash your own hands for the past 6 years. Since there are 365 days in each year, 6 years equals $365 \times 6 = 2190$ days. At 60 seconds, or 1 minute per day, you will have spent 2190 minutes washing your hands since you were 4 years old. There are 60 minutes in an hour, so this is equivalent to $2190 \div 60 = 36.5$ hours, or 36 hours and 30 minutes.

d) If you live to be 75, you will have spent $(75 - 4) = 71$ years washing your hands. Since you spend 1 minute per day, this is $71 \times 365 \times 1 = 25915$ minutes, or $25915 \div 60 \approx 432$ hours, which is $432 \div 24 = 18$ days of hand washing.
Problem of the Week
Problem B
Chipping Away

In the popular game video game Minecraft™, players have to get materials to build the tools they need. Marsha is playing and wants to make sure she has a good stock of the following tools:

- wood pick axes, each requiring 5 sticks;
- swords, each requiring 1 stick and 2 cobblestone blocks;
- shovels, each requiring 2 sticks and 1 cobblestone block;
- hoes, each requiring 2 sticks and 2 cobblestone blocks.

a) If she wants to have 6 of each tool, how many cobblestone blocks and how many sticks will she need?

b) Marsha has decided to build a house with a square base 8 blocks long by 8 blocks wide, and 4 blocks high. How many blocks will she need to build all the walls of her house? Be careful with the corners!

c) Once the walls are constructed, Marsha decides to build a pyramid-shaped roof on her house. For example, the first layer will have dimensions of 7 by 7 blocks; think about why this layer will require 24 blocks. The second layer will be 6 by 6 blocks, etcetera, until there is only 1 block on top. How many blocks will she need for the roof?

d) If each pick axe can mine 60 blocks, how many pick axes are needed to build the walls and roof of Marsha’s house?

Strands  Number Sense and Numeration,  Geometry and Spatial Sense
Problem of the Week
Problem B and Solution
Chipping Away

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d) If each pick axe can mine 60 blocks, how many pick axes are needed to build the walls and roof of Marsha’s house?

Solution
a) Examining the given information, we see that one of each tool would require $5 + 1 + 2 + 2 = 10$ sticks, and $0 + 2 + 1 + 2 = 5$ cobblestone blocks. Thus, if Marsha wants to have 6 of each tool, she will need $10 \times 6 = 60$ sticks, and $5 \times 6 = 30$ cobblestone blocks.

b) From the diagram, one layer of Marsha’s house will require $8 + 8 + 6 + 6 = 28$ blocks. (Note that you can think of the square base as two ‘lengths’ of 8 blocks, with two ‘widths’ of 6 blocks tucked between them.) Thus if her house is to be 4 blocks high, all the walls will require $28 \times 4 = 112$ blocks.
c) Each layer of the pyramidal roof will be a square, similar to the base walls. The table below lists the dimensions of each layer in blocks (from the bottom up), and the total blocks required for each layer, and for the entire roof. The terms ‘width’ and ‘length’ are used here in the same manner as in the solution for part b).

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>‘Width’</th>
<th>‘Length’</th>
<th>Layer Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 × 7</td>
<td>5</td>
<td>7</td>
<td>24</td>
</tr>
<tr>
<td>6 × 6</td>
<td>4</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>5 × 5</td>
<td>3</td>
<td>5</td>
<td>16</td>
</tr>
<tr>
<td>4 × 4</td>
<td>2</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>3 × 3</td>
<td>1</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>2 × 2</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>1 × 1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Total Blocks 85</td>
</tr>
</tbody>
</table>

Here’s how the roof would look:

![Pyramidal Roof Diagram](image)

d) Since the total number of blocks for both walls and roof is $112 + 85 = 197$, and each pick axe can mine 60 blocks, she would need 4 pick axes. (Three is too few, since they would only mine $3 \times 60 = 180$ blocks.)
Problem of the Week
Problem B
Go! Go! Es Cargo

A determined snail named Es Cargo climbs the trunk of a tree, seeking a juicy leaf 3 m above the ground. He climbs at a steady rate of 8 cm per minute, but every couple of minutes, he slips back down 4 cm on the steep trunk.

a) How long will it take Es Cargo to reach his goal?

<table>
<thead>
<tr>
<th>Time</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>8 + 8 − 4 = 12</td>
</tr>
<tr>
<td>3</td>
<td>12 + 8 = 20</td>
</tr>
<tr>
<td>4</td>
<td>20 + 8 − 4 = 24</td>
</tr>
<tr>
<td>5</td>
<td></td>
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<tr>
<td>6</td>
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</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

b) If he slips only 2 cm every 2 minutes, will it take Es Cargo half the time you found in part a) to reach his goal? Explain your reasoning.
Problem of the Week

Problem B

Go! Go! Es Cargo

Problem

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</tr>
<tr>
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<td>24 + 8 = 32</td>
</tr>
<tr>
<td>6</td>
<td>32 + 8 − 4 = 36</td>
</tr>
<tr>
<td>7</td>
<td>36 + 8 = 44</td>
</tr>
<tr>
<td>8</td>
<td>44 + 8 − 4 = 48</td>
</tr>
</tbody>
</table>

b) If he slips only 2 cm every 2 minutes, will it take Es Cargo half the time you found in part a) to reach his goal? Explain your reasoning.

Solution

a) Examining the table entries above, we see that a pattern emerges: Es Cargo climbs a net distance of 12 cm every 2 minutes. Since 48 minutes equals 24 × 2 minutes, by then he will have climbed 24 × 12 = 288 cm. In the 49th minute, he climbs to 288 + 8 = 296 cm. Thus in the first half of the 50th minute he climbs the last 4 cm, achieving his goal of 3 m, or 300 cm, in 49 1/2 minutes. Now he can munch away on his juicy leaf!

b) Since Es Cargo still climbs at 8 cm per minute, the difference of 2 cm in slippage will not be large enough to reduce his total time by half. (Constructing a similar table reveals that he climbs a net distance of 14 cm every 2 minutes in this case, taking a total of 42 3/4 minutes for 300 cm.)
Problem of the Week
Problem B
A Zombie Add-pocalypse

Zombies have arrived! Officials are strongly advising people to remain in their homes and away from anyone who may be infected. Infected people turn into zombies and must infect one other person each day to stay alive. An antidote is currently undergoing clinical trials, but will not be ready for use for 30 days.

It all started with one zombie which arrived one day, then infected one other person on the second day. On the third day, each of those zombies infected one other person, and so on.

a) The Centre for Zombie Control (CZC) needs to know how many people are going to be affected. How many zombies should they expect to be roaming around at the end of each day for the next two weeks?

<table>
<thead>
<tr>
<th>Day</th>
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</tr>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

b) How many zombies will there be by the time the antidote is ready?

c) If the antidote is ineffective, on what day will the population of zombies exceed that of the people on earth?
Problem of the Week
Problem B
A Zombie Add-pocalypse

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<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
</tr>
</tbody>
</table>

Solution
The following tree diagram illustrates the growth of the zombie population over 5 days.
a) The completed tables below reveal that the population of zombies after 14 days is 8,192.

b) After 30 days, by the time the antidote is ready, it is 536,870,912.

c) Their population will exceed the population of people on earth (between 7 and 8 billion) on day 34.

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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</tr>
<tr>
<td>2</td>
<td>2</td>
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<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
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<tr>
<td>6</td>
<td>32</td>
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<tr>
<td>7</td>
<td>64</td>
</tr>
<tr>
<td>8</td>
<td>128</td>
</tr>
<tr>
<td>9</td>
<td>256</td>
</tr>
<tr>
<td>10</td>
<td>512</td>
</tr>
<tr>
<td>11</td>
<td>1,024</td>
</tr>
<tr>
<td>12</td>
<td>2,048</td>
</tr>
<tr>
<td>13</td>
<td>4,096</td>
</tr>
<tr>
<td>14</td>
<td>8,192</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Day</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>16,384</td>
</tr>
<tr>
<td>16</td>
<td>32,768</td>
</tr>
<tr>
<td>17</td>
<td>65,536</td>
</tr>
<tr>
<td>18</td>
<td>131,072</td>
</tr>
<tr>
<td>19</td>
<td>262,144</td>
</tr>
<tr>
<td>20</td>
<td>524,288</td>
</tr>
<tr>
<td>21</td>
<td>1,048,576</td>
</tr>
<tr>
<td>22</td>
<td>2,097,152</td>
</tr>
<tr>
<td>23</td>
<td>4,194,304</td>
</tr>
<tr>
<td>24</td>
<td>8,388,608</td>
</tr>
<tr>
<td>25</td>
<td>16,777,216</td>
</tr>
<tr>
<td>26</td>
<td>33,554,432</td>
</tr>
<tr>
<td>27</td>
<td>67,108,864</td>
</tr>
<tr>
<td>28</td>
<td>134,217,728</td>
</tr>
<tr>
<td>29</td>
<td>268,435,456</td>
</tr>
<tr>
<td>30</td>
<td>536,870,912</td>
</tr>
<tr>
<td>31</td>
<td>1,073,741,824</td>
</tr>
<tr>
<td>32</td>
<td>2,147,483,648</td>
</tr>
<tr>
<td>33</td>
<td>4,294,967,296</td>
</tr>
<tr>
<td>34</td>
<td>8,589,934,592</td>
</tr>
</tbody>
</table>
Dr. Chapman has 4000 letters to mail to her clients. She has determined that she can

1. place a stamp, a sending label, and a return label on the envelope in 13 seconds,
2. sign, fold, and place the letter in the envelope in 15 seconds, and
3. seal and place the envelope in a box in 8 seconds.

a) How many hours will it take her to complete the task?

b) If Dr. Chapman hired two other people to help, with her doing step 2., and the others doing steps 1. and 3., how long will the task take?

c) Suppose she notices that she can sign and fold each letter in 10 seconds, but that it then takes the third person 13 seconds to place the letter in the envelope, seal it, and place it in a box. Would this be a more efficient way to do this task?
Problem of the Week
Problem B and Solution
Postal Processing

Problem
Dr. Chapman has 4 000 letters to mail to her clients. She has determined that she can

1. place a stamp, a sending label, and a re-turn label on the envelope in 13 seconds,

2. sign, fold, and place the letter in the en-velope in 15 seconds, and

3. seal and place the envelope in a box in 8 seconds.

a) How many hours will it take her to complete the task?

b) If Dr. Chapman hired two other people to help, with her doing step 2., and the others doing steps 1. and 3., how long will the task take?

c) Suppose she notices that she can sign and fold each letter in 10 seconds, but that it then takes the third person 13 seconds to place the letter in the envelope, seal it, and place it in a box. Would this be a more efficient way to do this task?

Solution
a) The three tasks will take the following times:

   Step 1. Stamping and labelling: 4 000 \times 13 = 52 000 s;
   Step 2. Signing, folding, and stuffing: 4 000 \times 15 = 60 000 s;
   Step 3. Sealing and boxing: 4 000 \times 8 = 32 000 s

   Thus the total time is 144 000 s, which is 144 000 \div 3 600 = 40 \text{ hr}, since there are 60 \times 60 = 3 600 s in 1 hr.

b) Step 2. is the slowest: Dr. Chapman will have to wait 13 s for the first envelope from Step 1; and the person doing Step 3 will have to wait 28 s for the first letter to seal, but will make up that time quickly since Step 3 takes only 8 s. So the whole task will take 13 s, plus the 60 000 s of Step 2., plus 8 s for the final envelope to be sealed and placed in a box, giving a total time of 60 000 s= 16\frac{2}{3} \text{ hr (or 16 hr, 40 min), plus 21 s.}

c) Since the longest task is now 13 s instead of 15 s, this would be a more efficient way to do this task. Further, the first two tasks are now independent and can be done simultaneously. Thus the total time would now be 4 000 \times 13 = 52 000 s, plus 13 s for the final envelope, or about 14\frac{1}{2} \text{ hr.}
Problem of the Week
Problem B
"Palindr"-homes

Jorge and Phyllis live on a street where houses are numbered between 10001 and 10997, with 10001 being the first house number, and 10997 the last. However, only every sixth number is used, i.e., 10001 is used, but 10002, 10003, 10004, 10005, 10006 are not, then 10007 is used, and so on.

a) How many homes are on this street?

b) How many of the house numbers on this street are palindromes (i.e., the digits occur in the same order both forward and reversed, such as 909)?
Problem of the Week
Problem B and Solution
"Palindr"-homes

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a) How many homes are on this street?

b) How many of the house numbers on this street are palindromes (i.e., the digits occur in the same order both forward and reversed, such as 909)?

Solution

a) Since \(10997 - 10001 = 996\), and only every sixth number is used, there are \(966 \div 6 = 166\) possible house numbers after 10001. Thus, including the first house (numbered 10001), the number of homes on this street is \(166 + 1 = 167\) homes.

b) Palindromes are numbers which appear the same when the digits are reversed. Thus, between 10001 and 10997, there are ten palindromes, 10001, 10101, 10201, 10301, 10401, 10501, 10601, 10701, 10801, and 10901. (The next palindrome would be 11011, which is greater than 10997.) However, the desired house numbers must also be such that the palindrome minus 10001 is divisible by 6, since only every sixth number is used. The ten differences are

\[0, 100, 200, 300, 400, 500, 600, 700, 800, \text{ and } 900.\]

Of these differences, we see that only four of the numbers, namely 0, 300, 600, and 900 are divisible by 6. It follows that only the four house numbers 10001, 10301, 10601, and 10901 are palindromes.
Problem of the Week

Problem B

Ms Orlando “Takes the Cake”

Ms. Orlando’s Grade 5 class has baked her a birthday cake. They wish to divide the 12 cm by 18 cm cake evenly among the 29 students and the teacher.

a) If each person receives a piece of the same size, what would be its top surface area?

b) Assuming the pieces are rectangular, with length being a whole number of cm, what would be the width of piece which is closest to a square? (Measurements of widths need not be whole numbers.)

c) Could the cake be cut evenly into 30 pieces which are NOT rectangular? Illustrate your answer with diagram(s).

STRANDS  Number Sense and Numeration, Measurement
Problem of the Week
Problem B and Solution
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c) Could the cake be cut evenly into 30 pieces which are NOT rectangular? Illustrate your answer with diagram(s).

Solution

a) The total surface area of the top of the cake is $18 \times 12 = 216 \text{ cm}^2$. Thus the top surface area of each equal piece would be $216 \div 30 = 7.2 \text{ cm}^2$.

b) With length a whole number and area 7.2 cm$^2$, possible dimensions of the pieces, in cm, are $1 \times 7.2, 2 \times 3.6, 3 \times 2.4, 4 \times 1.8, 5 \times 1.44, 6 \times 1.2, 8 \times 0.9, 12 \times 0.6$, and $18 \times 0.4$. Thus the one closest to a square would be 3 cm in length, with width 2.4 cm.

c) There are many possibilities for non-rectangular equal pieces. Here are three.
Problem of the Week
Problem B
Cell Phone Dilemma

You’re excited: your parents are permitting you to get your first cell phone! They don’t want to spend too much money on your plan, but they don’t want to do the research about different cell phone plans. It’s your phone; they expect you to make sure they pay the least amount possible.

Having done some digging, you’ve discovered you can get the \( \pi \)Phone 3.14, or an equivalent cool phone, with a two-year contract on any of the following plans from Telem, Smell, or Doggers. However, the cost of the phone varies.

<table>
<thead>
<tr>
<th>Telem</th>
<th>Phone: $200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data Plan</td>
<td>Cost/Month</td>
</tr>
<tr>
<td>500 MB</td>
<td>$45</td>
</tr>
<tr>
<td>1 GB</td>
<td>$70</td>
</tr>
<tr>
<td>Unlimited</td>
<td>$80</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Smell</th>
<th>Phone: $100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data Plan</td>
<td>Cost/Month</td>
</tr>
<tr>
<td>500 MB</td>
<td>$70</td>
</tr>
<tr>
<td>1 GB</td>
<td>$80</td>
</tr>
<tr>
<td>2 GB</td>
<td>$90</td>
</tr>
<tr>
<td>3 GB</td>
<td>$100</td>
</tr>
<tr>
<td>4 GB</td>
<td>$110</td>
</tr>
<tr>
<td>6 GB</td>
<td>$120</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Doggers</th>
<th>Phone: Free</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data Plan</td>
<td>Cost/Month</td>
</tr>
<tr>
<td>500 MB</td>
<td>$60</td>
</tr>
<tr>
<td>1 GB</td>
<td>$85</td>
</tr>
<tr>
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<td>$95</td>
</tr>
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<td>$130</td>
</tr>
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<td>10 GB</td>
<td>$140</td>
</tr>
</tbody>
</table>

A fourth possibility is the Hurricane Network, with which you can get a phone for $648, and pay the monthly fees shown at the right, but without a contract.

<table>
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<tr>
<th>Hurricane Network</th>
<th>Data Plan</th>
<th>Cost/Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 GB</td>
<td>$50</td>
<td></td>
</tr>
<tr>
<td>3 GB</td>
<td>$60</td>
<td></td>
</tr>
<tr>
<td>5 GB</td>
<td>$70</td>
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<tr>
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<td>$80</td>
<td></td>
</tr>
</tbody>
</table>
Which of these four plans should you choose in order to minimize your parents' costs over two years, assuming you wish 1 GB of data per month?

Use the following steps to guide your reasoning:

1. Make a table for each plan showing the costs for one, two, and three months.
2. Write a rule which gives the cost for the phone after any number of months, for each plan. Describe clearly the meaning of each symbol in your rule.
Problem of the Week
Problem B
Cell Phone Dilemma

Problem
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1. Make a table for each plan showing the costs for one, two, and three months.
2. Write a rule which gives the cost for the phone after any number of months, for each plan. Describe clearly the meaning of each symbol in your rule.

Solution

Assuming a 1 GB data plan (using ‘unlimited’ where necessary), the costs for 1, 2, and 3 months are shown in the table, plus the cost of the phone.

<table>
<thead>
<tr>
<th>Vendor</th>
<th>Phone Cost</th>
<th>Cost for 1 month</th>
<th>Cost for 2 months</th>
<th>Cost for 3 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>Telem</td>
<td>$200</td>
<td>$70</td>
<td>$140</td>
<td>$210</td>
</tr>
<tr>
<td>Smell</td>
<td>$100</td>
<td>$80</td>
<td>$160</td>
<td>$320</td>
</tr>
<tr>
<td>Doggers</td>
<td>$0</td>
<td>$85</td>
<td>$170</td>
<td>$255</td>
</tr>
<tr>
<td>Hurricane</td>
<td>$648</td>
<td>$50</td>
<td>$100</td>
<td>$150</td>
</tr>
</tbody>
</table>

General rules for the cost $C$ for $N$ months, and for 24 months, for 1 GB of data are

- Telem: $C = 200 + N \times 70$, so $N = 24$ gives $C = 200 + 24 \times 70 = 1880$.
- Smell: $C = 100 + N \times 80$, so $N = 24$ gives $C = 100 + 24 \times 80 = 2020$.
- Doggers: $C = 0 + N \times 85$, so $N = 24$ gives $C = 0 + 24 \times 85 = 2040$.
- Hurricane: $C = 648 + N \times 50$, so $N = 24$ gives $C = 648 + 24 \times 50 = 1848$.

Thus the plan of least cost is with Hurricane, due to the fact that over a long time period, the much lower monthly cost offsets the initially large cost for the phone.

Telem is a close second, only $32$ more, so it would also be a sensible choice. This second choice would require a much smaller initial outlay of money towards the plan.

A discussion could follow about the merits of each plan and why someone might still choose the Dogger's plan.
Consider the following sum:

\[
(2+3) + (32+23) + (232+323) + (3232+2323) + (23232+32323) + \cdots + (32323232+23232323)
\]

a) Add the bracketed pairs of numbers to simplify the sum.
b) What is the digit in the ones place in the sum?
c) What is the digit in the tens place in the sum?
d) What is the digit in the hundreds place in the sum?
e) Find the sum.
Problem of the Week
Problem B
High Fives!

Problem
Consider the following sum:
\[(2+3)+(32+23)+(232+323)+(3232+2323)+(23232+32323) + \cdots + (32323232+23232323) .\]

a) Add the bracketed pairs of numbers to simplify the sum.
b) What is the digit in the ones place in the sum?
c) What is the digit in the tens place in the sum?
d) What is the digit in the hundreds place in the sum?
e) Find the sum.

Solution

a) Adding the bracketed pairs gives
\[5 + 55 + 555 + 5555 + 55555 + \cdots + 55555555.\]

b) Thus the digit in the ones place in the sum must be a 0, since there are 8 terms in total from part a), each with a ones digit of 5, and \(8 \times 5 = 40\).

c) For the tens digit, there are 7 terms with a 5 in the tens place, which sum to \(7 \times 5 = 35\), plus the 4 carried from the ones sum of 40, giving \(35 + 4 = 39\). Hence the tens digit is a 9.

d) By similar reasoning, there are 6 terms with a 5 in the hundreds place, summing to \(6 \times 5 = 30\), plus the 3 carried from the tens column, giving \(30 + 3 = 33\). Thus the hundreds digit is a 3.

e) There are 5 terms with a 5 in the \(1000\)s place, summing to \(5 \times 5 = 25\), plus the 3 carried from the hundreds column, giving \(25 + 3 = 28\). Hence the \(1000\)s digit is 8.

The \(10000\)s place has sum \(4 \times 5 = 20\), plus the carried 2 from the \(1000\)s column, giving sum 22. Hence the \(10000\)s digit is a 2.
The 100 000 s place has sum $3 \times 5 = 15$, plus the carried 2 from the 10 000 s column, giving sum 17. Hence the 100 000 s digit is a 7.

The 1 000 000 s place has sum $2 \times 5 = 10$, plus the carried 1 from the 100 000 s column, giving sum 11. Hence the 1 000 000 s digit is a 1.

Finally, the 10 000 000 s place has sum $1 \times 5 = 5$, plus the carried 1 from the 1 000 000 s column, giving sum 6.

Placing the digits in order, we see that the total sum is 61 728 390.
Problem of the Week
Problem B
Licensed to Thrill!

McGregor School bought a movie license for $450. They want to show the newest animated movie, “Thawed”.

a) If they charge $5 per family to view the film, how many families would have to see the movie in order for them to recoup the cost of the license?

b) If all 400 seats in the gym are filled, and the school will break even on this showing, what is the average (mean) number of people in each family watching the film?

c) What is the maximum profit the school could make if more families were to attend?
Problem of the Week
Problem B
Licensed to Thrill!

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b) If all 400 seats in the gym are filled, and the school will break even on this showing, what is the average (mean) number of people in each family watching the film?

c) What is the maximum profit the school could make if more families were to attend?

Solution

a) Since the license cost $450, if they charge $5 per family, then $450 ÷ $5 = 90 families would have to see the movie.

b) If these 90 families occupy all 400 seats in the theatre, then the average (mean) number of people in each family is $400 ÷ 90 = 4 \frac{4}{9} \approx 4.4$ people.

c) The answer here depends on what is considered to be a ‘family’. The profit will be maximized for the least number of people per family, say one parent and one child.

With 400 seats, this would mean $400 ÷ 2 = 200$ families could attend. The maximal revenue would thus be $5 \times 200 = 1000$, giving a profit of $1000 - 450 = 550$. 
Problem of the Week
Problem B
Two’s Company

a) The dimensions A cm by B cm, and N cm by M cm, of the two rectangles shown below are four different single digits from 1, 2, \ldots, 9.

If the two rectangles have the same area, what digits CANNOT be A, B, N, or M?

\[ \text{A} \quad \text{B} \quad \text{M} \quad \text{N} \]

b) Does your answer change if the rectangles are not congruent, but the digits can be repeated, i.e., A, B, M, and N need not be distinct?
Problem of the Week
Problem B
Two’s Company

Problem
a) The dimensions A cm by B cm, and N cm by M cm, of the two rectangles shown below are four different single digits from 1, 2, · · ·, 9.

If the two rectangles have the same area, what digits CANNOT be A, B, N, or M?

b) Does your answer change if the rectangles are not congruent, but the digits can be repeated, i.e., A, B, M, and N need not be distinct?

Solution
a) The table on the right displays all products of distinct digits; the shaded entries are repeated. For example, A × B could be 1 × 8 and M × N could be 2 × 4.

Those products involve digits 1, 2, 3, 4, 6, 8, 9. Thus we see that the only digits which cannot be A, B, M, or N are 5 and 7.

b) If the digits can be repeated, then we also have the possibilities 1 × 4 = 2 × 2, 1 × 9 = 3 × 3, 2 × 8 = 4 × 4, and 4 × 9 = 6 × 6, i.e., the perfect squares. However, neither 5 nor 7 appears, so the answer does not change.
Problem of the Week
Problem B
Triple Trouble!

Ahmed wonders how many 3-digit whole numbers he can create which are not multiples of 10, and which have no repeating digits.

a) How many such numbers are there with a 1 in the hundreds place?
b) How many such numbers are there with a 2 in the hundreds place?
c) How many such numbers are there in total?

**STRANDS** Number Sense and Numeration, Patterning and Algebra
Problem of the Week
Problem B and Solution
Triple Trouble!

Problem
Ahmed wonders how many 3-digit whole numbers he can create which are not multiples of 10, and which have no repeating digits.

a) How many such numbers are there with a 1 in the hundreds place?

b) How many such numbers are there with a 2 in the hundreds place?

c) How many such numbers are there in total?

Solution
a) Three-digit numbers with a 1 in the hundreds place can be organized according to the tens digit as follows.

If the tens digit is 0, they are 102, 103, 104, 105, 106, 107, 108, 109.
If the tens digit is 1, there are none, since the 1 would be repeated.
If the tens digit is 2, they are 123, 124, 125, 126, 127, 128, 129.
If the tens digit is 3, they are 132, 134, 135, 136, 137, 138, 139.
If the tens digit is 4, they are 142, 143, 145, 146, 147, 148, 149.

Etcetera.

Observing the pattern we see there are 8 such numbers with tens digit 0, since the ones digit can be anything except 1 or 0. But for the possible tens digits 2, 3, 4, · · · , 9, there are only 7 possible numbers in each group, since the ones digit can’t be 0 (no multiples of 10), nor 1, nor the specific tens digit for that group.

Thus there are \(8 + (8 \times 7) = 8 + 56 = 64\) such numbers with a 1 in the hundreds place.

b) For numbers with a 2 in the hundreds place, the outcome would be the same, namely one group of 8 (201, 203, 204, 205, 206, 207, 208, 209), and 8 groups of 7 (e.g., 231, 234, 235, 236, 237, 238, 239), for a total of 64 such numbers.

c) The same pattern would occur for hundreds digits 3, 4, 5, · · · , 9, with the greatest possible such number being 987. Thus the grand total of the 9 groups of such 3-digit numbers is \(64 \times 9 = 576\).
The triangle has four rows. The top row is currently empty. The numbers in each of the two middle rows of this triangle are related to those in the row immediately below by a simple rule involving quotients and products.

a) Find the specific rule.

b) If this rule is applied to determine the topmost number, will it be a whole number?

c) If not, change ONE number in the bottom row of the triangle so that when you apply the rule you found to create a new number triangle, the topmost number will be a whole number.
Problem of the Week

Problem B

Terrible Threes!

Problem

The triangle has four rows. The top row is currently empty. The numbers in each of the two middle rows of this triangle are related to those in the row immediately below by a simple rule involving quotients and products.

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b) If this rule is applied to determine the topmost number, will it be a whole number?

c) If not, change ONE number in the bottom row of the triangle so that when you apply the rule you found to create a new number triangle, the topmost number will be a whole number.

Solution

a) The product of adjacent numbers in the row below, divided by 3, gives the number above. For example, $1 \times 3 = 3$, and $3 \div 3 = 1$, the number above. Similarly, $6 \times 2 = 12$, and $12 \div 3 = 4$, the number above. And so on.

b) By this rule, the top number would be $2 \times 4 = 8$, which is then divided by 3 to give $\frac{8}{3}$, which is not a whole number.

An alternative possibility is to note that the product of the numbers in the bottom row is 18, in the next row is 12, and in the third row is 8. Thus the product decreases by 6 from the bottom to the next row, and by 4 from that row to the third. So one could deduce that the product difference is decreasing by 2 as we move upward, so the next difference should be 2, and hence the top number should be a 6.

c) There are many solutions. Here are three possibilities.
Problem of the Week
Problem B
A-Maze-ing Mice!

Two mice, Charlie and Priscilla, run at the same speed in a small maze with two loops, as shown below.

They can run counter-clockwise around loop 1 in 10 s, and clockwise around loop 2 in 4 s.

They start together from $S$, where the corners of loop 1 and loop 2 meet, at time $t = 0$ s, with Charlie running on loop 1 and Priscilla on loop 2.

a) If Charlie continues to run around loop 1, and Priscilla continues to run around loop 2, when will they first meet again at $S$? When will the next meeting occur?

b) Suppose that, whenever they meet at $S$, they get confused and switch to the other loop, continuing at the same speed. In which loop will Charlie be after 32 s? After 45 s?

c) If they continue as in part b), is the length of time between their meetings at $S$ always the same? Explain.
Problem of the Week
Problem B
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c) If they continue as in part b), is the length of time between their meetings at $S$ always the same? Explain.

Solution
a) The table on the right gives the times for both Charlie and Priscilla after each of 5 laps. To meet at $S$, they must have matching times at the end of a lap. Clearly they will first meet at $S$ after 20 seconds, and will continue to meet every 20 seconds thereafter, since Priscilla always runs 5 laps (in 20 seconds) around Loop 2 while Charlie runs 2 laps (in 20 seconds) around Loop 1.

<table>
<thead>
<tr>
<th>Laps</th>
<th>C’s Time</th>
<th>P’s Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>20</td>
</tr>
</tbody>
</table>
b) The table on the right shows the times at which Charlie and Priscilla complete laps, assuming they switch loops whenever they meet at $S$. Their locations are indicated by $L1$ for Loop 1, $L2$ for Loop 2, and $S$ when they meet. Thus we see that after 32 seconds, Charlie is in Loop 2, and after 45 seconds, he is in Loop 1.

<table>
<thead>
<tr>
<th>Laps</th>
<th>C’s Time</th>
<th>P’s Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10 ($L1$)</td>
<td>4 ($L2$)</td>
</tr>
<tr>
<td>2</td>
<td>20 ($S$)</td>
<td>8 ($L2$)</td>
</tr>
<tr>
<td>3</td>
<td>24 ($L2$)</td>
<td>12 ($L2$)</td>
</tr>
<tr>
<td>4</td>
<td>28 ($L2$)</td>
<td>16 ($L2$)</td>
</tr>
<tr>
<td>5</td>
<td>32 ($L2$)</td>
<td>20 ($S$)</td>
</tr>
<tr>
<td>6</td>
<td>36 ($L2$)</td>
<td>30 ($L1$)</td>
</tr>
<tr>
<td>7</td>
<td>40 ($S$)</td>
<td>40 ($S$)</td>
</tr>
<tr>
<td>8</td>
<td>50 ($L1$)</td>
<td>44 ($L2$)</td>
</tr>
<tr>
<td>9</td>
<td>60 ($S$)</td>
<td>48 ($L2$)</td>
</tr>
<tr>
<td>10</td>
<td>64 ($L2$)</td>
<td>52 ($L2$)</td>
</tr>
<tr>
<td>11</td>
<td>68 ($L2$)</td>
<td>56 ($L2$)</td>
</tr>
<tr>
<td>12</td>
<td>72 ($L2$)</td>
<td>60 ($L2$)</td>
</tr>
</tbody>
</table>

c) The table entries indicate that Charlie and Priscilla continue to meet every 20 seconds. This is due to the fact that they both run at the same speed, so it’s as if they did not switch loops upon meeting at $S$, but just continued in the same loop as in part a).

That they meet every 20 seconds is due to the fact that Loop 1 takes either mouse 10 seconds, while Loop 2 takes four seconds. Thus whichever mouse is running in Loop 2 can do five laps while the other mouse does two laps of Loop 1. Since the smallest number that is a whole number (of laps) times both 4 and 10 is 20, they meet every 20 seconds.
The new professional hockey team in Toronto, the Toronto Oak Leafs, started the season with the following record:

- 4 wins (a win is worth 2 points)
- 2 losses (a loss is worth 0 points)
- 2 overtime losses (an overtime loss is worth 1 point)

a) How many points have they earned after these 8 games?

b) If they continued this pattern of wins and losses repeatedly, how many points would they have after an 80 game schedule?

c) On average, how many points do they earn every 4 games?

d) If 95 points guarantees them a playoff spot, and they continue this win/loss pattern, after what game would they be guaranteed a spot in the playoffs?
Problem of the Week  
Problem B and Solution  
Who Makes the Playoffs?

Problem  
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Solution

a) The 4 wins give the Oak Leafs $4 \times 2 = 8$ points, and the 2 overtime losses give them 2 points, so after the first 8 games they have $8 + 2 = 10$ points.

b) Since an 80 game schedule would have 10 sets of this 8 game sequence, they would accumulate $10 \times 10 = 100$ points by the end of this schedule.

c) Since they earn 10 points every 8 games, on average they earn $10 \div 2 = 5$ points every 4 games.

d) Since they earn 10 points every 8 games, they would have 90 points after $9 \times 8 = 72$ games. The number of games they need to make 95 points then depends on the order in which the next wins and losses occur. The worst-case scenario would be if their 4 losses all occurred before any wins, accumulating only 92 points after 76 games. They would then win games 77 and 78, accumulating 96 points, one more than the 95 needed. The best-case scenario would be if they won their next 3 games, OR won any two of these games and lost the third in overtime. In either case they would have at least 95 points after 75 games.
Patterning
&
Algebra

TAKE ME TO THE COVER
Problem of the Week

Problem B

Gimme Five!

Sports teams often celebrate a good play or a winning goal by giving one another ‘high fives’.

a) Emelia’s basketball team won their game. Everyone gave everyone else on the team (5 players in total) a single high five. How many high fives were exchanged?

b) If the ‘spare’ player on Emelia’s team were included in the high fives, how many exchanges would occur?

c) Yousef scored the winning goal for his soccer team. If all 11 players gave high fives to each other, how many high fives were exchanged?

Strands: Patterning and Algebra, Geometry and Spatial Sense
Problem of the Week
Problem B
Gimme Five!

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c) Yousef scored the winning goal for his soccer team. If all 11 players gave high fives to each other, how many high fives were exchanged?

Solution

a) This is a great example of a problem that students should “act out” to understand. Let the five players (students) be represented by A, B, C, D, and E. Then, thinking of the high fives as being done in order, proceed as follows.
• A gives 4 high fives, one to each of B, C, D, and E;
• B gives 3 high fives, one to each of C, D, and E (she’s already done A);
• C gives 2 high fives, one to each of D, and E (she’s already done A and B);
• D gives 1 high five to E (she’s already done the others).

Thus a total of \( 4 + 3 + 2 + 1 = 10 \) high fives were exchanged.
This is illustrated by the diagram at the right. Each dashed line represents one high five.

b) Let the ‘spare’ be represented by F.
Then, following the same reasoning as in part a), we see from the diagram that A would give 5 high fives (to B, C, D, E, and F), B would give 4 high fives (to C, D, E, and F), C would give 3, D would give 2, and E would give 1.
Thus a total of \( 5 + 4 + 3 + 2 + 1 = 15 \) high fives would be exchanged.

c) The reasoning in a) and b) generalizes to the high fives among the 11 players on Yousef’s soccer team, giving a total exchange of
\[
10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 55 \text{ high fives.}
\]

Note that each of these can be visualized as a geometry problem: the number of high fives is equal to the number of sides plus the number of diagonals of a regular polygon.
Problem of the Week

Problem B

Plus Primes

The diagram below shows a set of ‘plus signs’ of different sizes, each formed from a set of unit squares. These unit squares can be rearranged to form rectangles of various sizes; for example, the unit squares in the first plus sign can form one rectangle with dimensions 1 unit by 5 units.

a) Rearrange the unit squares of each of the other plus signs to form as many rectangles as possible. Then construct the next three plus signs in this sequence and do the same for them.

b) Construct a table (or label your diagrams) showing, for each of the six ‘plus signs’, the number of unit squares and the dimensions of the different rectangles which can be formed.

c) What is special about the number of unit squares in those ‘plus signs’ which can only be rearranged to form ONE rectangle?

d) Some of the ‘plus signs’ permit rearrangements that are square, a special type of rectangle. What is special about the number of unit squares in those ‘plus signs’?

e) What kind of number of unit squares are in the ‘plus signs’ which can be rearranged into more than one rectangle?

f) After the sixth ‘plus sign’, what will be the next one with a perfect square number of unit squares?

Strands
Number Sense and Numeration, Patterning and Algebra
Problem of the Week  
Problem B  
Plus Primes

Problem  
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e) What kind of number of unit squares are in the ‘plus signs’ which can be rearranged into more than one rectangle?

f) After the sixth ‘plus sign’, what will be the next one with a perfect square number of unit squares?

Solution  
a),b) In the diagram below, the six ‘plus signs’ and their associated rectangles (including squares) are shown, numbered 1. to 6. The number of unit squares is shown at the bottom right of each ‘plus sign’, and the associated rectangles are labelled with their dimensions (e.g., for 5., there are two rectangles, one with dimensions 1 unit by 21 units, and one 3 units by 7 units.)
c) The ‘plus signs’ 1., 3., and 4. have only one possible rectangle. In this case, the number of unit squares is a prime number, 5, 13, and 17, respectively, each number having only two factors, 1 and itself.

d) The ‘plus signs’ 2. and 6., which permit squares (3 by 3 units and 5 by 5 units) have a perfect square, or square number of unit squares, 9 and 25, respectively.

e) The ‘plus signs’ 2., 5., and 6., which have two rectangles have 9, 21, and 25 unit squares respectively. These are composite numbers having more than two factors; 9 has factors 1, 3, 9, while 21 has factors 1, 3, 7, and 21, and 25 has factors 1, 5, and 25.

f) The next ‘plus signs’ with a perfect square number of unit squares will be the one with 49 (7 × 7) unit squares. Since it will have 4 ‘arms’ of length 12, it will be the 12th ‘plus sign’.

**Extension:**
What is the first ‘plus sign’ which can be rearranged to form three different rectangles?
A determined snail named Es Cargo climbs the trunk of a tree, seeking a juicy leaf 3 m above the ground. He climbs at a steady rate of 8 cm per minute, but every couple of minutes, he slips back down 4 cm on the steep trunk.

a) How long will it take Es Cargo to reach his goal?

<table>
<thead>
<tr>
<th>Time</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>8 + 8 – 4 = 12</td>
</tr>
<tr>
<td>3</td>
<td>12 + 8 = 20</td>
</tr>
<tr>
<td>4</td>
<td>20 + 8 – 4 = 24</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

b) If he slips only 2 cm every 2 minutes, will it take Es Cargo half the time you found in part a) to reach his goal? Explain your reasoning.
Problem of the Week

Problem B

Go! Go! Es Cargo

Problem

A determined snail named Es Cargo climbs the trunk of a tree, seeking a juicy leaf 3 m above the ground. He climbs at a steady rate of 8 cm per minute, but every couple of minutes, he slips back down 4 cm on the steep trunk.

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<td>2</td>
<td>$8 + 8 - 4 = 12$</td>
</tr>
<tr>
<td>3</td>
<td>$12 + 8 = 20$</td>
</tr>
<tr>
<td>4</td>
<td>$20 + 8 - 4 = 24$</td>
</tr>
<tr>
<td>5</td>
<td>$24 + 8 = 32$</td>
</tr>
<tr>
<td>6</td>
<td>$32 + 8 - 4 = 36$</td>
</tr>
<tr>
<td>7</td>
<td>$36 + 8 = 44$</td>
</tr>
<tr>
<td>8</td>
<td>$44 + 8 - 4 = 48$</td>
</tr>
</tbody>
</table>

b) If he slips only 2 cm every 2 minutes, will it take Es Cargo half the time you found in part a) to reach his goal? Explain your reasoning.

Solution

a) Examining the table entries above, we see that a pattern emerges: Es Cargo climbs a net distance of 12 cm every 2 minutes. Since 48 minutes equals $24 \times 2$ minutes, by then he will have climbed $24 \times 12 = 288$ cm. In the $49^{th}$ minute, he climbs to $288 + 8 = 296$ cm. Thus in the first half of the $50^{th}$ minute he climbs the last 4 cm, achieving his goal of 3 m, or 300 cm, in $49 \frac{1}{2}$ minutes. Now he can munch away on his juicy leaf!

b) Since Es Cargo still climbs at 8 cm per minute, the difference of 2 cm in slippage will not be large enough to reduce his total time by half. (Constructing a similar table reveals that he climbs a net distance of 14 cm every 2 minutes in this case, taking a total of $42 \frac{3}{4}$ minutes for 300 cm.)
Jon, Laura, Geoff, and Riley are seated around a circular card table ready to play a game.

a) How many different seating arrangements are possible?

b) If Katia joins the group, how many seating arrangements are now possible?
Problem of the Week
Problem B
Cards, Anyone?

Problem
Jon, Laura, Geoff, and Riley are seated around a circular card table ready to play a game.

a) How many different seating arrangements are possible?

b) If Katia joins the group, how many seating arrangements are now possible?

Solution

a) For the four people Laura (L), Jon (J), Geoff (G), and Riley (R), there are six possible seating arrangements, as determined by the tree below.

```
Seat 1
    L
  /   \   /
Seat 2  J  G  R
   / |  / |  / |
Seat 3 G  R  R  J  J  G
  / |  / |  / |  / |
Seat 4 R  G  J  R  G  J
```

This result can also be obtained as follows: Assign Seat 1 to one player, say Laura; then there are three choices of who can sit to her left, in Seat 2. Once that player is chosen, say Jon, there are two players for Seat 3. Say Geoff is chosen; then only one possible player, Riley, remains for the Seat 4. Thus the number of possible arrangements is $3 \times 2 \times 1 = 6$. 
b) When a fifth player, Katia (K) is added to the game, the number of possible arrangements increases dramatically, from 6 to 24, as shown in the table below.

Using a similar alternative argument to the above, we can also arrive at 24 possible arrangements by choosing a player for Seat 1, which leaves four possible choices for Seat 2, three for Seat 3, two for seat 4, and only one for Seat 5. Thus the number of possible arrangements is 4 × 3 × 2 × 1 = 24.

In the tree structure, these factors are portrayed by the number of lines connecting each level of the tree, 4 from Seat 1 to Seat 2, 3 from Seat 2 to Seat 3, 2 from Seat 3 to Seat 4, and 1 from Seat 4 to Seat 5.

Note that in either case, rotating the players around the table is not regarded as a new arrangement since they would still play the card game in the same order.
Problem of the Week
Problem B
"Palindr"-homes

Jorge and Phyllis live on a street where houses are numbered between 10001 and 10997, with 10001 being the first house number, and 10997 the last. However, only every sixth number is used, i.e., 10001 is used, but 10002, 10003, 10004, 10005, 10006 are not, then 10007 is used, and so on.

a) How many homes are on this street?

b) How many of the house numbers on this street are palindromes (i.e., the digits occur in the same order both forward and reversed, such as 909)?
Problem of the Week
Problem B and Solution
"Palindr"-homes

Problem
Jorge and Phyllis live on a street where houses are numbered between 10001 and 10997, with 10001 being the first house number, and 10997 the last. However, only every sixth number is used, i.e., 10001 is used, but 10002, 10003, 10004, 10005, 10006 are not, then 10007 is used, and so on.

a) How many homes are on this street?

b) How many of the house numbers on this street are palindromes (i.e., the digits occur in the same order both forward and reversed, such as 909)?

Solution
a) Since $10997 - 10001 = 996$, and only every sixth number is used, there are $966 \div 6 = 166$ possible house numbers after 10001. Thus, including the first house (numbered 10001), the number of homes on this street is $166 + 1 = 167$ homes.

b) Palindromes are numbers which appear the same when the digits are reversed. Thus, between 10001 and 10997, there are ten palindromes, 10001, 10101, 10201, 10301, 10401, 10501, 10601, 10701, 10801, and 10901. (The next palindrome would be 11011, which is greater than 10997.) However, the desired house numbers must also be such that the palindrome minus 10001 is divisible by 6, since only every sixth number is used. The ten differences are $0, 100, 200, 300, 400, 500, 600, 700, 800,$ and $900$.

Of these differences, we see that only four of the numbers, namely $0, 300, 600, \text{ and } 900$ are divisible by 6. It follows that only the four house numbers 10001, 10301, 10601, and 10901 are palindromes.
Problem of the Week
Problem B
High Fives!

Consider the following sum:

\[(2+3)+(32+23)+(232+323)+(3232+2323)+(23232+32323) + \cdots + (32323232+23232323)\].

a) Add the bracketed pairs of numbers to simplify the sum.
b) What is the digit in the ones place in the sum?
c) What is the digit in the tens place in the sum?
d) What is the digit in the hundreds place in the sum?
e) Find the sum.
Problem of the Week
Problem B
High Fives!

Problem
Consider the following sum:
\[(2+3)+(32+23)+(232+323)+(3232+2323)+(23232+32323) + \cdots + (32323232+23232323)\].

a) Add the bracketed pairs of numbers to simplify the sum.

b) What is the digit in the ones place in the sum?

c) What is the digit in the tens place in the sum?

d) What is the digit in the hundreds place in the sum?

e) Find the sum.

Solution

a) Adding the bracketed pairs gives
\[5 + 55 + 555 + 5555 + 55555 + \cdots + 55555555\].

b) Thus the digit in the ones place in the sum must be a 0, since there are 8 terms in total from part a), each with a ones digit of 5, and \(8 \times 5 = 40\).

c) For the tens digit, there are 7 terms with a 5 in the tens place, which sum to \(7 \times 5 = 35\), plus the 4 carried from the ones sum of 40, giving \(35 + 4 = 39\). Hence the tens digit is a 9.

d) By similar reasoning, there are 6 terms with a 5 in the hundreds place, summing to \(6 \times 5 = 30\), plus the 3 carried from the tens column, giving \(30 + 3 = 33\). Thus the hundreds digit is a 3.

e) There are 5 terms with a 5 in the \(1000\)s place, summing to \(5 \times 5 = 25\), plus the 3 carried from the hundreds column, giving \(25 + 3 = 28\). Hence the \(1000\)s digit is 8.

The \(10000\)s place has sum \(4 \times 5 = 20\), plus the carried 2 from the \(1000\)s column, giving sum 22. Hence the \(10000\)s digit is a 2.
The 100 000 s place has sum $3 \times 5 = 15$, plus the carried 2 from the 10 000 s column, giving sum 17. Hence the 100 000 s digit is a 7.

The 1 000 000 s place has sum $2 \times 5 = 10$, plus the carried 1 from the 100 000 s column, giving sum 11. Hence the 1 000 000 s digit is a 1.

Finally, the 10 000 000 s place has sum $1 \times 5 = 5$, plus the carried 1 from the 1 000 000 s column, giving sum 6.

Placing the digits in order, we see that the total sum is 61 728 390.
Ahmed wonders how many 3-digit whole numbers he can create which are not multiples of 10, and which have no repeating digits.

a) How many such numbers are there with a 1 in the hundreds place?

b) How many such numbers are there with a 2 in the hundreds place?

c) How many such numbers are there in total?
Problem

Ahmed wonders how many 3-digit whole numbers he can create which are not multiples of 10, and which have no repeating digits.

a) How many such numbers are there with a 1 in the hundreds place?

b) How many such numbers are there with a 2 in the hundreds place?

c) How many such numbers are there in total?

Solution

a) Three-digit numbers with a 1 in the hundreds place can be organized according to the tens digit as follows.

If the tens digit is 0, they are 102, 103, 104, 105, 106, 107, 108, 109.

If the tens digit is 1, there are none, since the 1 would be repeated.

If the tens digit is 2, they are 123, 124, 125, 126, 127, 128, 129.

If the tens digit is 3, they are 132, 134, 135, 136, 137, 138, 139.

If the tens digit is 4, they are 142, 143, 145, 146, 147, 148, 149.

Etcetera.

Observing the pattern we see there are 8 such numbers with tens digit 0, since the ones digit can be anything except 1 or 0. But for the possible tens digits 2, 3, 4, …, 9, there are only 7 possible numbers in each group, since the ones digit can’t be 0 (no multiples of 10), nor 1, nor the specific tens digit for that group.

Thus there are $8 + (8 \times 7) = 8 + 56 = 64$ such numbers with a 1 in the hundreds place.

b) For numbers with a 2 in the hundreds place, the outcome would be the same, namely one group of 8 (201, 203, 204, 205, 206, 207, 208, 209), and 8 groups of 7 (e.g., 231, 234, 235, 236, 237, 238, 239), for a total of 64 such numbers.

c) The same pattern would occur for hundreds digits 3, 4, 5, …, 9, with the greatest possible such number being 987. Thus the grand total of the 9 groups of such 3-digit numbers is $64 \times 9 = 576$. 
Problem of the Week
Problem B
Nesting Triangles

The Sierpinski triangle is a famous example of a fractal. Starting with a black equilateral triangle, an equilateral triangle void (shown as a white triangle in the second diagram) one-fourth the size of the black triangle is placed symmetrically inside the black triangle. This process is then repeated in each of the remaining three black equilateral triangles, creating the third diagram. And so on, repeatedly.

How many voids (white triangles) exist in the sixth Sierpinski diagram? How many black triangles?
Problem of the Week
Problem B and Solution
Nesting Triangles

Problem
The Sierpinski triangle is a famous example of a fractal.

Starting with a black equilateral triangle, an equilateral triangle void (shown as a white triangle in the second diagram) one-fourth the size of the black triangle is placed symmetrically inside the black triangle.

This process is then repeated in each of the remaining three black equilateral triangles, creating the third diagram. And so on, repeatedly.

How many voids (white triangles) exist in the sixth Sierpinski diagram?
How many black triangles?

Solution
Careful observation and counting reveals the following patterns.

Each successive Sierpinski triangle has 3 times as many black triangles as the previous Sierpinski triangle, but they are one-fourth the size.

From the third triangle onward, each Sierpinski triangle has an additional number of white triangles equal to 3 times the number of white triangles which were added to the previous Sierpinski triangle, but they are one-fourth the size.

We summarize these observations numerically in the table below.

<table>
<thead>
<tr>
<th>Triangle No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>White △s</td>
<td>0</td>
<td>1</td>
<td>1 + 3</td>
<td>1 + 3 + (3 × 3)</td>
<td>1 + 3 + (3 × 3) + (3 × 3 × 3)</td>
</tr>
<tr>
<td>Black △s</td>
<td>1</td>
<td>3</td>
<td>3 × 3</td>
<td>3 × 3 × 3</td>
<td>3 × 3 × 3 × 3</td>
</tr>
</tbody>
</table>

Following these patterns, in the sixth Sierpinski Triangle, there will be

1 + 3 + (3 × 3) + (3 × 3 × 3) + (3 × 3 × 3 × 3) = 121 white triangles,
and 3 × 3 × 3 × 3 × 3 = 243 black triangles.

NOTE: Try using 1/2-inch grid paper to construct the 6th and 7th triangles.