Problem of the Week
Grade 5/6 (B)

Problem and Solutions
2014 - 2015

Strands
Data Management & Probability
Geometry & Spatial Sense
Measurement
Number Sense & Numeration
Patterning & Algebra

(Click the strand name above to jump to that section)

The problems in this booklet are organized into strands. A problem often appears in multiple strands. The problems are suitable for most students in Grade 5 or higher.
Data Management
&
Probability
You are playing the very scary video game Scaryman. Randomly generated rooms are rectangular, and each has a perimeter of 20 m. The length and width of each room is a whole number.

a) What are the dimensions of the room with the least area?

b) What are the dimensions of the room with the greatest area?

c) There are five silent monsters randomly placed in the room with the greatest area. Each monster takes up one square metre. If you step through the door into the room of greatest area, what is the probability that you will stumble into a monster?

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

STRAND: Measurement, Probability
Problem of the Week

Problem B and Solution

Scaryman

Problem

You are playing the very scary video game Scaryman. Randomly generated rooms are rectangular, and each has a perimeter of 20 m. The length and width of each room is a whole number.

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c) There are five silent monsters randomly placed in the room with the greatest area. Each monster takes up one square metre. If you step through the door into the room of greatest area, what is the probability that you will stumble into a monster?

Solution

The rooms are rectangular, each with a perimeter of 20 m. Since twice the sum of the side lengths, say \( l \) m and \( w \) m, must equal the perimeter, we have \( 2 \times (l + w) = 20 \), or \( l + w = 10 \) m. Thus the possible side lengths must sum to 10 m, and hence they are

\[ l + w = 10 \text{ m} = (1 + 9)\text{ m}, (2 + 8)\text{ m}, (3 + 7)\text{ m}, (4 + 6)\text{ m}, \text{ and } (5 + 5)\text{ m}. \]

The corresponding rectangles have areas of \( 1 \times 9 = 9 \text{ m}^2 \), \( 2 \times 8 = 16 \text{ m}^2 \), \( 3 \times 7 = 21 \text{ m}^2 \), \( 4 \times 6 = 24 \text{ m}^2 \), and \( 5 \times 5 = 25 \text{ m}^2 \).

(Note that a rectangle can be a square.)

a) The room with least area has dimensions 1 m by 9 m.

b) The room with greatest area has dimensions 5 m by 5 m.

c) Since each of the 5 monsters takes up 1 m\(^2\), and there are 25 squares of that size in the 5 m by 5 m room, the chance that you will stumble into a monster is 5 in 25, or \( \frac{5}{25} = \frac{1}{5} \).
Deepa and Parminder are playing a game with two regular dice (numbered 1 to 6). When they roll both dice, the smaller number is subtracted from the larger number. (If the numbers are the same, they roll the dice again.) Deepa gets 5 points if the difference is odd; Parminder gets 5 points if the difference is even.

a) Is this a fair game? Explain your reasoning.

b) If not, how would you change the rules to make it fair?
Problem of the Week
Problem B and Solution
Rolling Along

Problem
Deepa and Parminder are playing a game with two regular dice (numbered 1 to 6). When they roll both dice, the smaller number is subtracted from the larger number. (If the numbers are the same, they roll the dice again.) Deepa gets 5 points if the difference is odd; Parminder gets 5 points if the difference is even.

a) Is this a fair game? Explain your reasoning.

b) If not, how would you change the rules to make it fair?

Solution

a) We can use ‘trees’ to reveal all the possible outcomes of each roll.

In the diagrams below, O means the difference is odd, E means it is even, and R means roll again.

```
Die 1

<table>
<thead>
<tr>
<th>Die 2</th>
<th>Difference</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 2 3 4 5 6</td>
<td>O O O O</td>
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<tr>
<td>2</td>
<td>0 1 2 3 4 5</td>
<td>E E E E</td>
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<td>3</td>
<td>1 0 1 2 3 4</td>
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<tr>
<td></td>
<td>2 1 0 1 2 3</td>
<td>E E E E</td>
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<thead>
<tr>
<th>Die 2</th>
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<tr>
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<td>1 2 3 4 5 6</td>
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<td>5</td>
<td>3 2 1 0 1 2</td>
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</tr>
<tr>
<td>6</td>
<td>5 4 3 2 1 0</td>
<td>O O O O</td>
</tr>
</tbody>
</table>
```

Counting the outcomes, we see that there are 18 odd differences, 12 even differences, and 6 ‘roll again’ outcomes, a total of 36 possible outcomes.

Thus Deepa’s chances of winning are \(\frac{18}{36}\) or \(\frac{1}{2}\), while Parminder’s chances are \(\frac{12}{36}\) or \(\frac{1}{3}\). So this is not a fair game.

b) To make it fair, both players must have the same chance of winning. One way to do this would be to say that a 0 difference counts as ‘even’ rather than ‘roll again’. Then there would be 18 odd and 18 even differences.
Problem of the Week

Problem B

Charting a Course

Below is a chart showing Distances between Major Canadian Cities.

<table>
<thead>
<tr>
<th>CITIES</th>
<th>St. J NF</th>
<th>Chtn PEI</th>
<th>Hfax NS</th>
<th>Fred NB</th>
<th>QCty PQ</th>
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<th>Ottawa ON</th>
</tr>
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<tbody>
<tr>
<td>St. John’s, NF</td>
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<td>1503</td>
<td>1777</td>
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Source: http://gocanada.about.com/od/canadatravelplanner/a/drive_distances.htm

a) How far is it from Halifax to Ottawa? From Fredericton to Regina?

b) Which distance is greater: from Montreal to Charlottetown or from Quebec City to Halifax, and by how much?

c) Hannah travels from St. John’s to Montreal via Halifax; Jack goes from St. John’s to Montreal via Fredericton. Who travels the greater distance?

d) A few entries in the chart are not really necessary. Which ones? Why?

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Strand: Data Management
**Problem of the Week**

**Problem B and Solution**

**Charting a Course**

**Problem**

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Questions (repeated from the first page)

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c) Hannah travels from St. John’s to Montreal via Halifax; Jack goes from St. John’s to Montreal via Fredericton. Who travels the greater distance?

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Solution

a) The distance from Halifax to Ottawa is 1439 km; from Fredericton to Regina is 3813 km.

b) The distance from Montreal to Charlottetown is 1199 km; from Quebec City to Halifax is 982 km. Thus the distance from Montreal to Charlottetown is greater by $1199 - 982 = 217$ km.

c) Hannah traveled from St. John’s to Halifax, a distance of 1503 km, then on to Montreal, a distance of 1249 km. Thus Hannah travels a total of $1503 + 1249 = 2752$ km.

Jack travels from St. John’s to Fredericton, a distance of 1777 km, then on to Montreal, a distance of 834 km. Thus Jack travels a total of $1777 + 834 = 2611$ km.

Thus Hannah travels the greater distance, by $2752 - 2611 = 141$ km.

d) Examining the chart, we see that the distances given to the right above the diagonal (marked by dashes) are identical to those in the first six lines below that diagonal. In fact, the top seven lines of the chart display symmetry about the diagonal, because the cities on the horizontal match those on the vertical. Since the distance from St. John’s to Montreal, for example, is the same as the distance from Montreal to St. John’s, one set of these numbers could be deleted with no loss of information.

From the eighth line downward, the cities differ from those across the top of the chart. Thus, all the distances shown from the eighth line downward are necessary.
Problem of the Week

Problem B

Crayon Envy

Richard is proud of the vast array of colours in his new bag of 144 crayons. Dennis feels a little envious, having only 48 crayons in his pencil case. But he believes he has a better chance than Richard of choosing a red-shaded crayon if each boy pulls a crayon from his pencil case without looking.

Each boy has 12 different colours: brown, black, red, blue, yellow, green, orange, purple, white, grey, silver, and gold. Richard’s set contains 12 different shades of each colour, while Dennis’ set contains 4 different shades of each colour.

a) What is the theoretical probability of Richard choosing a red-shaded crayon?
b) What is the theoretical probability of Dennis choosing a red-shaded crayon?
c) Is Dennis correct in believing he stands a better chance than Richard?
d) To guarantee that he gets a red-shaded crayon, how many times would Dennis have to pull a crayon from his pencil case?

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Strand: Probability
Problem of the Week

Problem B and Solution

Crayon Envy

Problem

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c) Is Dennis correct in believing he stands a better chance than Richard?

d) To guarantee that he gets a red-shaded crayon, how many times would Dennis have to pull a crayon from his pencil case?

Solution

a) Since Richard has 12 red-shaded crayons out of 144, the theoretical probability of drawing a red-shaded crayon is \( \frac{12}{144} \) or \( \frac{1}{12} \).

b) Since Dennis has 4 red-shaded crayons out of 48, the theoretical probability of drawing a red-shaded crayon is \( \frac{4}{48} \) or \( \frac{1}{12} \).

c) Dennis is incorrect, since both boys have the same chance of drawing a red-shaded crayon.

d) Dennis has 44 crayons that are NOT red-shaded, all of which he could draw in theory before drawing a red-shaded crayon. Thus he would have to draw 45 crayons to be guaranteed a red-shaded one.
Problem of the Week

Problem B

Disc-O Toss

The game of Disc-O is played with three identically-sized discs. Each has one blank side which counts as 0, and the other side has a single digit number, one with the number 1, one with the number 2, and one with the number 3.

Players take turns shaking the discs in their hands and tossing them onto the floor. Their score is the sum of the numbers on the upward faces. The first player to reach 25 wins.

a) What are the possible scores with one toss of the three discs?

b) In how many ways can a score of 5 occur? A score of 3?

c) What is the theoretical probability of scoring 5 in a single toss?

d) What is the theoretical probability of scoring 3 in a single toss?

e) Experiment with this game. (For the discs, use a marker to write 1, 2, 3 on the ‘tails’ side of three pennies; use ‘heads’ as 0.) Work with a partner, and take 10 turns each; record your scores in the table at the right.

f) Mansour claims that the theoretical probability of scoring 5 after two turns is the same as the theoretical probability of scoring 6 after two turns. Is he correct? Explain your answer.

<table>
<thead>
<tr>
<th>Turn</th>
<th>Score Partner A</th>
<th>Score Partner B</th>
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</thead>
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Strand: Number Sense, Probability
Problem of the Week
Problem B and Solution
Disc-O Toss

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c) What is the theoretical probability of scoring 5 in a single toss?
d) What is the theoretical probability of scoring 3 in a single toss?
e) Experiment with this game. (For the discs, use a marker to write 1, 2, 3 on the ‘tails’ side of three pennies; use ‘heads’ as 0.) Work with a partner, and take 10 turns each; record your scores in the table at the right.
f) Mansour claims that the theoretical probability of scoring 5 after two turns is the same as the theoretical probability of scoring 6 after two turns. Is he correct? Explain your answer.

Solution
a) There are 8 possible ways to obtain scores with one toss of the three discs:

\[ 0 + 0 + 0 = 0; \quad 0 + 0 + 1 = 1; \quad 0 + 0 + 2 = 2; \quad 0 + 0 + 3 = 3; \]
\[ 0 + 1 + 2 = 3; \quad 0 + 1 + 3 = 4; \quad 0 + 2 + 3 = 5; \quad 1 + 2 + 3 = 6. \]

From these possibilities, we conclude that 7 scores are possible. One of the scores, namely 3, can be obtained two different ways.

b) A score of 5 can occur in only one way; a score of 3 can occur in two ways.

c) The probability of scoring 5 in a single toss is thus 1 in 8, or \( \frac{1}{8} \).

d) The probability of scoring 3 in a single toss is thus 2 in 8, or \( \frac{1}{4} \).

e) (Experiments will vary.)

f) The sum 5 could be from \( 0 + 5 \) or \( 1 + 4 \) or \( 2 + 3 \), in either order. Since there are two ways to get a score of 3 on a single turn, there are a total of 8 ways to get 5 in two turns. The sum 6 can be formed as \( 0 + 6 \), \( 1 + 5 \) or \( 2 + 4 \) (in either order), a total of 6 ways. But there is also \( 3 + 3 \). Each of the 3s can be formed in two ways on each turn. Thus there are an additional 4 ways to get 6 in two turns. There is a total of 10 ways to get 6 in two turns.

Since the number of possible outcomes for scoring a total of 5 in two turns is different than the number of possible outcomes for scoring a total of 6 in two turns, the probabilities would also be different. So Mansour is incorrect.
Geometry & Spatial Sense

TAKE ME TO THE COVER
Using your GPS app, you decide to plot your walk using the graphing option. Below is a description of your path; all turns are right angles (90°).

Walk north 100 m, then west 50 m, then south 150 m, then east 250 m, then north 100 m, then west 150 m, then south 50 m, and finally west 50 m.

a) On the grid given below, trace your path if your journey begins at point S. Determine the name of the resulting shape.
b) Determine the perimeter of the resulting shape.
c) Find the area enclosed by the map of your hike.
d) On graph paper, make another hike that starts and finishes at the same point. List the direction and distance of each side to share with your classmates. With your partner, figure out the name for the shape formed by each hike, and find its perimeter, and the area enclosed.

Check out other CEMC resources here:
[CEMC resources link]

**Strand:** Measurement, Geometry
Problem of the Week
Problem B and Solution

Take A Hike!

Problem
Using your GPS app, you decide to plot your walk using the graphing option. Below is a description of your path; all turns are right angles (90°).

Walk north 100 m, then west 50 m, then south 150 m, then east 250 m, then north 100 m, then west 150 m, then south 50 m, and finally west 50 m.

a) On the grid given below, trace your path if your journey begins at point S. Determine the name of the resulting shape.
b) Determine the perimeter of the resulting shape.
c) Find the area enclosed by the map of your hike.
d) On graph paper, make another hike that starts and finishes at the same point. List the direction and distance of each side to share with your classmates. With your partner, figure out the name for the shape formed by each hike, and find its perimeter, and the area enclosed.

Solution

a) The shape of the hike is an irregular octagon.

b) Using the given lengths of each side, we see that the perimeter of the octagon is

\[ 100 + 50 + 150 + 250 + 100 + 150 + 50 + 50 = 900 \text{ m}. \]

c) The area enclosed by the map of the hike is the sum of the areas of rectangles A, B, C. Since each rectangle has area equal to its length times its width, the total area is

\[ 50 \times 150 + 50 \times 50 + 150 \times 100 = 7500 + 2500 + 15000 = 25000 \text{ m}^2. \]

d) Answers will vary.
Teacher Ellen needs some small cubes to help her Grade 2 students learn some arithmetic. She has a 4 cm cube that is painted bright red on the outside. She plans to cut this cube into 64 cubes of equal size.

a) What will be the dimensions and volume of each of the smaller cubes?

b) How many of the smaller cubes will have exactly three red faces?

c) How many will have exactly two red faces? One red face?

d) How many of the smaller cubes will have no red paint?

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

**STRAND:** Geometry, Spatial Visualization
Problem of the Week
Problem B and Solution
Face Paint

Problem
Teacher Ellen needs some small cubes to help her Grade 2 students learn some arithmetic. She has a 4 cm cube that is painted bright red on the outside. She plans to cut this cube into 64 cubes of equal size.

a) What will be the dimensions and volume of each of the smaller cubes?
b) How many of the smaller cubes will have exactly three red faces?
c) How many will have exactly two red faces? One red face?
d) How many of the smaller cubes will have no red paint?

Solution (Building a model may be helpful.)

a) Since the cube is 4 cm by 4 cm by 4 cm, its volume is \(4 \times 4 \times 4 = 64 \text{ cm}^3\). Thus if Teacher Ellen cuts it into 64 cubes of equal size, the size of each of the smaller cubes will be 1 cm by 1 cm by 1 cm, i.e., 1 cm\(^3\).

b) To have exactly three red faces, the smaller cube would need to come from a corner of the larger cube. Thus eight of the smaller cubes will have exactly three red faces.

c) To have exactly two red faces, the smaller cube would need to come from an edge (but not a corner) of the larger cube. There are 24 such ‘edge’ cubes, 2 on each of the 12 edges of the cube.

To have exactly one red face, the smaller cube would need to come from a face (but not an edge) of the larger cube. There are four of these on each of the 6 faces, and hence \(4 \times 6 = 24\) such cubes.

d) The ‘interior’ cubes, which make up a 2 cm by 2 cm by 2 cm portion of the larger cube, will have no red paint. Thus there are 8 such cubes. We can obtain this answer also by subtracting what we have counted already from 64. (i.e. \(64 - 8 - 24 - 24 = 8\))

Extension: Suppose Teacher Ellen had started with a 5 cm cube to be cut into 125 smaller pieces. Predict how many of the smaller cubes will have no red faces.
Problem of the Week

Problem B

Tessellation Dedication

A tessellation of the plane using a trapezoid is illustrated below. (In a tessellation, all of the plane is covered, with no empty spaces, by arranging the same shape repeatedly in some pattern.)

a) In a set of standard pattern blocks, as shown at the right, which of the blocks will tessellate?

b) Which of the shapes below will NOT tessellate?

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

Strand: Geometry, Spatial Sense
Problem of the Week
Problem B and Solution
Tessellation Dedication

Problem
A tessellation of the plane using a trapezoid is illustrated below. (In a tessellation, all of the plane is covered, with no empty spaces, by arranging the same shape repeatedly in some pattern.)

a) In a set of standard pattern blocks, as shown at the right, which of the blocks will tessellate?

b) Which of the shapes below will NOT tessellate?
Solution

a) All the pattern blocks tessellate. The trapezoid tessellation is given in the problem. Below are some creative displays which reveal how the other blocks tessellate.

![Hexagon, Triangle, Argyle, and Kaleidoscope patterns]

b) The oval will not tessellate because the round, convex edges will always leave empty spaces between each copy of the oval.

The scalene triangle will tessellate, since two copies with matching long sides form a parallelogram, which we know tessellates as above.

The irregular trapezoid also will tessellate, by repeating the left-hand group of four trapezoids shown in the diagram at the right, inverting every other two trapezoids.

Peaked Shapes 1 and 3 will tessellate, as shown at the right. However, Peak Shape 2 will not tessellate; while the top and bottom peaks can be interlocked, the peak on the left side has no indentation to match on the right side.

For rectangular shapes such as these, whatever shape (e.g., triangle, semi-circle) is added to one side must be taken from the opposite side for a fit.
Problem of the Week

Problem B

Shapes and Symmetry

In the set of figures below, sides of equal length are indicated by a ‘|’ or ‘∥’.

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12.

Use the enlarged diagrams on the following page in answering questions a) to d).

a) Give a name for each figure.

b) Which figures have only one line of symmetry? Draw the line on each such figure.

c) Which figures have exactly two lines of symmetry? Draw the lines on each such figure.

d) Are there any figures with more than four lines of symmetry? If so, which figures?
Extension

Which figures would you describe as having a centre point? Why?

Check out other CEMC resources here:

cemc.uwaterloo.ca/resources/resources.html

**Strand:** Geometry
Problem of the Week
Problem B and Solution
Shapes and Symmetry

Problem and Solution
The diagrams below show all lines of symmetry for the set of figures given in the problem, except for the circle, which has infinitely many lines of symmetry (all diagonals).

1. 2. 3. 4. 
5. 6. 7. 8. 
9. 10. 11. 12.

a) Possible names for each figure are, in order: square, (equilateral) triangle, (regular) hexagon, rhombus or diamond, (scalene) triangle, rectangle, quadrilateral or kite, circle, (regular) pentagon, trapezoid or quadrilateral, quadrilateral, (non-regular) hexagon.

b) The figures with only one line of symmetry are Figures 7, 10, and 12.

c) The figures with exactly two lines of symmetry are Figures 4 and 6.

d) Figure 1 has 4 lines of symmetry, and Figures 3, 8, and 9 have more than 4 lines of symmetry.
Measurement

TAKE ME TO THE COVER
Problem of the Week

Problem B

Take a Hike!

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Walk north 100 m, then west 50 m, then south 150 m, then east 250 m, then north 100 m, then west 150 m, then south 50 m, and finally west 50 m.

a) On the grid given below, trace your path if your journey begins at point S. Determine the name of the resulting shape.
b) Determine the perimeter of the resulting shape.
c) Find the area enclosed by the map of your hike.
d) On graph paper, make another hike that starts and finishes at the same point. List the direction and distance of each side to share with your classmates. With your partner, figure out the name for the shape formed by each hike, and find its perimeter, and the area enclosed.

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STRAND: Measurement, Geometry
Problem of the Week

Problem B and Solution

Take A Hike!

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a) The shape of the hike is an irregular octagon.

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\[100 + 50 + 150 + 250 + 100 + 150 + 50 + 50 = 900 \text{ m}.\]

c) The area enclosed by the map of the hike is the sum of the areas of rectangles A, B, C.

Since each rectangle has area equal to its length times its width, the total area is

\[50 \times 150 + 50 \times 50 + 150 \times 100 = 7500 + 2500 + 15000 = 25000 \text{ m}^2.\]

d) Answers will vary.
Problem of the Week

Problem B

Scaryman

You are playing the very scary video game Scaryman. Randomly generated rooms are rectangular, and each has a perimeter of 20 m. The length and width of each room is a whole number.

a) What are the dimensions of the room with the least area?

b) What are the dimensions of the room with the greatest area?

c) There are five silent monsters randomly placed in the room with the greatest area. Each monster takes up one square metre. If you step through the door into the room of greatest area, what is the probability that you will stumble into a monster?

Check out other CEMC resources here:

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STRAND: Measurement, Probability
Problem of the Week
Problem B and Solution
Scaryman

Problem

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c) There are five silent monsters randomly placed in the room with the greatest area. Each monster takes up one square metre. If you step through the door into the room of greatest area, what is the probability that you will stumble into a monster?

Solution

The rooms are rectangular, each with a perimeter of 20 m. Since twice the sum of the side lengths, say \( l \) m and \( w \) m, must equal the perimeter, we have

\[ 2 \times (l + w) = 20, \text{ or } l + w = 10 \text{ m}. \]

Thus the possible side lengths must sum to 10 m, and hence they are

\[ l + w = 10 \text{ m} = (1 + 9)\text{m}, (2 + 8)\text{m}, (3 + 7)\text{m}, (4 + 6)\text{m}, \text{ and } (5 + 5)\text{m}. \]

The corresponding rectangles have areas of \( 1 \times 9 = 9 \text{ m}^2 \), \( 2 \times 8 = 16 \text{ m}^2 \), \( 3 \times 7 = 21 \text{ m}^2 \), \( 4 \times 6 = 24 \text{ m}^2 \), and \( 5 \times 5 = 25 \text{ m}^2 \).

(Note that a rectangle can be a square.)

a) The room with least area has dimensions 1 m by 9 m.
b) The room with greatest area has dimensions 5 m by 5 m.
c) Since each of the 5 monsters takes up 1 m\(^2\), and there are 25 squares of that size in the 5 m by 5 m room, the chance that you will stumble into a monster is 5 in 25, or \( \frac{5}{25} = \frac{1}{5} \).
Problem of the Week

Problem B

Tarts? Sweet!

In the left column of the table below are the ingredients required to make 48 Pecan Mini-tarts.

Belinda wants to make 24 tarts and is wondering what quantity of each ingredient she will need. Mr. Singh wants to feed his whole class, and needs to make 60 tarts.

In the chart below, fill in the required quantities of each ingredient. Are there any ingredients for which the exact quantity is unreasonable?

<table>
<thead>
<tr>
<th>Ingredients for 48 tarts</th>
<th>Ingredients for 24 tarts</th>
<th>Ingredients for 60 tarts</th>
</tr>
</thead>
<tbody>
<tr>
<td>250 g cream cheese</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 cup butter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{4}$ cup sugar</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 cups flour</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 eggs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{3}$ cup butter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 cup brown sugar</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 cup chopped pecans</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1\frac{1}{2}$ teaspoons vanilla</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{2}$ cup candied cherries</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Recipe instructions follow.. Yummm....

Pastry
Cream butter and cream cheese with sugar until light and fluffy. Add flour a bit at a time and blend well. Shape gently into a log and cut into 48 equal-sized pieces. Place each piece into a cup of a mini-tart pan, and press with your fingers to shape the tart shell. (Alternatively, on a lightly floured surface, roll each piece to a suitably-sized circle.)

Filling
Break the eggs into a mixing bowl. Add the brown sugar, pecans, vanilla, and cherries and mix well. Distribute the filling evenly among the tart shells. Bake at 350 degrees F (180 degrees C) until lightly browned, about 30 minutes.

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Strand: Number Sense, Measurement
Problem of the Week
Problem B and Solution
Tarts? Sweet!

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<th>Ingredients for 60 tarts</th>
</tr>
</thead>
<tbody>
<tr>
<td>250 g cream cheese</td>
<td>125 g</td>
<td>312.5 g</td>
</tr>
<tr>
<td>1 cup butter</td>
<td>½ cup</td>
<td>1½ cups</td>
</tr>
<tr>
<td>¼ cup sugar</td>
<td>⅛ cup</td>
<td>5/16 cup</td>
</tr>
<tr>
<td>2 cups flour</td>
<td>1 cup</td>
<td>2¼ cups</td>
</tr>
<tr>
<td>2 eggs</td>
<td>1 egg</td>
<td>2½ eggs</td>
</tr>
<tr>
<td>⅓ cup butter</td>
<td>¼ cup</td>
<td>5/12 cup</td>
</tr>
<tr>
<td>1 cup brown sugar</td>
<td>½ cup</td>
<td>1½ cups</td>
</tr>
<tr>
<td>1 cup chopped pecans</td>
<td>½ cup</td>
<td>1½ cups</td>
</tr>
<tr>
<td>1½ teaspoons vanilla</td>
<td>¾ tsp</td>
<td>1⅞ tsp</td>
</tr>
<tr>
<td>½ cup candied cherries</td>
<td>¼ cup</td>
<td>5/8 cup</td>
</tr>
</tbody>
</table>

Exact quantities which seem unreasonable:
- 2½ eggs is not reasonable; perhaps use two large eggs to obtain the extra ½ egg;
- while 312.5 g of cream cheese may seem tricky, it is actually just one 250 g package plus ¼ of another package.
Problem of the Week

Problem B

Wrecked-Angle

The area of a rectangle is equal to the product of its length times its width.

a) What are the possible dimensions of a rectangle of area 60 cm\(^2\)? Fill in the first two columns of the given table, using width as the lesser dimension and length as the greater.

b) Which of these possible rectangles, if its length is decreased by 4 cm and its width is increased by 1 cm, would result in a rectangle four times as long as it is wide? Complete the table to help find the answer.

<table>
<thead>
<tr>
<th>Width</th>
<th>Length</th>
<th>Width + 1</th>
<th>Length - 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
<td>2</td>
<td>56</td>
</tr>
</tbody>
</table>

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense, Measurement
Problem of the Week
Problem B and Solution
Wrecked-Angle

Problem

The area of a rectangle is equal to the product of its length times its width.

a) What are the possible dimensions of a rectangle of area $60 \text{ cm}^2$? Fill in the first two columns of the given table, using width as the lesser dimension and length as the greater.

b) Which of these possible rectangles, if its length is decreased by 4 cm and its width is increased by 1 cm, would result in a rectangle four times as long as it is wide? Complete the table to help find the answer.

Solution

Here is the completed table.

Table of Dimensions (all measurements in cm)

<table>
<thead>
<tr>
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<th>Width + 1</th>
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<tbody>
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<td>60</td>
<td>2</td>
<td>56</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>3</td>
<td>26</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

a) The first two columns reveal the six possible sets of dimensions.

b) The third row of the table shows that the 3 cm by 20 cm rectangle, upon increasing its width to 4 cm and decreasing its length to 16 cm, gives a rectangle with length four times its width.
Problem of the Week  
Problem B  
Dollars for Digits

McMillan’s Construction Company is just completing the construction of homes on a street in a new subdivision. Metal digits are to be placed above each front door to label the house numbers.

The total cost of each three-digit house number is shown in the table at the right.

The two digits which are congruent (i.e., have the same shape) cost exactly the same amount in whole dollars.

What is the cost of each metal digit 0 \ldots 9?

<table>
<thead>
<tr>
<th>North Side</th>
<th>Cost</th>
<th>South Side</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>688</td>
<td>$14.00</td>
<td>693</td>
<td>$12.25</td>
</tr>
<tr>
<td>692</td>
<td>$11.50</td>
<td>697</td>
<td>$11.50</td>
</tr>
<tr>
<td>696</td>
<td>$12.00</td>
<td>701</td>
<td>$8.50</td>
</tr>
<tr>
<td>700</td>
<td>$9.50</td>
<td>705</td>
<td>$11.00</td>
</tr>
<tr>
<td>704</td>
<td>$10.25</td>
<td></td>
<td></td>
</tr>
</tbody>
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*Check out other CEMC resources here:*  
cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense, Measurement
Problem of the Week
Problem B and Solution
Dollars for Digits

Problem

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<td></td>
<td></td>
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Solution

Since the digits 6 and 9 are congruent, they cost the same amount in whole dollars. Noting that the house number 696 costs $12.00, we thus deduce that both the 6 and the 9 cost $4.00 each.

Then, using the house number 688, we see that two 8’s cost $14.00 − $4.00 = $10.00, and hence each 8 costs $5.00.

From house numbers 692 and 697, we see that a 2 and a 7 each must cost $11.50 − $8.00 = $3.50. Similarly, from house number 693, a 3 must cost $12.25 − $8.00 = $4.25.

Now, from 700, two 0’s cost $9.50 − $3.50 = $6.00, and hence each 0 costs $3.00.

Finally, from 701, a 1 costs $8.50 − $3.50 − $3.00 = $2.00, while from 704, a 4 costs $10.25 − $3.50 − $3.00 = $3.75, and from 705, a 5 costs $11.00 − $3.50 − $3.00 = $4.50.
Problem of the Week

Problem B

Face Paint

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b) How many of the smaller cubes will have exactly three red faces?

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STRAND: Geometry, Spatial Visualization
Problem of the Week

Problem B and Solution

Face Paint

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c) How many will have exactly two red faces? One red face?

d) How many of the smaller cubes will have no red paint?

Solution (Building a model may be helpful.)

a) Since the cube is 4 cm by 4 cm by 4 cm, its volume is \(4 \times 4 \times 4 = 64\text{ cm}^3\).

Thus if Teacher Ellen cuts it into 64 cubes of equal size, the size of each of the smaller cubes will be 1 cm by 1 cm by 1 cm, i.e., \(1\text{ cm}^3\).

b) To have exactly three red faces, the smaller cube would need to come from a corner of the larger cube. Thus eight of the smaller cubes will have exactly three red faces.

c) To have exactly two red faces, the smaller cube would need to come from an edge (but not a corner) of the larger cube. There are 24 such ‘edge’ cubes, 2 on each of the 12 edges of the cube.

To have exactly one red face, the smaller cube would need to come from a face (but not an edge) of the larger cube. There are four of these on each of the 6 faces, and hence \(4 \times 6 = 24\) such cubes.

d) The ‘interior’ cubes, which make up a 2 cm by 2 cm by 2 cm portion of the larger cube, will have no red paint. Thus there are 8 such cubes. We can obtain this answer also by subtracting what we have counted already from 64. (i.e. \(64 - 8 - 24 - 24 = 8\))

Extension: Suppose Teacher Ellen had started with a 5 cm cube to be cut into 125 smaller pieces. Predict how many of the smaller cubes will have no red faces.
Problem of the Week

Problem B

A Piece of Cake

Suki is making a special chocolate cake for her friend Maya. To make this chocolate cake, she needs sugar, cocoa powder, and flour in the ratio 8 : 3 : 7.

a) Express the needed flour as a fraction of the needed sugar.

b) Express, in lowest terms, the needed cocoa powder as a fraction of the total of the three ingredients.

c) Suppose the recipe requires 400 g of sugar. How much cocoa powder does she need?

d) Another cake recipe requires 120 g of cocoa powder. If the ratios remain the same, how much sugar does she need? How much flour?

e) If she wished to make a cake weighing 1.8 kg, could she determine the required weight of each ingredient from the given ratios? Explain your answer.

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**Strand:** Number Sense, Measurement
Problem of the Week
Problem B and Solution
A Piece of Cake

Problem

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e) If she wished to make a cake weighing 1.8 kg, could she determine the required weight of each ingredient from the given ratios? Explain your answer.

Solution

a) Since the ratio of flour to sugar is 7:8, the amount of flour needed is $\frac{7}{8}$ the amount of sugar.
b) The ratio of cocoa powder to all three ingredients is 3:18. In lowest terms, the fraction $\frac{3}{18}$ is $\frac{1}{6}$.
c) Since the ratio of cocoa powder to sugar is $\frac{3}{8}$, if the recipe requires 400 g of sugar, then the amount of cocoa powder needed is $\frac{3}{8} \times 400 = 150$ g.
d) The ratio of sugar to cocoa powder is $\frac{8}{3}$, while that of flour to cocoa powder is $\frac{7}{3}$. Thus, for this new recipe with 120 g of cocoa powder, Suki will need $\frac{8}{3} \times 120 = 320$ g of sugar, and $\frac{7}{3} \times 120 = 280$ g of flour.
e) Since the ratio of each ingredient to the total weight of the ingredients is known, Suki could determine the required weight of each ingredient as follows, based on a total weight of 1800 g.

- The ratio of sugar to the total weight is $\frac{8}{18} = \frac{4}{9}$, so the amount of sugar required is $\frac{4}{9} \times 1800 = 800$ g.
- The ratio of cocoa powder to the total weight is $\frac{3}{18} = \frac{1}{6}$, so the amount of cocoa powder required is $\frac{1}{6} \times 1800 = 300$ g.
- The ratio of flour to the total weight is $\frac{7}{18}$, so the amount of flour required is $\frac{7}{18} \times 1800 = 700$ g.
Problem of the Week

Problem B

Skott’s Spots

Skott was painting the walls in his living room and den. Unfortunately, he accidentally got some small dots of paint on the ceiling near the wall. So he decided to put a decorative rope along the ceiling, where it meets the walls.

To do this, Skott has to measure the perimeter of the ceilings to make sure he has enough of the decorative rope to go around each ceiling.

Both ceilings have an area of 24 square metres, with dimensions that are whole numbers.

Skott realizes that his 20 m roll of decorative rope is only enough for one ceiling but not the other.

a) Find the dimensions of the room for which Skott has enough decorative rope.

b) Given that the other room has different dimensions, and its perimeter is less than 50 m, how much more decorative rope does he need to ensure he can do the other room?

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense, Measurement
Problem of the Week
Problem B and Solution
Skott’s Spots

Problem

Skott was painting the walls in his living room and den. Unfortunately, he accidentally got some small dots of paint on the ceiling near the wall. So he decided to put a decorative rope along the ceiling, where it meets the walls.

To do this, Skott has to measure the perimeter of the ceilings to make sure he has enough of the decorative rope to go around each ceiling.

Both ceilings have an area of 24 square metres, with dimensions that are whole numbers.

Skott realizes that his 20 m roll of decorative rope is only enough for one ceiling but not the other.

a) Find the dimensions of the room for which Skott has enough decorative rope.

b) Given that the other room has different dimensions, and its perimeter is less than 50 m, how much more decorative rope does he need to ensure he can do the other room?

Solution

a) From the table at the right, we see that the only room of area 24\,\text{m}^2 for which Skott has enough decorative rope is one that is 4\,\text{m} by 6\,\text{m}.

b) Given that its perimeter is less than 50\,\text{m} and its dimensions are different than the other room, the other room could have dimensions of either 3\,\text{m} by 8\,\text{m} or 2\,\text{m} by 12\,\text{m}, with perimeters of 22\,\text{m} and 28\,\text{m} respectively. Thus to ensure he can do the other room, Skott needs 28\,\text{m} more of the decorative rope.

For rooms of area 24\,\text{m}^2:

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 \times 24</td>
<td>50</td>
</tr>
<tr>
<td>2 \times 12</td>
<td>28</td>
</tr>
<tr>
<td>3 \times 8</td>
<td>22</td>
</tr>
<tr>
<td>4 \times 6</td>
<td>20</td>
</tr>
</tbody>
</table>
Richard and Dennis would like to make boxes to hold their crayons more compactly, rather than carrying them around in pencil cases. Their crayons have the shape of rectangular prisms, each 8 cm long and 1 cm thick. Using quality materials from MaryLynne’s Mercantile, Richard has constructed a rectangular box with dimensions as shown below.

![Box Diagram](image)

a) Will Richard’s pencil box hold all of Dennis’ 48 crayons? Will it hold Richard’s 144 crayons?

b) What are possible dimensions of a box 8 cm long that will exactly hold Dennis’ crayons?

c) Richard would prefer a box in which his 144 crayons will stand on end, for easy access. What would be a suitable set of dimensions for such a box?

*Check out other CEMC resources here: cemc.uwaterloo.ca/resources/resources.html*

**Strand:** Number Sense, Measurement
Problem of the Week

Problem B and Solution

Box It Up!

Problem

Richard and Dennis would like to make boxes to hold their crayons more compactly, rather than carrying them around in pencil cases. Their crayons have the shape of rectangular prisms, each 8 cm long and 1 cm thick.

Using quality materials from MaryLynne’s Mercantile, Richard has constructed a rectangular box with dimensions 16 cm long, 6 cm high, and 6 cm deep.

a) Will Richard’s pencil box hold all of Dennis’ 48 crayons? Will it hold Richard’s 144 crayons?

b) What are possible dimensions of a box 8 cm long that will exactly hold Dennis’ crayons?

c) Richard would prefer a box in which his 144 crayons will stand on end, for easy access. What would be a suitable set of dimensions for such a box?

Solution

The diagram above reveals that the dimensions of the box that will admit a layer of 12 crayons is 16 cm long by 6 cm deep. Since the box is 6 cm high, it will hold 6 such layers, with no space left over. Thus the capacity of the box is $12 \times 6 = 72$ crayons.

a) We thus conclude that Richard’s box will hold all of Dennis’s 48 crayons, but will not hold Richard’s 144 crayons.

b) A box to exactly hold Dennis’s 48 crayons would need to have a volume of

$$48 \times 8 \times 1 \times 1 = 384 \text{ cm}^3.$$  

Since the box is to be 8 cm long, its cross section would need to have an area of

$$A = 384 \div 8 = 48 \text{ cm}^2,$$

i.e., the height times the depth must be 48 cm².
Thus the possible dimensions are:

- 1 cm by 48 cm (1 layer of 48 crayons, or 48 layers of 1 crayon);
- 2 cm by 24 cm (2 layers of 24 crayons, or 24 layers of 2 crayons);
- 3 cm by 16 cm (3 layers of 16 crayons, or 16 layers of 3 crayons);
- 4 cm by 12 cm (4 layers of 12 crayons, or 12 layers of 4 crayons);
- 6 cm by 8 cm (6 layers of 8 crayons, or 8 layers of 6 crayons).

(Think about which of these possibilities are unsuitable for a crayon box.)

c) Since each crayon has a cross section of area 1 cm\(^2\), for the crayons to stand on end, the base area of Richard’s box will need to be 144 cm\(^2\), and its height must be 8 cm.

Thus the possible dimensions of the base are:

- 1 cm by 144 cm (highly unsuitable);
- 2 cm by 72 cm (also unsuitable);
- 3 cm by 48 cm (still a little long);
- 4 cm by 36 cm (might work, but still a bit thin and long);
- 6 cm by 24 cm (reasonably suitable).
- 8 cm by 18 cm (suitable)
- 12 cm by 12 cm (suitable)

(The criteria for suitability are worth discussing.)
Laura is attending a wedding at which 300 guests are present. The guests begin to leave the celebration at 9:00 p.m., 20 at a time, and thereafter, groups of 20 guests leave at 20-minute intervals.

The bride and groom depart once two-fifths of the guests have left. Right after that, a late-night buffet is served to the remaining guests.

At what time is the buffet served?
Problem of the Week
Problem B and Solution
When Do We Eat?

Problem
Laura is attending a wedding at which 300 guests are present. The guests begin to leave the celebration at 9:00 p.m., 20 at a time, and thereafter, groups of 20 guests leave at 20-minute intervals.

The bride and groom depart once two-fifths of the guests have left. Right after that, a late-night buffet is served to the remaining guests.

At what time is the buffet served?

Solution
The buffet is served once $\frac{2}{5}$ of the guests have left, i.e., after $\frac{2}{5} \times 300 = 120$ guests have left, in groups of 20. Since $120 = 6 \times 20$, this means that six groups of 20 guests must leave before the remaining guests get to eat.

The groups of 20 guests at a time start leaving at 9:00 p.m., and exit every 20 minutes after that, the second group at 9:20, the third at 9:40, the fourth at 10:00, the fifth at 10:20, and the sixth at 10:40.

Thus the buffet will be served at 10:40 p.m.
Problem of the Week
Problem B
Penny-Less

Larry Min is frugal (careful with his money), so he is always looking for a way to save a few pennies. Unfortunately for Larry, pennies are no longer used in Canada. Even so, he knows that every time he buys gas, the total gets rounded to the nearest nickel, so he decides to take advantage of this.

a) If Larry buys 50 L of gas priced at $1.20 per L, how much will he pay?

b) Since rounding occurs to the nearest nickel, there are different fuel pump purchases which will cost the same amount. For example, a fill-up which shows $30.52 on the pump will require a cash payment of only $30.50; similarly, one which comes to $30.50 will require the same cash payment, $30.50, as will pump totals of $30.48, or $30.51. What is the set of possible totals shown on the pump that will all require the same cash payment as Larry paid in part a)?

c) Which of the possible pump totals in part b) will save Larry the greatest amount of money?

d) If penny-wise Larry saves this much every week of the year, how much will he save over one year?

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

Strand: Measurement, Number Sense
Problem of the Week
Problem B and Solution
Penny-Less

Problem
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a) If Larry buys 50 L of gas priced at $1.20 per L, how much will he pay?

b) Since rounding occurs to the nearest nickel, there are different fuel pump purchases which will cost the same amount. For example, a fill-up which shows $30.52 on the pump will require a cash payment of only $30.50; similarly, one which comes to $30.49 will require the same cash payment, $30.50, as will pump totals of $30.48, or $30.51. What is the set of possible totals shown on the pump that will all require the same cash payment as Larry paid in part a)?

c) Which of the possible pump totals in part b) will save Larry the greatest amount of money?

d) If penny-wise Larry saves this much every week of the year, how much will he save over one year?

Solution

a) If Larry buys 50 L of gas priced at $1.20 per L, he will pay

\[ 50 \times 1.20 = 60.00 \, \text{.} \]

b) Pump totals which would all require a cash payment of $60.00 are $59.98, $59.99, $60.00, $60.01, and $60.02.

c) To get the greatest amount of gas possible for $60.00, Larry should put in enough gas to show $60.02 on the pump, saving $0.02.

d) If penny-wise Larry saves this much every week of the year, he will save a total of

\[ 52 \times 0.02 = 1.04 \, \text{ over the year.} \]
Number Sense & Numeration
Problem of the Week

Problem B

Extremely Sporty!

Mr. Jones’ class (25 students) and Ms. Rodriguez’ class (24 students) are off to the Extreme Sports Camp on a field trip. Their expenses will be as follows:

- the program costs $10 per student;
- the bus costs $235.20;
- the group will have lunch at the Camp, at a charge of $2.75 per student.

If every student participates in the field trip, how much do the teachers have to charge each student in order to pay for the trip?

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense
Problem of the Week
Problem B and Solution
Extremely Sporty

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- the bus costs $235.20;
- the group will have lunch at the Camp, at a charge of $2.75 per student.

If every student participates in the field trip, how much do the teachers have to charge each student in order to pay for the trip?

Solution
In total, 24 + 25 = 49 students are going on the field trip. To find the cost to each student, we need to find cost of the bus per student, which is

\[ \frac{235.20}{49} = 4.80 \]

per student.
Thus the total cost per student is

\[ \$10 \text{ (program cost)} + \$2.75 \text{ (lunch)} + \$4.80 \text{ (bus cost)} = \$17.55. \]
Problem of the Week

Problem B

Dizzying Distances

Dizzy the indecisive rabbit is sitting by his favourite tree stump when he hears a dog barking nearby.

In a panic, he runs 1 m to the right, then turns around and runs 3 m to the left, then turns around and runs 5 m to the right, then 7 m left, etcetera.

a) If he continues to run in this pattern, how far will he be from the stump as he makes his 10th turn?

b) What is the total distance Dizzy has run at his 3rd turn? At his 6th turn? How are these distances related to the number of turns?

c) Using your results from part b), predict the total distance Dizzy will have run as he makes his 10th turn. Check your prediction by completing the table at the right.

<table>
<thead>
<tr>
<th>Turn No.</th>
<th>Position R, L (right or left of stump)</th>
<th>Total Distance Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 m R</td>
<td>1 m</td>
</tr>
<tr>
<td>2</td>
<td>2 m L</td>
<td></td>
</tr>
</tbody>
</table>

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

Strand: Number Sense, Patterning and Algebra
Problem of the Week
Problem B and Solution
Dizzying Distances

Problem
Dizzy the indecisive rabbit is sitting by his favourite tree stump when he hears a dog barking nearby.

In a panic, he runs 1 m to the right, then turns around and runs 3 m to the left, then turns around and runs 5 m to the right, then 7 m left, etcetera.

a) If he continues to run in this pattern, how far will he be from the stump as he makes his 10th turn?

b) What is the total distance Dizzy has run at his 3rd turn? At his 6th turn? How are these distances related to the number of turns?

c) Using your results from part b), predict the total distance Dizzy will have run as he makes his 10th turn. Check your prediction by completing the table at the right.

Solution
a) As he makes his 10th turn, Dizzy will be 10 m from the stump.

b) The total distance Dizzy has run at his 3rd turn is 9 m; at his 6th turn, he has run 36 m. The total distance run is the square of the number of the turn.

c) The relation in b) would predict Dizzy has run 100 m when he makes his 10th turn, as is confirmed by the last entry in the completed table.

<table>
<thead>
<tr>
<th>Turn No.</th>
<th>Position R, L (right or left of stump)</th>
<th>Total Distance Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 m R</td>
<td>1 m</td>
</tr>
<tr>
<td>2</td>
<td>2 m L</td>
<td>4 m</td>
</tr>
<tr>
<td>3</td>
<td>3 m R</td>
<td>9 m</td>
</tr>
<tr>
<td>4</td>
<td>4 m L</td>
<td>16 m</td>
</tr>
<tr>
<td>5</td>
<td>5 m R</td>
<td>25 m</td>
</tr>
<tr>
<td>6</td>
<td>6 m L</td>
<td>36 m</td>
</tr>
<tr>
<td>7</td>
<td>7 m R</td>
<td>49 m</td>
</tr>
<tr>
<td>8</td>
<td>8 m L</td>
<td>64 m</td>
</tr>
<tr>
<td>9</td>
<td>9 m R</td>
<td>81 m</td>
</tr>
<tr>
<td>10</td>
<td>10 m L</td>
<td>100 m</td>
</tr>
</tbody>
</table>
Problem of the Week
Problem B
Chock Full o’ Chocolates

For their school’s recent fundraiser, Ivan sold $24 worth of white chocolate almond bars on the first day, and $2 worth each day thereafter. Sonia sold $12 worth of caramel crunchies on the first day, and $4 worth each day thereafter. The person who sells the most wins an iMuse.

a) If the campaign ran for 5 days, who sold the greater total value of chocolate bars, and how much greater was the total?

b) Suppose the campaign is extended, and each person continues to sell at the same rate. On which day would they both have sold the same value of chocolate bars?

c) On what day will Sonia’s total be $10 greater than Ivan’s?

Source: chocablog

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

Strand: Pattern and Algebra, Number Sense
Problem of the Week

Problem B and Solution

Chock Full o’ Chocolates

Problem

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b) Suppose the campaign is extended, and each person continues to sell at the same rate. On which day would they both have sold the same value of chocolate bars?

c) On what day will Sonia’s total be $10 greater than Ivan’s?

Solution

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ivan</td>
<td>$24</td>
<td>$26</td>
<td>$28</td>
<td>$30</td>
<td>$32</td>
<td>$34</td>
<td>$36</td>
<td>$38</td>
<td>$40</td>
<td>$42</td>
<td>$44</td>
<td>$46</td>
</tr>
<tr>
<td>Sonia</td>
<td>$12</td>
<td>$16</td>
<td>$20</td>
<td>$24</td>
<td>$28</td>
<td>$32</td>
<td>$36</td>
<td>$40</td>
<td>$44</td>
<td>$48</td>
<td>$52</td>
<td>$56</td>
</tr>
</tbody>
</table>

The table of sales reveals the desired solutions.

a) At the end of 5 days, Ivan has sold the greater total value by $32 – $28 = $4.

b) Both Ivan and Sonia have sold $36 worth of chocolate bars on day 7.

c) Sonia’s total is $10 more than Ivan’s on day 12 of the campaign.

Alternatively: This problem can be solved algebraically by more advanced students. If \( n \) numbers the days, then the total sales in $ on day \( n \) are:

Ivan: \( I = 24 + 2 \times (n - 1) \), and Sonia: \( S = 12 + 4 \times (n - 1) \).

For part a), let \( n = 5 \); for b), let \( I = S \) and solve the equation for \( n \); and for c), let \( I + 10 = S \) and solve the equation for \( n \).
In the left column of the table below are the ingredients required to make 48 Pecan Mini-tarts.

Belinda wants to make 24 tarts and is wondering what quantity of each ingredient she will need. Mr. Singh wants to feed his whole class, and needs to make 60 tarts.

In the chart below, fill in the required quantities of each ingredient. Are there any ingredients for which the exact quantity is unreasonable?

<table>
<thead>
<tr>
<th>Ingredients for 48 tarts</th>
<th>Ingredients for 24 tarts</th>
<th>Ingredients for 60 tarts</th>
</tr>
</thead>
<tbody>
<tr>
<td>250 g cream cheese</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 cup butter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{4}$ cup sugar</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 cups flour</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 eggs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{3}$ cup butter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 cup brown sugar</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 cup chopped pecans</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1\frac{1}{2}$ teaspoons vanilla</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{2}$ cup candied cherries</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Recipe instructions follow... Yummm....

Pastry
Cream butter and cream cheese with sugar until light and fluffy. Add flour a bit at a time and blend well. Shape gently into a log and cut into 48 equal-sized pieces. Place each piece into a cup of a mini-tart pan, and press with your fingers to shape the tart shell. (Alternatively, on a lightly floured surface, roll each piece to a suitably-sized circle.)

Filling
Break the eggs into a mixing bowl. Add the brown sugar, pecans, vanilla, and cherries and mix well. Distribute the filling evenly among the tart shells. Bake at 350 degrees F (180 degrees C) until lightly browned, about 30 minutes.

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

STRAND: Number Sense, Measurement
Problem of the Week

Problem B and Solution

Tarts? Sweet!

Problem

In the left column of the table below are the ingredients required to make 48 Pecan Mini-tarts. Belinda wants to make 24 tarts and is wondering what quantity of each ingredient she will need. Mr. Singh wants to feed his whole class, and needs to make 60 tarts.

In the chart below, fill in the required quantities of each ingredient. Are there any ingredients for which the exact quantity is unreasonable?

Solution

<table>
<thead>
<tr>
<th>Ingredients for 48 tarts</th>
<th>Ingredients for 24 tarts</th>
<th>Ingredients for 60 tarts</th>
</tr>
</thead>
<tbody>
<tr>
<td>250 g cream cheese</td>
<td>125 g</td>
<td>312.5 g</td>
</tr>
<tr>
<td>1 cup butter</td>
<td>1/2 cup</td>
<td>1 1/4 cups</td>
</tr>
<tr>
<td>1/4 cup sugar</td>
<td>1/8 cup</td>
<td>5/16 cup</td>
</tr>
<tr>
<td>2 cups flour</td>
<td>1 cup</td>
<td>2 1/2 cups</td>
</tr>
<tr>
<td>2 eggs</td>
<td>1 egg</td>
<td>2 1/2 eggs</td>
</tr>
<tr>
<td>1/3 cup butter</td>
<td>1/6 cup</td>
<td>5/12 cup</td>
</tr>
<tr>
<td>1 cup brown sugar</td>
<td>1/2 cup</td>
<td>1 1/4 cups</td>
</tr>
<tr>
<td>1 cup chopped pecans</td>
<td>1/2 cup</td>
<td>1 1/4 cups</td>
</tr>
<tr>
<td>1 1/2 teaspoons vanilla</td>
<td>3/4 tsp</td>
<td>1 7/8 tsp</td>
</tr>
<tr>
<td>1/2 cup candied cherries</td>
<td>1/4 cup</td>
<td>5/8 cup</td>
</tr>
</tbody>
</table>

Exact quantities which seem unreasonable:

- 2 1/2 eggs is not reasonable; perhaps use two large eggs to obtain the extra 1/2 egg;
- while 312.5 g of cream cheese may seem tricky, it is actually just one 250 g package plus 1/4 of another package.
Problem of the Week

Problem B

A Full House Rocks!

The town of Blue Mountains is building a new theatre to hold the Grand Finals for the annual Elvis Festival. This is a very popular show, with 6000 tickets sold. The mayor is concerned that the architect’s strange seating plan will not have enough seats to hold the crowd.

According to the architect’s seating plan, the first row in each section has 176 seats, the second row has 188, and the third row has 200 seats. (Note that the number of seats in each row increases by the same number.)

a) If each section has 6 rows, how many seats will there be in the last row?

b) If the theatre has a total of 5 sections, A, B, C, D, and E, will there be enough seats for all of the ticket holders? How many extra seats will there be?

Check out other CEMC resources here:

cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense
Problem of the Week

Problem B and Solution

A Full House Rocks!

Problem

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b) If the theatre has a total of 5 sections, A, B, C, D, and E, will there be enough seats for all of the ticket holders? How many extra seats will there be?

Solution

a) According to the given information, the number of seats in each row increases by $188 - 176 = 12$ seats over the previous row. From the table, the sixth row will have 236 seats, and seats per section total 1236.

<table>
<thead>
<tr>
<th>Row</th>
<th>Seats</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>176</td>
</tr>
<tr>
<td>2</td>
<td>188</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
</tr>
<tr>
<td>4</td>
<td>212</td>
</tr>
<tr>
<td>5</td>
<td>224</td>
</tr>
<tr>
<td>6</td>
<td>236</td>
</tr>
<tr>
<td>Total</td>
<td>1236</td>
</tr>
</tbody>
</table>

b) Since there are five sections, the total number of seats is $5 \times 1236 = 6180$, which is enough for all the ticket holders with 180 seats to spare.
Problem of the Week

Problem B

Pedicures for Pooches

Anderson’s Doggy Daycare has decided to offer a Dog Grooming Service to their customers.

This service will be available 5 days per week.

Suppose they wash 12 dogs per day. To entice clients, they offer nail clipping for only $8.00 with every doggy wash.

If 75% of their clients take advantage of this offer, how much money will the nail clipping generate in the month of February?

Check out other CEMC resources here:

cemc.uwaterloo.ca/resources/resources.html

STRAND: Number Sense
Problem of the Week
Problem B and Solution
Pedicures for Pooches

Problem
Anderson’s Doggy Daycare has decided to offer a Dog Grooming Service to their customers.
This service will be available 5 days per week.
Suppose they wash 12 dogs per day. To entice clients, they offer nail clipping for only $8.00 with every doggy wash.
If 75% of their clients take advantage of this offer, how much money will the nail clipping generate in the month of February?

Solution
There are normally 20 work days in the four weeks of the month of February. At a rate of 12 dogs per day, the groomers will wash a total of $20 \times 12 = 240$ dogs in February.
If 75% of their customers take advantage of the nail clipping service, then $0.75 \times 240 = 180$ dogs will have their nails clipped.
At $8.00 per clipping, the revenue generated will be $8.00 \times 180 = $1440 for nail clipping in February.

Something to Think About
How might your answer change if Anderson’s made this offer during a leap year?
Problem of the Week

Problem B

Wrecked-Angle

The area of a rectangle is equal to the product of its length times its width.

a) What are the possible dimensions of a rectangle of area 60 cm\(^2\)? Fill in the first two columns of the given table, using width as the lesser dimension and length as the greater.

b) Which of these possible rectangles, if its length is decreased by 4 cm and its width is increased by 1 cm, would result in a rectangle four times as long as it is wide? Complete the table to help find the answer.

<table>
<thead>
<tr>
<th>Table of Dimensions (all measurements in cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
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<tr>
<td></td>
</tr>
</tbody>
</table>

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense, Measurement
Problem of the Week
Problem B and Solution
Wrecked-Angle

Problem
The area of a rectangle is equal to the product of its length times its width.

a) What are the possible dimensions of a rectangle of area 60 cm$^2$? Fill in the first two columns of the given table, using width as the lesser dimension and length as the greater.

b) Which of these possible rectangles, if its length is decreased by 4 cm and its width is increased by 1 cm, would result in a rectangle four times as long as it is wide? Complete the table to help find the answer.

Solution
Here is the completed table.

<table>
<thead>
<tr>
<th>Width</th>
<th>Length</th>
<th>Width + 1</th>
<th>Length − 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>2</td>
<td>56</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
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<td>16</td>
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<tr>
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<td>15</td>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

a) The first two columns reveal the six possible sets of dimensions.

b) The third row of the table shows that the 3 cm by 20 cm rectangle, upon increasing its width to 4 cm and decreasing its length to 16 cm, gives a rectangle with length four times its width.
Problem of the Week

Problem B

Numb Sums

Many whole numbers can be written as sums of consecutive numbers, i.e., numbers which follow one after the other. For example, $5 = 2 + 3$, $24 = 7 + 8 + 9$, $30 = 4 + 5 + 6 + 7 + 8$, etcetera.

a) Determine all the numbers from 1 to 16 which can be written as sums of 2 or more consecutive numbers.

b) Which number from 1 to 16 can be written as a sum of consecutive numbers in the greatest number of ways?

c) Which number(s) from 1 to 16 cannot be written as a sum of consecutive numbers? What are the factors of these numbers?

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

Strand: Number Sense
Problem of the Week
Problem B and Solution
Numb Sums

Problem

Many whole numbers can be written as sums of consecutive numbers, i.e., numbers which follow one after the other.

For example, $5 = 2 + 3$, $24 = 7 + 8 + 9$, $30 = 4 + 5 + 6 + 7 + 8$, etcetera.

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b) Which number from 1 to 16 can be written as a sum of consecutive numbers in the greatest number of ways?

c) Which number(s) from 1 to 16 cannot be written as a sum of consecutive numbers? What are the factors of these numbers?

Solution

a) Numbers which can be written as sums of 2 or more consecutive numbers are:

1 = 0+1, \hspace{0.5cm} 3 = 1+2, \hspace{0.5cm} 5 = 2+3, \hspace{0.5cm} 6 = 1+2+3, \hspace{0.5cm} 7 = 3+4, \hspace{0.5cm} 9 = 2+3+4 = 4+5, \hspace{0.5cm} 10 = 1+2+3+4, \hspace{0.5cm} 11 = 5+6, \hspace{0.5cm} 12 = 3+4+5, \hspace{0.5cm} 13 = 6+7, \hspace{0.5cm} 14 = 2+3+4+5, \hspace{0.5cm}
and \hspace{0.5cm} 15 = 1 + 2 + 3 + 4 + 5 = 4 + 5 + 6 = 7 + 8.

b) The number 15 can be written as a sum in three different ways, the greatest number of ways for the numbers 1 to 16.

c) The numbers 2, 4, 8, and 16 cannot be written as a sum of consecutive numbers. Their factors are:

2: \{1,2\}, \hspace{0.5cm} 4: \{1,2,4\}, \hspace{0.5cm} 8: \{1,2,4,8\}, \hspace{0.5cm} 16: \{1,2,4,8,16\}.

Things to Think About

Why is it that all the odd numbers can be written as sums of consecutive numbers? Is this true of any odd number?

How do the factors of the even numbers which can be written as a sum of consecutive numbers differ from those of the numbers in part c)?
Problem of the Week

Problem B

Dollars for Digits

McMillan’s Construction Company is just completing the construction of homes on a street in a new subdivision. Metal digits are to be placed above each front door to label the house numbers.

The total cost of each three-digit house number is shown in the table at the right.

The two digits which are congruent (i.e., have the same shape) cost exactly the same amount in whole dollars.

What is the cost of each metal digit 0\ldots9?

<table>
<thead>
<tr>
<th>North Side</th>
<th>Cost</th>
<th>South Side</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>688</td>
<td>$14.00</td>
<td>693</td>
<td>$12.25</td>
</tr>
<tr>
<td>692</td>
<td>$11.50</td>
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<td>705</td>
<td>$11.00</td>
</tr>
<tr>
<td>704</td>
<td>$10.25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

**STRAND:** Number Sense, Measurement
Problem

McMillan’s Construction Company is just completing the construction of homes on a street in a new subdivision. Metal digits are to be placed above each front door to label the house numbers.

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</tr>
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<td>$10.25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Solution

Since the digits 6 and 9 are congruent, they cost the same amount in whole dollars. Noting that the house number 696 costs $12.00, we thus deduce that both the 6 and the 9 cost $4.00 each.

Then, using the house number 688, we see that two 8’s cost $14.00 − $4.00 = $10.00, and hence each 8 costs $5.00.

From house numbers 692 and 697, we see that a 2 and a 7 each must cost $11.50 − $8.00 = $3.50. Similarly, from house number 693, a 3 must cost $12.25 − $8.00 = $4.25.

Now, from 700, two 0’s cost $9.50 − $3.50 = $6.00, and hence each 0 costs $3.00.

Finally, from 701, a 1 costs $8.50 − $3.50 − $3.00 = $2.00, while from 704, a 4 costs $10.25 − $3.50 − $3.00 = $3.75, and from 705, a 5 costs $11.00 − $3.50 − $3.00 = $4.50.
Problem of the Week

Problem B

A Piece of Cake

Suki is making a special chocolate cake for her friend Maya. To make this chocolate cake, she needs sugar, cocoa powder, and flour in the ratio 8 : 3 : 7.

a) Express the needed flour as a fraction of the needed sugar.

b) Express, in lowest terms, the needed cocoa powder as a fraction of the total of the three ingredients.

c) Suppose the recipe requires 400 g of sugar. How much cocoa powder does she need?

d) Another cake recipe requires 120 g of cocoa powder. If the ratios remain the same, how much sugar does she need? How much flour?

e) If she wished to make a cake weighing 1.8 kg, could she determine the required weight of each ingredient from the given ratios? Explain your answer.
Problem of the Week
Problem B and Solution
A Piece of Cake

Problem

Suki is making a special chocolate cake for her friend Maya.
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d) Another cake recipe requires 120 g of cocoa powder. If the ratios remain the same, how much sugar does she need? How much flour?
e) If she wished to make a cake weighing 1.8 kg, could she determine the required weight of each ingredient from the given ratios? Explain your answer.

Solution

a) Since the ratio of flour to sugar is 7:8, the amount of flour needed is $\frac{7}{8}$ the amount of sugar.
b) The ratio of cocoa powder to all three ingredients is 3:18. In lowest terms, the fraction $\frac{3}{18}$ is $\frac{1}{6}$.
c) Since the ratio of cocoa powder to sugar is $\frac{3}{8}$, if the recipe requires 400 g of sugar, then the amount of cocoa powder needed is $\frac{3}{8} \times 400 = 150$ g.
d) The ratio of sugar to cocoa powder is $\frac{8}{3}$, while that of flour to cocoa powder is $\frac{7}{3}$. Thus, for this new recipe with 120 g of cocoa powder, Suki will need $\frac{8}{3} \times 120 = 320$ g of sugar, and $\frac{7}{3} \times 120 = 280$ g of flour.
e) Since the ratio of each ingredient to the total weight of the ingredients is known, Suki could determine the required weight of each ingredient as follows, based on a total weight of 1800 g.

- The ratio of sugar to the total weight is $\frac{8}{18} = \frac{4}{9}$, so the amount of sugar required is $\frac{4}{9} \times 1800 = 800$ g.
- The ratio of cocoa powder to the total weight is $\frac{3}{18} = \frac{1}{6}$, so the amount of cocoa powder required is $\frac{1}{6} \times 1800 = 300$ g.
- The ratio of flour to the total weight is $\frac{7}{18}$, so the amount of flour required is $\frac{7}{18} \times 1800 = 700$ g.
Problem of the Week

Problem B

Twitter Litter

Ben Bird’s Twitter account has 500 followers. When he sends a tweet, \(\frac{1}{4}\) of his followers retweet his message, and 40\% just read it.

**Among his remaining followers**, 15 out of every 25 just ignore his chatter, while the rest choose to stop following him completely.

When Ben sends this tweet, how many of his original 500 followers:

a) retweet it?

b) read it?

c) ignore it?

d) stop following him?

*Check out other CEMC resources here:*

cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense
Problem of the Week
Problem B and Solution
Twitter Litter

Problem
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Among his remaining followers, 15 out of every 25 just ignore his chatter, while the rest choose to stop following him completely.

When Ben sends this tweet, how many of his original 500 followers:

a) retweet it?
b) read it?
c) ignore it?
d) stop following him?

Solution

a) Since \(\frac{1}{4}\) of 500 is \(\frac{1}{4} \times 500 = 125\), we see that 125 of Ben’s followers retweet his message.

b) Since 40% of 500 is \(\frac{40}{100} \times 500 = 0.4 \times 500 = 200\), we see that 200 of Ben’s followers just read his tweet.

c) Since \(500 - 125 - 200 = 175\), there are 175 followers remaining. Of these, \(\frac{15}{25}\) just ignore his chatter. Since there are 7 sets of 25 in 175 (i.e., \(175 = 7 \times 25\)), and 15 ignore his tweet in each of these sets, we see that \(15 \times 7 = 105\) ignore his message.

(OR: \(\frac{15}{25}\) of 175 is \(\frac{15}{25} \times 175 = \frac{3}{5} \times 175 = 0.6 \times 175 = 105\) followers ignore his tweet.)

d) This leaves \(175 - 105 = 70\) of his original followers who stop following him.
Problem of the Week

Problem B

Skott’s Spots

Skott was painting the walls in his living room and den. Unfortunately, he accidentally got some small dots of paint on the ceiling near the wall. So he decided to put a decorative rope along the ceiling, where it meets the walls.

To do this, Skott has to measure the perimeter of the ceilings to make sure he has enough of the decorative rope to go around each ceiling.

Both ceilings have an area of 24 square metres, with dimensions that are whole numbers.

Skott realizes that his 20 m roll of decorative rope is only enough for one ceiling but not the other.

a) Find the dimensions of the room for which Skott has enough decorative rope.

b) Given that the other room has different dimensions, and its perimeter is less than 50 m, how much more decorative rope does he need to ensure he can do the other room?

Check out other CEMC resources here: cemc.uwaterloo.ca/resources/resources.html

STRAND: Number Sense, Measurement
Problem of the Week
Problem B and Solution
Skott’s Spots

Problem

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a) Find the dimensions of the room for which Skott has enough decorative rope.

b) Given that the other room has different dimensions, and its perimeter is less than 50 m, how much more decorative rope does he need to ensure he can do the other room?

Solution

a) From the table at the right, we see that the only room of area $24 \text{ m}^2$ for which Skott has enough decorative rope is one that is 4 m by 6 m.

b) Given that its perimeter is less than 50 m and its dimensions are different than the other room, the other room could have dimensions of either 3 m by 8 m or 2 m by 12 m, with perimeters of 22 m and 28 m respectively. Thus to ensure he can do the other room, Skott needs 28 m more of the decorative rope.

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 × 24</td>
<td>50</td>
</tr>
<tr>
<td>2 × 12</td>
<td>28</td>
</tr>
<tr>
<td>3 × 8</td>
<td>22</td>
</tr>
<tr>
<td>4 × 6</td>
<td>20</td>
</tr>
</tbody>
</table>
Problem of the Week

Problem B

Chaining Around

According to the Book of World Records, the longest paper chain made by an individual measures 378.63 m in length, and was made in 2012.

Vanessa has decided that she would like to break the existing world record.

a) If there are 20 paper links per metre in her chain, estimate the number of links needed to make a chain 378.63 m in length.

b) About how many more paper strips would Vanessa need to make a chain 605 m long?

c) Is your answer to b) an overestimate or an underestimate of the actual number of links needed? (Your method of rounding will affect your answer.)

---

Check out other CEMC resources here:

cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense (Estimation)
Problem of the Week
Problem B and Solution

Chaining Around

Problem

According to the Book of World Records, the longest paper chain made by an individual measures 378.63 m in length, and was made in 2012. Vanessa has decided that she would like to break the existing world record.

a) If there are 20 paper links per metre in her chain, estimate the number of links needed to make a chain 378.63 m in length.

b) About how many more paper strips would Vanessa need to make a chain 605 m long?

c) Is your answer to b) an overestimate or an underestimate of the actual number of links needed? (Your method of rounding will affect your answer.)

Solution

a) If there are 20 links per metre, then a chain 378.63 m in length would require \(20 \times 378.63 = 7572.6\) links, i.e., \(7573\) links.

Alternatively, we could round the length to 380 m, and hence estimate the number of links required as \(20 \times 380 = 7600\), an overestimate.

b) Using the rounded length of the original chain as 380 m, we see that a 605 m chain would be \(605 - 380 = 225\) m longer. Thus an estimate of the extra strips Vanessa needs is \(20 \times 225 = 4500\) links.

Alternatively, using \(605 - 378.63 = 226.37\) as the difference in links gives the required links as \(20 \times 226.37 = 4527.4\), or \(4527\) extra strips.

c) The first answer in b) is an underestimate due to rounding up 378.63 to 380 and then subtracting, so 225 links is an underestimate. The second answer is an underestimate as well, due to rounding down 4527.4 to 4527. The actual number of extra links needed is 4528, since you could not do a ‘partial’ link.
Problem of the Week

Problem B

Box It Up!

Richard and Dennis would like to make boxes to hold their crayons more compactly, rather than carrying them around in pencil cases. Their crayons have the shape of rectangular prisms, each 8 cm long and 1 cm thick. Using quality materials from MaryLynne’s Mercantile, Richard has constructed a rectangular box with dimensions as shown below.

![Diagram of a rectangular box with dimensions 16 cm x 6 cm x 6 cm]

a) Will Richard’s pencil box hold all of Dennis’ 48 crayons? Will it hold Richard’s 144 crayons?

b) What are possible dimensions of a box 8 cm long that will exactly hold Dennis’ crayons?

c) Richard would prefer a box in which his 144 crayons will stand on end, for easy access. What would be a suitable set of dimensions for such a box?

Check out other CEMC resources here:
[cemc.uwaterloo.ca/resources/resources.html]

**Strand:** Number Sense, Measurement
Problem of the Week

Problem B and Solution

Box It Up!

Problem

Richard and Dennis would like to make boxes to hold their crayons more compactly, rather than carrying them around in pencil cases. Their crayons have the shape of rectangular prisms, each 8 cm long and 1 cm thick.

Using quality materials from MaryLynne’s Mercantile, Richard has constructed a rectangular box with dimensions 16 cm long, 6 cm high, and 6 cm deep.

a) Will Richard’s pencil box hold all of Dennis’ 48 crayons? Will it hold Richard’s 144 crayons?

b) What are possible dimensions of a box 8 cm long that will exactly hold Dennis’ crayons?

c) Richard would prefer a box in which his 144 crayons will stand on end, for easy access. What would be a suitable set of dimensions for such a box?

Solution

The diagram above reveals that the dimensions of the box that will admit a layer of 12 crayons is 16 cm long by 6 cm deep. Since the box is 6 cm high, it will hold 6 such layers, with no space left over. Thus the capacity of the box is $12 \times 6 = 72$ crayons.

a) We thus conclude that Richard’s box will hold all of Dennis’s 48 crayons, but will not hold Richard’s 144 crayons.

b) A box to exactly hold Dennis’s 48 crayons would need to have a volume of $48 \times 8 \times 1 \times 1 = 384 \text{ cm}^3$.

Since the box is to be 8 cm long, its cross section would need to have an area of $A = 384 \div 8 = 48 \text{ cm}^2$, i.e., the height times the depth must be 48 cm$^2$. 
Thus the possible dimensions are:

- 1 cm by 48 cm (1 layer of 48 crayons, or 48 layers of 1 crayon);
- 2 cm by 24 cm (2 layers of 24 crayons, or 24 layers of 2 crayons);
- 3 cm by 16 cm (3 layers of 16 crayons, or 16 layers of 3 crayons);
- 4 cm by 12 cm (4 layers of 12 crayons, or 12 layers of 4 crayons);
- 6 cm by 8 cm (6 layers of 8 crayons, or 8 layers of 6 crayons).

(Think about which of these possibilities are unsuitable for a crayon box.)

c) Since each crayon has a cross section of area 1 cm$^2$, for the crayons to stand on end, the base area of Richard’s box will need to be 144 cm$^2$, and its height must be 8 cm.

Thus the possible dimensions of the base are:

- 1 cm by 144 cm (highly unsuitable);
- 2 cm by 72 cm (also unsuitable);
- 3 cm by 48 cm (still a little long);
- 4 cm by 36 cm (might work, but still a bit thin and long);
- 6 cm by 24 cm (reasonably suitable).
- 8 cm by 18 cm (suitable)
- 12 cm by 12 cm (suitable)

(The criteria for suitability are worth discussing.)
Problem of the Week

Problem B

When Do We Eat?

Laura is attending a wedding at which 300 guests are present. The guests begin to leave the celebration at 9:00 p.m., 20 at a time, and thereafter, groups of 20 guests leave at 20-minute intervals.

The bride and groom depart once two-fifths of the guests have left. Right after that, a late-night buffet is served to the remaining guests.

At what time is the buffet served?

Check out other CEMC resources here: cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense, Measurement
Problem of the Week

Problem B and Solution

When Do We Eat?

Problem

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The bride and groom depart once two-fifths of the guests have left. Right after that, a late-night buffet is served to the remaining guests.

At what time is the buffet served?

Solution

The buffet is served once $\frac{2}{5}$ of the guests have left, i.e., after $\frac{2}{5} \times 300 = 120$ guests have left, in groups of 20. Since $120 = 6 \times 20$, this means that six groups of 20 guests must leave before the remaining guests get to eat.

The groups of 20 guests at a time start leaving at 9:00 p.m., and exit every 20 minutes after that, the second group at 9:20, the third at 9:40, the fourth at 10:00, the fifth at 10:20, and the sixth at 10:40.

Thus the buffet will be served at 10:40 p.m.
Problem of the Week

Problem B

Disc-O Toss

The game of Disc-O is played with three identically-sized discs. Each has one blank side which counts as 0, and the other side has a single digit number, one with the number 1, one with the number 2, and one with the number 3.

Players take turns shaking the discs in their hands and tossing them onto the floor. Their score is the sum of the numbers on the upward faces. The first player to reach 25 wins.

a) What are the possible scores with one toss of the three discs?

b) In how many ways can a score of 5 occur? A score of 3?

c) What is the theoretical probability of scoring 5 in a single toss?

d) What is the theoretical probability of scoring 3 in a single toss?

e) Experiment with this game. (For the discs, use a marker to write 1, 2, 3 on the ‘tails’ side of three pennies; use ‘heads’ as 0.) Work with a partner, and take 10 turns each; record your scores in the table at the right.

f) Mansour claims that the theoretical probability of scoring 5 after two turns is the same as the theoretical probability of scoring 6 after two turns. Is he correct? Explain your answer.

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Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

Strand: Number Sense, Probability
Problem of the Week

Problem B and Solution

Disc-O Toss

Problem

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f) Mansour claims that the theoretical probability of scoring 5 after two turns is the same as the theoretical probability of scoring 6 after two turns. Is he correct? Explain your answer.

Solution

a) There are 8 possible ways to obtain scores with one toss of the three discs:

0 + 0 + 0 = 0; 0 + 0 + 1 = 1; 0 + 0 + 2 = 2; 0 + 0 + 3 = 3; 0 + 1 + 2 = 3; 0 + 1 + 3 = 4; 0 + 2 + 3 = 5; 1 + 2 + 3 = 6.

From these possibilities, we conclude that 7 scores are possible. One of the scores, namely 3, can be obtained two different ways.

b) A score of 5 can occur in only one way; a score of 3 can occur in two ways.

c) The probability of scoring 5 in a single toss is thus 1 in 8, or \(\frac{1}{8}\).

d) The probability of scoring 3 in a single toss is thus 2 in 8, or \(\frac{1}{4}\).

e) (Experiments will vary.)

f) The sum 5 could be from 0 + 5 or 1 + 4 or 2 + 3, in either order. Since there are two ways to get a score of 3 on a single turn, there are a total of 8 ways to get 5 in two turns. The sum 6 can be formed as 0 + 6, 1 + 5 or 2 + 4 (in either order), a total of 6 ways. But there is also 3 + 3. Each of the 3s can be formed in two ways on each turn. Thus there are an additional 4 ways to get 6 in two turns. There is a total of 10 ways to get 6 in two turns.

Since the number of possible outcomes for scoring a total of 5 in two turns is different than the number of possible outcomes for scoring a total of 6 in two turns, the probabilities would also be different. So Mansour is incorrect.
The three numbers 54, 321, and 8765 all share something special: their digits are in decreasing order from left to right, with a difference of 1 from one digit to the next.

a) How many such numbers are there from 10 to 100? From 100 to 1 000? From 1 000 to 10 000?

b) How many such numbers would you predict there are from 10 000 to 100 000?

c) Jeff claims the greatest such number is less than 10 billion. Is he correct? Explain your answer.

d) How many such numbers are there in total?

Check out other CEMC resources here:
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**Strand:** Number Sense
Problem of the Week

Problem B and Solution

Countdown

Problem

The three numbers 54, 321, and 8765 all share something special: their digits are in decreasing order from left to right, with a difference of 1 from one digit to the next.

a) How many such numbers are there from 10 to 100? From 100 to 1 000? From 1 000 to 10 000?

b) How many such numbers would you predict there are from 10 000 to 100 000?

c) Jeff claims the greatest such number is less than 10 billion. Is he correct? Explain your answer.

d) How many such numbers are there in total?

Solution

a) From 10 to 100, we have the numbers
10, 21, 32, 43, 54, 65, 76, 87, and 98, i.e., 9 such numbers.

From 100 to 1 000, we have the numbers
210, 321, 432, 543, 654, 765, 876, and 987, i.e., 8 such numbers.

From 1 000 to 10 000, we have the numbers
3210, 4321, 5432, 6543, 7654, 8765, and 9876, i.e., 7 such numbers.

b) From the pattern emerging in part a), we would expect that from 10 000 to 100 000 there will be 6 such numbers.

c) Since 10 billion = 10 000 000 000 has 11 digits, and we only have 10 digits with which to form our special numbers, the greatest such number must be less than 10 billion. In fact, it is 9 876 543 210.

d) Following the established pattern, in total, there will be
$9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$ such numbers.
Problem of the Week

Problem B

Penny-Less

Larry Min is frugal (careful with his money), so he is always looking for a way to save a few pennies. Unfortunately for Larry, pennies are no longer used in Canada. Even so, he knows that every time he buys gas, the total gets rounded to the nearest nickel, so he decides to take advantage of this.

a) If Larry buys 50 L of gas priced at $1.20 per L, how much will he pay?

b) Since rounding occurs to the nearest nickel, there are different fuel pump purchases which will cost the same amount. For example, a fill-up which shows $30.52 on the pump will require a cash payment of only $30.50; similarly, one which comes to $30.50 will require the same cash payment, $30.50, as will pump totals of $30.48, or $30.51. What is the set of possible totals shown on the pump that will all require the same cash payment as Larry paid in part a)?

c) Which of the possible pump totals in part b) will save Larry the greatest amount of money?

d) If penny-wise Larry saves this much every week of the year, how much will he save over one year?

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

Strand: Measurement, Number Sense
Problem of the Week

Problem B and Solution

Penny-Less

Problem
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c) Which of the possible pump totals in part b) will save Larry the greatest amount of money?

d) If penny-wise Larry saves this much every week of the year, how much will he save over one year?

Solution

a) If Larry buys 50 L of gas priced at $1.20 per L, he will pay

\[ 50 \times 1.20 = 60.00 \, \text{.} \]

b) Pump totals which would all require a cash payment of $60.00 are $59.98, $59.99, $60.00, $60.01, and $60.02.

c) To get the greatest amount of gas possible for $60.00, Larry should put in enough gas to show $60.02 on the pump, saving $0.02.

d) If penny-wise Larry saves this much every week of the year, he will save a total of

\[ 52 \times 0.02 = 1.04 \, \text{ over the year.} \]
Problem of the Week

Problem B

Food Daze

The Parent Council at Blackburn Elementary School is raising money to buy a new climber for the school yard. Starting on the second day of the school year, they will sell smoothies every other day. Starting on the third day, they will sell pizza every third day.

a) Is it possible for students to buy smoothies and pizza on the 100th day of school? Why, or why not?

b) During the first 100 days of school, on how many days do they sell smoothies and pizza on the same day?

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

Strand: Number Sense
Problem of the Week
Problem B and Solution

Food Daze

Problem

The Parent Council at Blackburn Elementary School is raising money to buy a new climber for the school yard. Starting on the second day of the school year, they will sell smoothies every other day. Starting on the third day, they will sell pizza every third day.

a) Is it possible for students to buy smoothies and pizza on the 100\textsuperscript{th} day of school? Why, or why not?

b) During the first 100 days of school, on how many days do they sell smoothies and pizza on the same day?

Solution

a) Since pizza is only sold every third day, and 100 is not a multiple of 3, it is not possible to buy both pizza and smoothies on the 100\textsuperscript{th} day.

b) In the hundreds chart at the right, the diagonal hatching up to the right represents a day on which smoothies are sold, while up to the left represents a day on which pizza is sold. When they overlap, both are sold, namely on multiples of 6, since they must be divisible by both 2 and 3. This occurs 16 times, since the last one, 96, is $6 \times 16$. 
Patterning
&
Algebra

TAKE ME TO THE COVER
Problem of the Week

Problem B

Dizzying Distances

Dizzy the indecisive rabbit is sitting by his favourite tree stump when he hears a dog barking nearby.

In a panic, he runs 1 m to the right, then turns around and runs 3 m to the left, then turns around and runs 5 m to the right, then 7 m left, etcetera.

a) If he continues to run in this pattern, how far will he be from the stump as he makes his 10th turn?

b) What is the total distance Dizzy has run at his 3rd turn? At his 6th turn? How are these distances related to the number of turns?

c) Using your results from part b), predict the total distance Dizzy will have run as he makes his 10th turn. Check your prediction by completing the table at the right.

<table>
<thead>
<tr>
<th>Turn No.</th>
<th>Position R, L (right or left of stump)</th>
<th>Total Distance Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 m R</td>
<td>1 m</td>
</tr>
<tr>
<td>2</td>
<td>2 m L</td>
<td></td>
</tr>
</tbody>
</table>

Check out other CEMC resources here:

cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense, Patterning and Algebra
Problem of the Week
Problem B and Solution
Dizzying Distances

Problem
Dizzy the indecisive rabbit is sitting by his favourite tree stump when he hears a dog barking nearby.

In a panic, he runs 1 m to the right, then turns around and runs 3 m to the left, then turns around and runs 5 m to the right, then 7 m left, etcetera.

a) If he continues to run in this pattern, how far will he be from the stump as he makes his 10th turn?
b) What is the total distance Dizzy has run at his 3rd turn? At his 6th turn? How are these distances related to the number of turns?
c) Using your results from part b), predict the total distance Dizzy will have run as he makes his 10th turn. Check your prediction by completing the table at the right.

Solution
a) As he makes his 10th turn, Dizzy will be 10 m from the stump.
b) The total distance Dizzy has run at his 3rd turn is 9 m; at his 6th turn, he has run 36 m. The total distance run is the square of the number of the turn.
c) The relation in b) would predict Dizzy has run 100 m when he makes his 10th turn, as is confirmed by the last entry in the completed table.

<table>
<thead>
<tr>
<th>Turn No.</th>
<th>Position R, L (right or left of stump)</th>
<th>Total Distance Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 m R</td>
<td>1 m</td>
</tr>
<tr>
<td>2</td>
<td>2 m L</td>
<td>4 m</td>
</tr>
<tr>
<td>3</td>
<td>3 m R</td>
<td>9 m</td>
</tr>
<tr>
<td>4</td>
<td>4 m L</td>
<td>16 m</td>
</tr>
<tr>
<td>5</td>
<td>5 m R</td>
<td>25 m</td>
</tr>
<tr>
<td>6</td>
<td>6 m L</td>
<td>36 m</td>
</tr>
<tr>
<td>7</td>
<td>7 m R</td>
<td>49 m</td>
</tr>
<tr>
<td>8</td>
<td>8 m L</td>
<td>64 m</td>
</tr>
<tr>
<td>9</td>
<td>9 m R</td>
<td>81 m</td>
</tr>
<tr>
<td>10</td>
<td>10 m L</td>
<td>100 m</td>
</tr>
</tbody>
</table>
Problem of the Week

Problem B

Chock Full o’ Chocolates

For their school’s recent fundraiser, Ivan sold $24 worth of white chocolate almond bars on the first day, and $2 worth each day thereafter. Sonia sold $12 worth of caramel crunchies on the first day, and $4 worth each day thereafter. The person who sells the most wins an iMuse.

a) If the campaign ran for 5 days, who sold the greater total value of chocolate bars, and how much greater was the total?

b) Suppose the campaign is extended, and each person continues to sell at the same rate. On which day would they both have sold the same value of chocolate bars?

c) On what day will Sonia’s total be $10 greater than Ivan’s?

Source: chocablog

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

Strand: Pattern and Algebra, Number Sense
Problem of the Week
Problem B and Solution
Chock Full o’ Chocolates

Problem
For their school’s recent fundraiser, Ivan sold $24 worth of white chocolate almond bars on the first day, and $2 worth each day thereafter. Sonia sold $12 worth of caramel crunchies on the first day, and $4 worth each day thereafter. The person who sells the most wins an iMuse.

a) If the campaign ran for 5 days, who sold the greater total value of chocolate bars, and how much greater was the total?

b) Suppose the campaign is extended, and each person continues to sell at the same rate. On which day would they both have sold the same value of chocolate bars?

c) On what day will Sonia’s total be $10 greater than Ivan’s?

Solution

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ivan</td>
<td>$24</td>
<td>$26</td>
<td>$28</td>
<td>$30</td>
<td>$32</td>
<td>$34</td>
<td>$36</td>
<td>$38</td>
<td>$40</td>
<td>$42</td>
<td>$44</td>
<td>$46</td>
</tr>
<tr>
<td>Sonia</td>
<td>$12</td>
<td>$16</td>
<td>$20</td>
<td>$24</td>
<td>$28</td>
<td>$32</td>
<td>$36</td>
<td>$40</td>
<td>$44</td>
<td>$48</td>
<td>$52</td>
<td>$56</td>
</tr>
</tbody>
</table>

The table of sales reveals the desired solutions.

a) At the end of 5 days, Ivan has sold the greater total value by $32 − $28 = $4.

b) Both Ivan and Sonia have sold $36 worth of chocolate bars on day 7.

c) Sonia’s total is $10 more than Ivan’s on day 12 of the campaign.

Alternatively: This problem can be solved algebraically by more advanced students. If \( n \) numbers the days, then the total sales in $ on day \( n \) are:

\[
\text{Ivan: } I = 24 + 2 \times (n - 1), \quad \text{and Sonia: } S = 12 + 4 \times (n - 1).
\]

For part a), let \( n = 5 \); for b), let \( I = S \) and solve the equation for \( n \); and for c), let \( I + 10 = S \) and solve the equation for \( n \).
Problem of the Week

Problem B

Just Bead It!

Serena is making a necklace, using different shapes and types of beads in the arrangement shown below, and then repeating this arrangement.

![Necklace with various beads](image)

a) If she continues stringing the beads in this manner, what will be the type and shape of the 40th bead on the string? (Try to find the answer without listing all 40 objects.)

b) Serena has some crystal beads she wants to add to the pattern in her necklace, giving a basic arrangement of 7 beads. At what position in the pattern should she insert each crystal bead so that the 52nd bead is a crystal?

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

**Strand:** Pattern and Algebra
Problem of the Week
Problem B and Solution
Just Bead It!

Problem

Serena is making a necklace, using different shapes and types of beads in the arrangement shown below, and then repeating this arrangement.

a) If she continues stringing the beads in this manner, what will be the type and shape of the 40th bead on the string? (Try to find the answer without listing all 40 objects.)

b) Serena has some crystal beads she wants to add to the pattern in her necklace, giving a basic arrangement of 7 beads. At what position in the pattern should she insert each crystal bead so that the 52nd bead is a crystal?

Solution

a) There are 6 beads in Serena’s basic arrangement. After seven repeats of this set, the 42nd bead will be a heart. So the 40th bead will be the same as two before the heart, namely the plain ball (a sphere).

b) After seven repeats of the new pattern, there will be $7 \times 7 = 49$ beads in the necklace. So the 52nd bead will be the third bead in the pattern, since $49 + 3 = 52$. Hence Serena should make the third bead in each pattern a crystal in order that the 52nd bead be a crystal.
Problem of the Week
Problem B
Watch Out Pat, Turn!

Consider the pattern illustrated below.

a) What are the three pattern attributes?
b) How does each attribute change?
c) What shape is the 50th figure in the pattern? the 75th figure?
d) Describe (or draw) the 10th figure in this pattern.
e) When will the first figure reappear?
f) Describe (or draw) the 100th figure in the pattern.

Be prepared to explain your answers to each question.

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

Strand: Pattern and Algebra
Problem of the Week

Problem B and Solution

Watch Out Pat, Turn!

Problem

Consider the pattern illustrated below.

a) What are the three pattern attributes?
b) How does each attribute change?
c) What shape is the $50^{th}$ figure in the pattern? the $75^{th}$ figure?
d) Describe (or draw) the $10^{th}$ figure in this pattern.
e) When will the first figure reappear?
f) Describe (or draw) the $100^{th}$ figure in the pattern.

Be prepared to explain your answers to each question.

Solution

a) The three pattern attributes are shape, shading, and (arrow) direction.
b) The shape alternates between circle and square, odd numbered figures being circles and even being squares. The shading goes from light to dark to ‘hatched’, repeating after each three figures. The direction turns by $90^\circ$ with each shape, repeating after each four figures.
c) Since 50 is an even number, the shape of the $50^{th}$ figure will be a square, while that of the $75^{th}$ figure will be a circle because 75 is odd.
d) Since the even numbers in the pattern are squares, the $10^{th}$ figure will be a square. Since the shading repeats every three figures, numbers 1, 4, 7, and 10 will have light shading. Since the direction repeats every four figures, numbers 2, 6, and 10 will point to the right. Thus the $10^{th}$ figure will be a light square with direction arrow to the right.
e) Since the shape repeats after every light figure, the shading after every three, and the direction after every four, the entire pattern cannot repeat until after 12 figures (12 being the least multiple of 2, 3, and 4). So the the $13^{th}$ figure will be the same as the first.
f) Since $12 \times 8 = 96$, the pattern starts again after that. So the $100^{th}$ figure will be the same as the $4^{th}$, a light square with direction arrow to the left.
Problem of the Week
Problem B
Un Beau Scarecrow

Nevaeh is creating a scarecrow for the display at the local fall fair. She wants to
dress her scarecrow with a shirt, pants, hat, and scarf.

She goes through her old wardrobe box and her
closet and finds the following items:

- one pair of torn blue jeans;
- a plaid shirt and a flowered shirt;
- an orange hat and a striped hat;
- a red scarf, a striped scarf, and a green scarf.

a) In how many different ways can Nevaeh dress her scarecrow?

b) What is the probability that her scarecrow is wearing jeans? No jeans?

c) What is the probability that her scarecrow is wearing an orange hat?

d) Find a combination of clothing items for the scarecrow which has a
   probability of \( \frac{1}{4} \). [There are several answers.]

Check out other CEMC resources here:
cemc.uwaterloo.ca/resources/resources.html

**Strand:** Probability
Problem of the Week
Problem B and Solution

Un Beau Scarecrow

Problem
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a) In how many different ways can Nevaeh dress her scarecrow?
b) What is the probability that her scarecrow is wearing jeans? No jeans?
c) What is the probability that her scarecrow is wearing an orange hat?
d) Find a combination of clothing items for the scarecrow which has a
probability of \(\frac{1}{4}\). [There are several answers.]

Solution

a) Here is a ‘tree’ which displays all possible outfits; there are 12 in total.

b) Since blue jeans are the only pants, the scarecrow is certain to be wearing
jeans, i.e., a probability of \(\frac{12}{12} = 1\), or 100%. On the other hand, unless her
scarecrow goes ‘pantless’, the probability of it not wearing jeans is 0, or 0%.

c) There are 6 outfits out of 12 with an orange hat, so the probability is \(\frac{6}{12} = \frac{1}{2}\),
or 50%. (OR, since there are only 2 possible hats, the probability is 1 in 2.)

d) For a probability of \(\frac{1}{4}\), we need 3 possibilities out of 12. So, for example,
jeans, the plaid shirt, the striped hat, and any scarf has a probability of \(\frac{1}{4}\).