The problems in this booklet are organized into strands. A problem often appears in multiple strands. The problems are suitable for most students in Grade 5 or higher.
Data Management & Probability
Problem of the Week
Problem B
Going to the Dogs

The points marked on the graph below show how much it will cost the Wilson family to board their family dog Fifi at Dennis’ Doggie Hotel while they go on holidays. The fees are given for 1, 2, 3, and 4 days; for example, it will cost $75 to board Fifi for 2 days.

![Graph showing cost of boarding Fifi for different numbers of days]

Dennis' Doggie Hotel Fees

a) What would it cost the Wilsons to board their dog at the Doggie Hotel for a week?

b) In March, the family is planning to go south for two weeks. The Dog Emporium offers a 2-week rate of $500 during the month of March. Is this a better deal than Dennis’ Doggie Hotel? Explain your reasoning.

Check out other CEMC resources at:
http://cemc.uwaterloo.ca/resources/resources.html

**Strand:** Data Management, Number Sense
Problem of the Week
Problem B and Solution

Going to the Dogs

Problem

The points marked on the graph below show how much it will cost the Wilson family to board their family dog Fifi at Dennis’ Doggie Hotel while they go on holidays. The fees are given for 1, 2, 3, and 4 days; for example, it will cost $75 to board Fifi for 2 days.

![Graph showing the costs for different number of days.]

a) What would it cost the Wilsons to board their dog at the Doggie Hotel for a week?

b) In March, the family is planning to go south for two weeks. The Dog Emporium offers a 2-week rate of $500 during the month of March. Is this a better deal than Dennis’ Doggie Hotel? Explain your reasoning.

Solution

Week 1 at DDHotel

<table>
<thead>
<tr>
<th>No. of Days</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$40</td>
</tr>
<tr>
<td>2</td>
<td>$75</td>
</tr>
<tr>
<td>3</td>
<td>$110</td>
</tr>
<tr>
<td>4</td>
<td>$145</td>
</tr>
<tr>
<td>5</td>
<td>$180</td>
</tr>
<tr>
<td>6</td>
<td>$215</td>
</tr>
<tr>
<td>7</td>
<td>$250</td>
</tr>
</tbody>
</table>

a) The graph reveals that the first day at Dennis’ Doggie Hotel costs $40, and each of the six additional days is $35. Thus the cost to board Fifi for one week will be $40 + (6 \times $35) = $250, matching the total revealed by the large dots on the extended graph above.

Week 2 at DDHotel

<table>
<thead>
<tr>
<th>No. of Days</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>$285</td>
</tr>
<tr>
<td>9</td>
<td>$320</td>
</tr>
<tr>
<td>10</td>
<td>$355</td>
</tr>
<tr>
<td>11</td>
<td>$390</td>
</tr>
<tr>
<td>12</td>
<td>$425</td>
</tr>
<tr>
<td>13</td>
<td>$460</td>
</tr>
<tr>
<td>14</td>
<td>$495</td>
</tr>
</tbody>
</table>

b) The table at the right shows the cost of the second week at Dennis’ Doggie Hotel, seven more days at $35 per day, giving a total of $495. Since the Dog Emporium costs $500 for two weeks, the Wilsons would save $500-$495=$5 by using Dennis’ Doggie Hotel.
Problem of the Week
Problem B

Name Change

Ned, Ted, and Fred each have some nickels, dimes, and/or quarters. Given the information below, what are the two possible sets of coins each person could have?

a) Fred has 6 coins which have a mean (average) value of 10 cents per coin.
b) Ted has 5 coins which have a mean value of 10 cents per coin.
c) Ned has 4 coins which have a mean value of 10 cents per coin.

Check out other CEMC resources at:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Number Sense, Data Management
Problem of the Week
Problem B and Solution

Name Change

Problem
Ned, Ted, and Fred each have some nickels, dimes, and/or quarters. Given the information below, what are the two possible sets of coins each person could have?

a) Fred has 6 coins which have a mean (average) value of 10 cents per coin.
b) Ted has 5 coins which have a mean value of 10 cents per coin.
c) Ned has 4 coins which have a mean value of 10 cents per coin.

Solution

a) Since Fred has 6 coins with a mean value of 10 cents per coin, he must have a total of $6 \times 10$ cents $= 60$ cents. Clearly he could have 6 dimes, or, since

$$5 + 5 + 5 + 10 + 10 + 25 = 60,$$

he could also have 3 nickels, 2 dimes, and 1 quarter.

b) Since Ted has 5 coins with a mean value of 10 cents per coin, he must have a total of $5 \times 10$ cents $= 50$ cents. Thus he could have 5 dimes, or, since

$$5 + 5 + 5 + 10 + 25 = 50,$$

he could also have 3 nickels, 1 dime, and 1 quarter.

c) Since Ned has 4 coins with a mean value of 10 cents per coin, he must have a total of $4 \times 10$ cents $= 40$ cents. Thus he could have 4 dimes, or, since

$$5 + 5 + 5 + 25 = 40,$$

he could also have 3 nickels, no dimes, and 1 quarter.
Problem of the Week
Problem B
Pitter Platter

The staff at Grey Valley Private School were preparing for their Holiday Social. Mr. Van Der Pas found a caterer who would provide a variety of platters for the event, priced as follows:

Cheese - $60, Mediterranean - $50, Antipasto - $60, Fresh Fruit - $50, Vegetable Tray - $40.

He orders seven platters costing exactly $360. If he orders at most two of any platter, what are the possible combinations of platters Mr. Van Der Pas could order?

Check out other CEMC resources here:
http://cemc.uwaterloo.ca/resources/resources.html

STRAND: Data Management, Number Sense
Problem of the Week
Problem B and Solution

Pitter Platter

Problem
The staff at Grey Valley Private School were preparing for their Holiday Social. Mr. Van Der Pas found a caterer who would provide a variety of platters, priced as follows:

Cheese - $60, Mediterranean - $50, Antipasto - $60, Fresh Fruit - $50, Vegetable Tray - $40.

He orders seven platters costing exactly $360. If he orders at most two of any platter, what are the possible combinations of platters Mr. Van Der Pas could order?

Solution
There are nine possible combinations of platters which cost a total of $360 such that there is at most two of any platter type. The combinations are shown in the charts below. Six combinations are shown in the first chart and the other three are shown in the second chart.

<table>
<thead>
<tr>
<th>Type</th>
<th>Combinations and Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antipasto</td>
<td>2 $120 2 $120 2 $120 1 $60 1 $60 1 $60</td>
</tr>
<tr>
<td>Cheese</td>
<td>1 $60 1 $60 1 $60 2 $120 2 $120 2 $120</td>
</tr>
<tr>
<td>Fresh Fruit</td>
<td>2 $100 1 $50 2 $100 1 $50</td>
</tr>
<tr>
<td>Mediterranean</td>
<td>1 $50 2 $100 1 $50 2 $100</td>
</tr>
<tr>
<td>Vegetable Tray</td>
<td>2 $80 2 $80 2 $80 2 $80 2 $80 2 $80</td>
</tr>
<tr>
<td>Total</td>
<td>7 $360 7 $360 7 $360 7 $360 7 $360</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type</th>
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<tr>
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<td>2 $120 1 $60</td>
</tr>
<tr>
<td>Cheese</td>
<td>2 $120 1 $60</td>
</tr>
<tr>
<td>Fresh Fruit</td>
<td>2 $100 2 $100 2 $100</td>
</tr>
<tr>
<td>Mediterranean</td>
<td>2 $100 2 $100 2 $100</td>
</tr>
<tr>
<td>Vegetable Tray</td>
<td>1 $40 1 $40 1 $40</td>
</tr>
<tr>
<td>Total</td>
<td>7 $360 7 $360 7 $360</td>
</tr>
</tbody>
</table>
Problem of the Week
Problem B
Mean Numbers

a) Pick any three consecutive numbers (i.e., three numbers in a row such as 25, 26, 27) and find their mean (average). Compare your answer with your neighbour.

b) Now find the mean of a set of four consecutive numbers and compare answers.

c) What happens when you find the mean of five consecutive numbers? And for six consecutive numbers?

d) What trend do you notice for the average of an odd quantity of numbers? An even quantity? Why do you think this happens?

e) Will the mean (average) of eleven consecutive numbers be a whole number?

Check out other CEMC resources at:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Data Management, Number Sense
Problem of the Week
Problem B and Solution
Mean Numbers

Problem

a) Pick any three consecutive numbers (i.e., three numbers in a row such as 25, 26, 27) and find their mean (average). Compare your answer with your neighbour.
b) Now find the mean of a set of four consecutive numbers and compare answers.
c) What happens when you find the mean of five consecutive numbers? And for six consecutive numbers?
d) What trend do you notice for the mean (average) of an odd quantity of numbers? An even quantity? Why do you think this happens?
e) Will the mean (average) of eleven consecutive numbers be a whole number?

Solution

a) Consider, for example, the three consecutive numbers 35, 36, 37. If we picture them on a number line as below,

\[
\begin{array}{c}
35 \\
\hline
36 \\
\hline
37
\end{array}
\]

we see that 36 is exactly the mid-point between 35 and 37.

If we find their mean (average), we have

\[
\frac{35 + 36 + 37}{3} = \frac{108}{3} = 36,
\]

the middle number. Similarly, the average of 36, 37, 38 is

\[
\frac{36 + 37 + 38}{3} = \frac{111}{3} = 37,
\]

again the middle number.

In fact, the mean (average) of any three consecutive whole numbers will always be the middle number since it is exactly the midpoint on the number line between the first and last numbers, i.e., the ‘balance point’ (think of a teeter-totter).
b) If we add a fourth number to our sample in a), then their mean is

\[
\frac{35 + 36 + 37 + 38}{4} = \frac{146}{4} = 36 \frac{1}{2},
\]

which is clearly NOT a whole number. Picturing this on a number line,

\[35 \quad 36 \quad 36\frac{1}{2} \quad 37 \quad 38\]

we see that the average is, however, still the mid-point between the first and last numbers, 35 and 38. Thus for four consecutive numbers, the mean (average) will always be half way between the middle two numbers, which is again the ‘balance point’.

c) A set of five consecutive numbers will have the middle number as its mean. For example, the average of 4, 5, 6, 7, 8 is

\[
\frac{4 + 5 + 6 + 7 + 8}{5} = \frac{30}{5} = 6.
\]

A set of six consecutive numbers will have the mid-point on the number line as its mean. For example, the average of 3, 4, 5, 6, 7, 8 is

\[
\frac{3 + 4 + 5 + 6 + 7 + 8}{6} = \frac{33}{6} = 5 \frac{1}{2},
\]

half way between the middle two numbers 5 and 6.

d) We observe that the mean (average) of an odd quantity of whole consecutive numbers seems to always be the middle number, while the average of an even quantity of whole consecutive numbers is always half way between the middle two numbers. In all cases, the average is the midpoint between the first and last numbers when pictured on the number line.

e) Based on our observations, since 11 is an odd quantity of numbers, we predict that the mean (average) of eleven consecutive numbers will be the middle number, a whole number.
Problem of the Week

Problem B

Pony Up!

A horse’s height \( h \) is measured by an ancient unit known as a ‘hand’.

The measurement is done from the ground to the withers (the tip of the shoulder blade at the base of the horse’s neck).

Suppose there are two ponies and three horses in a field, with heights 12, 13, 14, 15, and 16 hands. If you select four animals randomly, what is the probability that their average height is at least 14 hands?

**Historical Note:** While it is now agreed that one hand equals about 10 cm (4 inches), in various cultures at various times in history it was the width of four fingers, or four fingers and a thumb, or a fist.

*Check out other CEMC resources at:
http://cemc.uwaterloo.ca/resources/resources.html*

**Strand:** Probability
Problem of the Week
Problem B and Solution

Pony Up!

Problem

A horse’s height $h$ is measured by an ancient unit known as a ‘hand’.
The measurement is done from the ground to the withers (the tip of the shoulder blade at the base of the horse’s neck).
Suppose there are two ponies and three horses in a field, with heights 12, 13, 14, 15, and 16 hands. If you select four animals randomly, what is the probability that their average height is at least 14 hands?

Solution

There are five different ways to select 4 animals, as shown in the table at the right. Their average heights are also calculated and shown in the table. For example, the average of the first selection is

$$\frac{12 + 13 + 14 + 15}{4} = \frac{54}{4} = 13.5 \text{ hands.}$$

The other averages are calculated in a similar manner.

Recall that the probability of an event is the ratio

$$P = \frac{\text{Number of desired outcomes}}{\text{Total number of possible outcomes}}.$$ 

Since there are only 3 ways to select 4 animals such that their average height is at least 14 hands, the probability is $P = \frac{3}{5}$.
Problem of the Week
Problem B

Fairly Fair

You and a friend are playing a game at the fair. It involves a big spinner divided evenly into 10 sections, each containing a different digit from 0 to 9. Observing 10 spins of the spinner, you both realize that it landed on the digit 0 six times.

a) What is the theoretical probability of the spinner landing on the digit 0? (i.e., What outcome would you expect given the design of the spinner?)

b) What is the experimental probability of landing on the digit 0? (i.e., What do your observations show?)

c) Theoretically, how many times would you expect the spinner to land on 0 if it is spun 160 times?

d) What would your observations predict for 160 spins, if 0 continues to occur with the same experimental probability as in b)?

e) Your friend wants to put her last $1.00 on the number 0 because she believes it is more likely to win than not to win. Is your friend correct? Why, or why not?

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Strand: Probability
Problem of the Week
Problem B and Solution

Fairly Fair

Problem

You and a friend are playing a game at the fair. It involves a big spinner divided evenly into 10 sections, each containing a different digit from 0 to 9. Observing 10 spins of the spinner, you both realize that it landed on the digit 0 six times.

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b) What is the experimental probability of landing on the digit 0? (i.e., What do your observations show?)

c) Theoretically, how many times would you expect the spinner to land on 0 if it is spun 160 times?

d) What would your observations predict for 160 spins, if 0 continues to occur with the same experimental probability as in b)?

e) Your friend wants to put her last $1.00 on the number 0 because she believes it is more likely to win than not to win. Is your friend correct? Why, or why not?

Solution

a) The theoretical probability of landing on the digit 0 is \( \frac{1}{10} \), since there are 10 digits, evenly spaced (and thus equally likely), and only one is a 0.

b) The experimental probability (observed frequency) is \( \frac{6}{10} \) or \( \frac{3}{5} \).

c) Since \( \frac{1}{10} = \frac{16}{160} \), theoretically we expect a 0 result 16 times in 160 spins.

d) Since \( \frac{6}{10} = \frac{96}{160} \), experimentally we expect a 0 result 96 times in 160 spins.

e) Since all the digits have an equal chance \( \left( \frac{1}{10} \right) \) of winning, your friend is not correct. (Remember that previous spins of the spinner have no bearing on future spins.)
Geometry
&
Spatial Sense
Problem of the Week
Problem B
Don’t Get Vexed by This Hex!

There are six basic pattern blocks, as shown below.


Suppose you have the following collection of some of these blocks:

- 11 green triangles;
- 5 red trapezoids;
- 5 blue rhombi.

a) How many hexagons identical in shape and size (i.e., congruent) to the yellow hexagon can be made using these pieces?

b) How many and what shapes, if any, are left over?

Check out other CEMC resources at:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Geometry and Spatial Sense
Problem of the Week
Problem B and Solution

Don’t Get Vexed by This Hex!

Problem
There are six basic pattern blocks, as shown at the right. Suppose you have the following collection of some of these blocks:

- 11 green triangles;
- 5 red trapezoids;
- 5 blue rhombi.

a) How many hexagons identical in shape and size (i.e., congruent) to the yellow hexagon can be made using these pieces?

b) How many and what shapes, if any, are left over?

Solution
a) Six hexagons can be made, as shown below.

b) This combination of pattern blocks gives no leftover pieces.

Something to Think About
An interesting way to explain why there are no leftover pieces is as follows.
Assign an area of 1 unit to the green triangle. Then the rhombus has area 2 units, the trapezoid area 3 units, and the hexagon area 6 units. Thus, with 11 triangles, 5 trapezoids, and 5 rhombi, we have a total area of

\[(11 \times 1) + (5 \times 2) + (5 \times 3) = 36 \text{ units.}\]

Since 36 is a multiple of 6, we can create the equivalent of exactly six yellow hexagons.
Problem of the Week
Problem B
Don’t Be So Picky

Four toothpicks can make a square. Eight toothpicks can make two squares.

a) Can you make two squares using seven toothpicks? If so, demonstrate how either with a diagram or toothpicks.

b) Can you use eight toothpicks to form exactly three squares? If so, demonstrate how either with a diagram or toothpicks.

c) Can you use eight toothpicks to form more than three squares?
   HINT: Squares can be of different sizes.

Check out other CEMC resources at:
http://cemc.uwaterloo.ca/resources/resources.html

STRAND: Geometry and Spatial Sense
Don’t Be So Picky

Four toothpicks can make a square. Eight toothpicks can make two squares.

a) Can you make two squares using seven toothpicks? If so, describe how with a diagram.

b) Can you use eight toothpicks to form exactly three squares? If so, describe how with a diagram.

c) Can you use eight toothpicks to form more than three squares?

Hint: Squares can be of different sizes.

Solution

Each of parts a), b), c) is possible. The solutions are shown in the diagram below.
Problem of the Week
Problem B

Can You Draw It?

Use a protractor and ruler to draw triangles with the following properties, if you can. Find the measure of each angle on your triangles in parts a) to d), and label the side lengths for parts e) to h). If you can’t draw such a triangle, explain why not.

a) A triangle with three acute angles.
b) A triangle with two right angles.
c) A triangle with one obtuse angle.
d) A triangle with two obtuse angles.
e) A triangle with side lengths 6 cm, 4 cm, and 4 cm.
f) A triangle with side lengths 6 cm, 4 cm, and 3 cm.
g) A triangle with side lengths 6 cm, 4 cm, and 2 cm.
h) A triangle with side lengths 3 cm, 4 cm, and 5 cm.

Check out other CEMC resources here:
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Strand: Geometry and Spatial Sense
Problem of the Week
Problem B and Solution
Can You Draw It?

Problem
Use a protractor and ruler to draw triangles with the following properties, if you can. Find the measure of each angle on your triangles in parts a) to d'), and label the side lengths for parts e) to h). If you can’t draw such a triangle, explain why not.

a) A triangle with three acute angles.
b) A triangle with two right angles.
c) A triangle with one obtuse angle.
d) A triangle with two obtuse angles.
e) A triangle with side lengths 6 cm, 4 cm, and 4 cm.
f) A triangle with side lengths 6 cm, 4 cm, and 3 cm.
g) A triangle with side lengths 6 cm, 4 cm, and 2 cm.
h) A triangle with side lengths 3 cm, 4 cm, and 5 cm.

Solution

a) Solutions will vary; all angles must be less than 90°.
b) No such triangle can be formed; since AC is parallel to BD, they will never meet.
c) Solutions will vary; a single obtuse angle must occur, as indicated.
d) Such a triangle cannot be formed. AC and BD will never meet because the angles at A and C are greater than 90°.

e) The solution triangle is shown (to scale).

f) The solution triangle is shown (to scale).

g) Such a triangle cannot be formed. AC and BD will never meet because the length of AC plus BD exactly equals the third side of the triangle, AB. Thus C and D won’t meet until AC and BD lie right on top of AB.

h) The solution triangle is shown. The angle at C is a right angle.
Problem of the Week
Problem B

Edges With Nothing in Common

A famous problem, only recently solved, is the "Four Colour Problem", which asks whether any map can be coloured with at most four distinct colours so that no two countries with a common boundary are the same colour. Here are some similar questions in three dimensions.

a) The surface of a cube is to be painted so that no two adjacent faces (i.e., faces which have an edge in common) are the same colour. What is the minimum number of different colours required to paint the six faces of the cube so as to satisfy this rule? Explain your reasoning.

b) What is the answer to this question for the surface of a tetrahedron, which has four faces? Explain your reasoning.

c) What is the answer to this question for a hexagonal prism, which has eight faces? Explain your reasoning.

d) Do you find anything puzzling in your answers to these questions?

Check out other CEMC resources here:
http://cemc.uwaterloo.ca/resources/resources.html

STRAND: Geometry and Spatial Sense
Problem of the Week
Problem B and Solution

Edges with Nothing in Common

Problem

a) The surface of a cube is to be painted so that no two adjacent faces (i.e., faces which have an edge in common) are the same colour. What is the minimum number of different colours required to paint the six faces of the cube so as to satisfy this rule? Explain your reasoning.

b) What is the answer to this question for the surface of a tetrahedron, which has four faces? Explain your reasoning.

c) What is the answer to this question for a hexagonal prism, which has eight faces? Explain your reasoning.

d) Do you find anything puzzling in your answers to these questions?

Solution

a) The cube requires three colours, one for each pair of opposite faces A1 and A2 (front and back), B1 and B2 (left and right), C1 and C2 (top and bottom).

b) The tetrahedron requires four colours, since every face S1 (front), S2 (left), S3 (top), and S4 (back) shares an edge with every other face.

c) The hexagonal prism requires three colours, one for the two hexagons on the top and the bottom, another for the vertical rectangular faces S1, S3, S5, and a third for S2, S4, and S6.

d) It seems puzzling that the three-dimensional shape with the fewest faces (the tetrahedron) requires the greatest number of colours.
Problem of the Week
Problem B

Tumbling T’s

A green pattern block (the triangle) has side lengths of 2 cm. Suppose it is rolled to the right (without sliding) a number of times. If the triangle stops so that the letter T is in the upright position, what possible distances $d$ could it have rolled?

Check out other CEMC resources here:
http://cemc.uwaterloo.ca/resources/resources.html

**Strand:** Geometry, Measurement
Problem of the Week
Problem B and Solution

Tumbling T’s

Problem

A green pattern block (the triangle) has side lengths of 2 cm. Suppose it is rolled to the right (without sliding) a number of times. If the triangle stops so that the letter T is in the upright position, what possible distances \( d \) could it have rolled?

![Diagram of triangle rolling](image)

Solution

If the pattern block is rotated 3 times to the right, pivoting first on vertex A, then B, then C without sliding, it will land with the T in the upright position. Since the sides of the triangle are 2 cm in length, it will have rolled a distance \( d = 6 \) cm.

If it is rotated another 3 times, it will again have the T in the upright position, and will have rolled a total of \( d = 12 \) cm.

Thus, for every 3 rolls, the distance \( d \) increases by 6 cm, as shown in the table below. Hence \( d \) will always be a multiple of 6 cm; the table continues indefinitely.

<table>
<thead>
<tr>
<th>No. of rolls</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
<th>\ldots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance ( d )</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>\ldots</td>
</tr>
</tbody>
</table>
Problem of the Week
Problem B
Reflect on These Numbers

Suppose that all ten single digits are written in the font shown below:

\[
0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9
\]

a) Which four digits are symmetric about a horizontal line through the middle?

b) What three-digit numbers are symmetric about a horizontal line through the middle? You may use any digit more than once in total, but only once in each three-digit number.

c) Of these numbers, which are also symmetric about a vertical line through the middle? Explain your thinking.

Check out other CEMC resources here:
http://cemc.uwaterloo.ca/resources/resources.html

**Strand:** Geometry/Spatial Sense
Problem of the Week
Problem B and Solution
Reflect on These Numbers

Problem

Suppose that all ten single digits are written in the font shown below:

\[
\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\end{array}
\]

a) Which four digits are symmetric about a horizontal line through the middle?

b) What three-digit numbers are symmetric about a horizontal line through the middle? You may use each digit more than once in total, but only once in each three-digit number.

c) Of the three-digit numbers found in part b), which are also symmetric about a vertical line through the middle? Explain your thinking.

Solution

a) The only numbers in the given set with a horizontal line of symmetry are \(0, 1, 3, 8\).

b) Any three-digit combination of the numbers in part a) that does not start with \(0\) will have a horizontal line of symmetry.

Those beginning with \(1\) are 103, 108, 130, 138, 180, and 183.

Those beginning with \(3\) are 301, 308, 310, 318, 380, and 381.

Those beginning with \(8\) are 801, 803, 810, 813, 830, and 831.

c) There are three digits with both a vertical and a horizontal line of symmetry, \(0, 1, 8\). But for a three-digit number to have both horizontal and vertical symmetry, it would have to be a palindrome such as 808, 101, or 888. Since we can use each digit only once in each three-digit number, there are no such numbers in our set.

Something to Think About

How would your answer to part b) change if you could use the digits more than once in any three-digit number?
Problem of the Week
Problem B

Linking Up is Hard to Do!

a) How many cubes of different sizes could you make if you have 30 multilink cubes? You do not need to use all of the cubes.

b) How many rectangular prisms could you make with up to 8 multilink cubes? Indicate any that might be identical to another if you changed their orientation, i.e., any that would be the same if you rotated them.

Check out other CEMC resources here:
http://cemc.uwaterloo.ca/resources/resources.html

**Strand:** Geometry
Problem of the Week
Problem B and Solution

Linking Up is Hard to Do!

Problem

a) How many cubes of different sizes could you make if you have 30 multilink cubes? You do not need to use all of the cubes.

b) How many rectangular prisms could you make with up to 8 multilink cubes? Indicate any that might be identical to another if you changed their orientation, i.e., any that would be the same if you rotated them.

Solution

a) With 30 multilink cubes, three cubes can be constructed, with dimensions 1 \times 1 \times 1 (a single cube), 2 \times 2 \times 2 (using 8 cubes), and 3 \times 3 \times 3 (using 27 cubes). There are not enough for a 4 \times 4 \times 4 cube, which needs 64 cubes.

b) With 8 multilink cubes, 12 different rectangular prisms can be constructed. There are 8 simple ones which are just a single row of cubes, with dimensions 1 \times 1 \times 1, 1 \times 2 \times 1, 1 \times 3 \times 1, 1 \times 4 \times 1, 1 \times 5 \times 1, 1 \times 6 \times 1, 1 \times 7 \times 1, and 1 \times 8 \times 1. There are also the 2 \times 2 \times 2 cube and 3 other prisms with dimensions 1 \times 2 \times 2, 1 \times 3 \times 2, and 1 \times 4 \times 2, as shown below.

Any other prisms are just different orientations of these 12.
Problem of the Week
Problem B

To Quilting Bee or Not to Quilting Bee

Sarah is working on a quilt created from square blocks which are 1 decimetre (dm) by 1 decimetre.

Each block has a white background and a dark triangle with the upper vertex at the midpoint of the top side, as shown in the diagram on the left.

a) Describe how Sarah used four of these basic blocks to create the windmill motif (design) shown at the right.

b) Sarah’s whole quilt requires 16 of these four-block motifs (windmill designs). Will she need more, less, or the same amount of dark fabric as white fabric?

Check out other CEMC resources at:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Geometry and Spatial Sense
Problem of the Week
Problem B and Solution

To Quilting Bee or Not to Quilting Bee

Sarah is working on a quilt created from square blocks which are 1 decimetre (dm) by 1 decimetre. Each block has a white background and a dark triangle with the upper vertex at the midpoint of the top side, as shown in the diagram above.

a) Describe how Sarah used four of these basic blocks to create the windmill motif (design) shown at the right.

b) Sarah’s whole quilt requires 16 of these four-block motifs (windmill designs). Will she need more, less, or the same amount of dark fabric as white fabric?

Solution

a) The basic block A (outlined by dashed lines in the diagram above) is rotated $90^\circ$ (i.e., $\frac{1}{4}$ turn) about the centre of the larger square motif to create B. It is rotated again by $90^\circ$ to create C, and once more for D, completing the design.

b) The height of the dark triangle in the basic block is $h = 1 \text{ dm} = 10 \text{ cm}$ and so is the base $b = 10 \text{ cm}$. Thus its area is $\frac{1}{2} \times b \times h = \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$. Each white triangle has the same height, but the base is 5 cm, giving area equal to half that of the dark triangle. Thus together the two white triangles equal the area of the dark triangle.

Since this is true for every basic block in the motif, and every motif in the whole quilt, Sarah will need the same amount of dark fabric as white fabric to make her quilt.

NOTE: Part b) can also be solved by dividing the dark triangle in half by a vertical line, and then showing that each white triangle is congruent to half of the dark triangle.

Something to Think About

If Sarah decides to vary her basic 1 dm by 1 dm block a bit, as shown in the diagram on the right, will your answer to part b) change? Explain your reasoning.
Problem of the Week
Problem B
Quilty as Charged!

Sarah is making more quilts (see the previous POTWB). She has three basic blocks in mind, each 1 dm by 1 dm.

(The dotted lines are not part of the basic block; they are provided to help to envision how each block is constructed. In each block, the dotted lines subdivide the block into equal areas.)

a) If Sarah creates three motifs, one from each basic block, using the same geometric operations as she used to create her windmill motif, what will each motif look like? Make three sketches to show your answers. Which motif do you find the most appealing?

b) For these quilts, Sarah plans to use yellow fabric for the background and blue for the shaded areas. If she has three times as much yellow fabric as blue fabric, could she make a quilt from every one of the three basic blocks? Explain your reasoning.

Check out other CEMC resources at:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Geometry and Spatial Sense, Measurement
Problem

Sarah is making more quilts (see the previous POTWB). She has three basic blocks in mind, each 1 dm by 1 dm. The dotted lines are not part of the basic block; they are provided to help to envision how each block is constructed. In each block, the dotted lines subdivide the block into equal areas.

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b) For these quilts, Sarah plans to use yellow fabric for the background and blue for the shaded areas. If she has three times as much yellow fabric as blue fabric, could she make a quilt from every one of the three basic blocks? Explain your reasoning.

Solution

a) Recall that Sarah obtained her motifs by rotating each basic block 90° three times in succession. Repeating this for these new basic blocks reveals the three motifs in the diagram below. The one found most appealing will depend on the eye of the beholder!
b) Since each basic block is replicated to form each motif, which is then replicated again many times to form the quilt, we need only look at the relative areas of yellow and blue fabric used in each block.

Consider the more lightly shaded versions of the three blocks shown in the diagram below.

In block 1, the two shaded (blue) triangles form exactly one quarter of the square. Thus \( \frac{1}{4} \) of the basic block is blue and \( \frac{3}{4} \) is yellow. This will be true of Motif 1 and of any quilt formed from such motifs. Since Sarah has three times as much yellow fabric as blue fabric, she will be able to make any such quilt.

In Block 2, it is clear by comparison that the shaded (blue) circular segments have greater area than the triangles of Block 1. Thus more than \( \frac{1}{4} \) of the area of basic Block 2 is blue, and so any quilt made from Motif 2 will be more than \( \frac{1}{4} \) blue as well. So Sarah would not have enough fabric for such a quilt.

In Block 3, the diamond shape in the centre consists of four triangles, each of which is half of one of the rectangles formed by the dotted lines. Thus together they form half of the area within the middle four rectangles, i.e., they are equal to \( \frac{1}{4} \) of the total area of all eight rectangles. So, by the same reasoning as for Block 1, Sarah will be able to make any quilt formed from Motif 3.
Problem of the Week
Problem B
Canadian Hero

In 1980, Terry Fox ran about 3 339 miles (5 374 km) across Canada during his Marathon of Hope to raise money for cancer research.

a) Each day he tried to run a marathon, which is 42 km. At this rate, how many days would it take to cover the total distance he ran?

b) His run ended in Thunder Bay, Ontario after 143 days. Was he able to run a marathon a day, as he had hoped?

c) By 2012, 500 million dollars had been raised in Terry’s name. How many dollars is that for each km that he ran?

d) Count the number of strides it takes you to run 10 m. Using this information, determine how many steps it would take you to run the same total distance as Terry ran.
[Remember that he did the whole run with an artificial leg!]

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STRAND: Number Sense, Measurement
Problem of the Week
Problem B and Solution
Canadian Hero

Problem

In 1980, Terry Fox ran about 3339 miles (5374 km) across Canada during his Marathon of Hope to raise money for cancer research.

a) Each day he tried to run a marathon, which is 42 km. At this rate, how many days would it take to cover the total distance he ran?

b) His run ended in Thunder Bay, Ontario after 143 days. Was he able to run a marathon a day, as he had hoped?

c) By 2012, 500 million dollars had been raised in Terry’s name. How many dollars is that for each km that he ran?

d) Count the number of strides it takes you to run 10 m. Using this information, determine how many steps it would take you to run the same total distance as Terry ran.

[Remember that he did the whole run with an artificial leg!]

Solution

a) If he ran 42 km per day, then in 100 days he would run 4200 km. In the next 20 days, he would run $20 \times 42 = 840$ km, giving a total of 5040 km in 120 days. Another 8 days would add $8 \times 42 = 336$ km, giving a total of $5040 + 336 = 5376$ km. Thus 5374 km would take him 128 days at 42 km per day.

OR, we can calculate the time taken by dividing the distance by the rate of speed, giving $5374 \div 42 \approx 128$ days.

b) Since his run ended after 143 days, we know he was not able to run a marathon a day; otherwise it would have ended after 128 days. (He actually ran $5374 \div 143 \approx 37.6$ km per day.)

c) The dollars raised per km is $500\,000\,000 \div 5374 \approx $93,041 per km.

d) An average stride is about 1 m. Thus, to run 5374 km = 5374000 m would take about 5374000 steps. Your solution will vary depending on the length of your stride.
Problem of the Week
Problem B

A Classic Race

The Hare and the Tortoise are running a 100 metre race. The Hare can run 20 m in 5 s, while the Tortoise ambles along at 2 m every 5 s. Confident of an easy win, the Hare stops to nibble some tasty wild carrots nearby. If the race ends in a tie, for how long did the Hare stop?
Problem of the Week
Problem B and Solution

A Classic Race

Problem

The Hare and the Tortoise are running a 100 metre race. The Hare can run 20 m in 5 s, while the Tortoise ambles along at 2 m every 5 s. Confident of an easy win, the Hare stops to nibble some tasty wild carrots nearby. If the race ends in a tie, for how long did the Hare stop?

Solution

Since $100 \div 2 = 50$, there are 50 groups of 2 m in 100 m. The tortoise took 5 s to go each 2 m group and did not stop. Thus the tortoise took $50 \times 5 \text{ s} = 250 \text{ s}$ to run the race.

Since $100 \div 20 = 5$, there are 5 groups of 20 m in 100 m. The hare can run each 20 m group in 5 s. Thus if the hare had run the race without stopping, it would have taken him $5 \times 5 \text{ s} = 25 \text{ s}$ to run the race.

The race ends in a tie, both animals took the same amount of time to reach the finish line. So the hare must have spent a total of 250 s running and eating. Since he only needed 25 s to run the 100 m, he must have spent $250 \text{ s} - 25 \text{ s} = 225 \text{ s}$ nibbling carrots.
Problem of the Week
Problem B
Cycling for Sweets

The time was 21:58:00 and Mrs. Arjmand was preparing her children’s lunches for school the next day. Suddenly she realized that she was supposed to send cupcakes to school for the bake sale the next day, for which she needed to buy icing sugar.

Mrs. Arjmand will have to ride her bicycle to one of the two convenience stores near her home, as shown in the diagram. Both stores are open until 10:00 p.m., but McShane’s clock is 5 minutes slow.

If she can cycle at 6 metres per second, to which convenience store should she go? Explain your thinking.

Check out other CEMC resources here:
http://cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense, Measurement
Problem of the Week
Problem B and Solution
Cycling for Sweets

Problem

The time was 21:58:00 and Mrs. Arjmand was preparing her children’s lunches for school the next day. Suddenly she realized that she was supposed to send cupcakes to school for the bake sale the next day, for which she needed to buy icing sugar.

Mrs. Arjmand will have to ride her bicycle to one of the two convenience stores near her home; McShane’s is 1.44 kilometres away, and Andersen’s is 798 metres in another direction. Both stores are open until 10:00 p.m., but McShane’s clock is 5 minutes slow.

If she can cycle at 6 metres per second, to which convenience store should she go? Explain your thinking.

Solution

To ride 1.44 km (or 1440 metres) to McShane’s store at a speed of 6 metres per second will take Mrs. Arjmand

\[
1440 \div 6 = 240 \text{ seconds, or 4 minutes.}
\]

To ride 798 metres to Andersen’s store will take her

\[
798 \div 6 = 133 \text{ seconds, or 2 minutes and 13 seconds.}
\]

Since the time is 21:58:00, or 2 minutes before 10:00, she cannot make it to Andersen’s in time, since the clock there is accurate, so he will close 13 seconds before she gets there.

But McShane’s clock is 5 minutes slow, so it shows 7 minutes to 10:00 when she leaves home. Thus she can make it to McShane’s with 3 minutes to spare.
Problem of the Week
Problem B

Tumbling T’s

A green pattern block (the triangle) has side lengths of 2 cm. Suppose it is rolled to the right (without sliding) a number of times. If the triangle stops so that the letter T is in the upright position, what possible distances $d$ could it have rolled?
Problem of the Week
Problem B and Solution

Tumbling T’s

Problem

A green pattern block (the triangle) has side lengths of 2 cm. Suppose it is rolled to the right (without sliding) a number of times. If the triangle stops so that the letter T is in the upright position, what possible distances \( d \) could it have rolled?

\[
\begin{array}{c}
\text{C} \\
\text{A} \\
\text{B}
\end{array}
\]

\[
\begin{array}{c}
\text{T} \\
\text{T}
\end{array}
\]

\[
\text{d}
\]

Solution

If the pattern block is rotated 3 times to the right, pivoting first on vertex A, then B, then C without sliding, it will land with the T in the upright position. Since the sides of the triangle are 2 cm in length, it will have rolled a distance \( d = 6 \) cm.

If it is rotated another 3 times, it will again have the T in the upright position, and will have rolled a total of \( d = 12 \) cm.

Thus, for every 3 rolls, the distance \( d \) increases by 6 cm, as shown in the table below. Hence \( d \) will always be a multiple of 6 cm; the table continues indefinitely.

<table>
<thead>
<tr>
<th>No. of rolls</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
<th>\ldots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance ( d )</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>\ldots</td>
</tr>
</tbody>
</table>
Farmer Jack wants to fence off one side of his driveway. He has 20 fenceposts, each 15 cm in diameter, and the driveway is 117 m long.

If the posts are to be evenly spaced, how far apart should they be (i.e., what is the distance $L$)?

*Check out other CEMC resources at:*  
http://cemc.uwaterloo.ca/resources/resources.html

**Strand:** Measurement
Problem of the Week
Problem B and Solution
Spaced Out Posts

Problem
Farmer Jack wants to fence off one side of his driveway. He has 20 fenceposts, each 15 cm in diameter, and the driveway is 117 m long.
If the posts are to be evenly spaced, how far apart should they be (i.e., what is the distance L)?

![Diagram of spaced out posts]

Solution
Think of the posts in a row along the 117 m driveway.

![Diagram of posts along driveway]

Between the 20 posts there will be 19 spaces of length L.
However, the posts themselves will take some space; since there are 20 posts of width 15 cm = 0.15 m, they will occupy $20 \times 0.15 \text{ m} = 3 \text{ m}$.
Thus the total distance remaining for the space between the posts is $117 \text{ m} - 3 \text{ m} = 114 \text{ m}$.
Since there are 19 spaces of length L m, we know that $19 \times L \text{ m} = 114 \text{ m}$.
Thus the length of each space must be $L = 114 \div 19 = 6 \text{ m}$. 
Number Sense & Numeration
Problem of the Week
Problem B

Canadian Hero

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[Remember that he did the whole run with an artificial leg!]

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Strand: Number Sense, Measurement
Problem of the Week
Problem B and Solution
Canadian Hero

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Solution

a) If he ran 42 km per day, then in 100 days he would run 4 200 km. In the next 20 days, he would run $20 \times 42 = 840$ km, giving a total of 5 040 km in 120 days. Another 8 days would add $8 \times 42 = 336$ km, giving a total of 5 040 + 336 = 5 376 km. Thus 5 374 km would take him 128 days at 42 km per day.

OR, we can calculate the time taken by dividing the distance by the rate of speed, giving $5 374 \div 42 \approx 128$ days.

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c) The dollars raised per km is $500 000 000 \div 5 374 \approx 93 041$ per km.

d) An average stride is about 1 m. Thus, to run 5 374 km = 5 374 000 m would take about 5 374 000 steps. Your solution will vary depending on the length of your stride.
Problem of the Week

Problem B

Money in the Bank

Ever since her $5^{th}$ birthday, lucky Chan has saved the $75.00 she receives each year from her aunt instead of a birthday toy. Next week, Chan will turn 14, and she has decided that after her $14^{th}$ birthday party, she will take all of her birthday money and open a savings account.

a) How much money will Chan have to open her account immediately following her $14^{th}$ birthday?

b) If the bank pays $2\%$ interest on the balance (i.e., 0.02 times the balance) at the end of each year, and she continues to bank all her birthday money and spends none, how much will Chan have in her bank account immediately after her $16^{th}$ birthday?

<table>
<thead>
<tr>
<th>Birthday</th>
<th>New Balance</th>
<th>Interest for Following Year</th>
<th>End-of-Year Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$14^{th}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$15^{th}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$16^{th}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Check out other CEMC resources for Grades 4 to 6 here:
http://cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense; Pattern/Algebra
Problem of the Week
Problem B and Solution

Money in the Bank

Problem

Ever since her 5\textsuperscript{th} birthday, lucky Chan has saved the $75.00 she receives each year from her aunt instead of a birthday toy. Next week, Chan will turn 14, and she has decided that after her 14\textsuperscript{th} birthday party, she will take all of her birthday money and open a savings account.

a) How much money will Chan have to open her account immediately following her 14\textsuperscript{th} birthday?

b) If the bank pays 2\% interest on the balance (i.e., 0.02 times the balance) at the end of each year, and she continues to bank all her birthday money and spends none, how much will Chan have in her bank account immediately after her 16\textsuperscript{th} birthday?

Solution

a) By just after her 14\textsuperscript{th} birthday, Chan will have accumulated $75.00 from each of the birthdays when she turned 5, 6, 7, 8, 9, 10, 11, 12, 13, and 14, i.e., from a total of 10 birthdays. Thus she will have $10 \times 75.00 = \$750.00 with which to open her savings account.

b) Chan will receive interest on the initial deposit of $750.00 after her 14\textsuperscript{th} birthday, and on her total amount after depositing the $75.00 she receives for her 15\textsuperscript{th}. Assuming she adds her 16\textsuperscript{th} birthday present, the amounts are:

<table>
<thead>
<tr>
<th>Birthday</th>
<th>New Balance</th>
<th>Interest for Following Year</th>
<th>End-of-Year Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>14\textsuperscript{th}</td>
<td>$750.00</td>
<td>0.02 \times 750 = 15.00</td>
<td>$765.00</td>
</tr>
<tr>
<td>15\textsuperscript{th}</td>
<td>$840.00</td>
<td>0.02 \times 840 = 16.80</td>
<td>$856.80</td>
</tr>
<tr>
<td>16\textsuperscript{th}</td>
<td>$931.80</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Thus immediately after her 16\textsuperscript{th} birthday, Chan has $931.80.
Fei has earned $8.00 this week helping his mother with housework. He wants to buy her some chocolates for her birthday, and has decided on orange creams at $0.60 each, or truffles at $0.80 each, or a mix of both.

If he spends all of his earnings, what are the possible different combinations of chocolates and/or truffles Fei can purchase?

Check out other CEMC resources at:
http://cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense
Problem of the Week
Problem B and Solution

Sweet Child of Mine

Problem

Fei has earned $8.00 this week helping his mother with housework. He wants to buy her some chocolates for her birthday, and has decided on orange creams at $0.60 each, or truffles at $0.80 each, or a mix of both.

If he spends all of his earnings, what are the possible different combinations of chocolates and/or truffles Fei can purchase?

Solution

Since he can only purchase ‘whole’ chocolates, we need to find whole numbers $n$ of truffles and $m$ of orange creams such that their cost is exactly $8.00. Expressed as a mathematical statement, this is $0.80 \times n + 0.60 \times m = 8.00$.

We construct a table showing all the possibilities, starting with $n = 10$, which is the maximum number of truffles Fei can purchase with $8.00.

<table>
<thead>
<tr>
<th>No. of Truffles</th>
<th>Cost of Truffles in $</th>
<th>Amount left for Creams in $</th>
<th>Number/Cost of Creams</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10 \times 0.80 = 8.00</td>
<td>0</td>
<td>0 \times 0.60 = 0 $</td>
<td>$8.00</td>
</tr>
<tr>
<td>9</td>
<td>9 \times 0.80 = 7.20</td>
<td>8.00 - 7.20 = 0.80</td>
<td>1 \times 0.60 = $0.60</td>
<td>$7.80</td>
</tr>
<tr>
<td>8</td>
<td>8 \times 0.80 = 6.40</td>
<td>8.00 - 6.40 = 1.60</td>
<td>2 \times 0.60 = $1.20</td>
<td>$7.60</td>
</tr>
<tr>
<td>7</td>
<td>7 \times 0.80 = 5.60</td>
<td>8.00 - 5.60 = 2.40</td>
<td>4 \times 0.60 = $2.40</td>
<td>$8.00</td>
</tr>
<tr>
<td>6</td>
<td>6 \times 0.80 = 4.80</td>
<td>8.00 - 4.80 = 3.20</td>
<td>5 \times 0.60 = $3.00</td>
<td>$7.80</td>
</tr>
<tr>
<td>5</td>
<td>5 \times 0.80 = 4.00</td>
<td>8.00 - 4.00 = 4.00</td>
<td>6 \times 0.60 = $3.60</td>
<td>$7.40</td>
</tr>
<tr>
<td>4</td>
<td>4 \times 0.80 = 3.20</td>
<td>8.00 - 3.20 = 4.80</td>
<td>8 \times 0.60 = $4.80</td>
<td>$8.00</td>
</tr>
<tr>
<td>3</td>
<td>3 \times 0.80 = 2.40</td>
<td>8.00 - 2.40 = 5.60</td>
<td>9 \times 0.60 = $5.40</td>
<td>$7.80</td>
</tr>
<tr>
<td>2</td>
<td>2 \times 0.80 = 1.60</td>
<td>8.00 - 1.60 = 6.40</td>
<td>10 \times 0.60 = $6.00</td>
<td>$7.60</td>
</tr>
<tr>
<td>1</td>
<td>1 \times 0.80 = 0.80</td>
<td>8.00 - 0.80 = 7.20</td>
<td>12 \times 0.60 = $7.20</td>
<td>$8.00</td>
</tr>
<tr>
<td>0</td>
<td>0 \times 0.80 = 0 $</td>
<td>8.00</td>
<td>13 \times 0.60 = $7.80</td>
<td>$7.80</td>
</tr>
</tbody>
</table>

Thus there are four possible combinations Fei could purchase: 10 truffles and no orange creams, 7 truffles and 4 creams, 4 truffles and 8 creams, or 1 truffle and 12 creams. (You may disagree with calling the first choice a ‘combination’.)

Extension:

Use the pattern in your results from this question to try to predict the number of possible combinations if Fei had $12.00 saved and spent all of it on chocolates as above. [You should find six possible combinations.]
Problem of the Week
Problem B

Name Change

Ned, Ted, and Fred each have some nickels, dimes, and/or quarters. Given the information below, what are the two possible sets of coins each person could have?

a) Fred has 6 coins which have a mean (average) value of 10 cents per coin.
b) Ted has 5 coins which have a mean value of 10 cents per coin.
c) Ned has 4 coins which have a mean value of 10 cents per coin.

Check out other CEMC resources at:
http://cemc.uwaterloo.ca/resources/resources.html

**STRAND:** Number Sense, Data Management
Problem of the Week
Problem B and Solution

Name Change

Problem

Ned, Ted, and Fred each have some nickels, dimes, and/or quarters. Given the information below, what are the two possible sets of coins each person could have?

a) Fred has 6 coins which have a mean (average) value of 10 cents per coin.

b) Ted has 5 coins which have a mean value of 10 cents per coin.

c) Ned has 4 coins which have a mean value of 10 cents per coin.

Solution

a) Since Fred has 6 coins with a mean value of 10 cents per coin, he must have a total of $6 \times 10$ cents $= 60$ cents. Clearly he could have 6 dimes, or, since

$$5 + 5 + 5 + 10 + 10 + 25 = 60,$$

he could also have 3 nickels, 2 dimes, and 1 quarter.

b) Since Ted has 5 coins with a mean value of 10 cents per coin, he must have a total of $5 \times 10$ cents $= 50$ cents. Thus he could have 5 dimes, or, since

$$5 + 5 + 5 + 10 + 25 = 50,$$

he could also have 3 nickels, 1 dime, and 1 quarter.

c) Since Ned has 4 coins with a mean value of 10 cents per coin, he must have a total of $4 \times 10$ cents $= 40$ cents. Thus he could have 4 dimes, or, since

$$5 + 5 + 5 + 25 = 40,$$

he could also have 3 nickels, no dimes, and 1 quarter.
Problem of the Week
Problem B
A Perfectly Odd Square

Let’s play "What’s My Number?"

a) I am a multiple of 3. I am an odd number. I am between 10 and 30. I am 2 more than a perfect square. What number am I?

b) If I am even, not odd, what number could I be?

c) Make up your own mystery number challenge. Test to make sure you have only one solution. Try it on a classmate.

Check out other CEMC resources here:
http://cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense
Problem of the Week
Problem B and Solution
A Perfectly Odd Square

Problem

Let’s play "What’s My Number?"

a) I am a multiple of 3. I am an odd number. I am between 10 and 30. I am 2 more than a perfect square. What number am I?

b) If I am even, not odd, what number could I be?

c) Make up your own mystery number challenge. Test to make sure you have only one solution. Try it on a classmate.

Solution

a) Multiples of 3 between 10 and 30 are 12, 15, 18, 21, 24, 27, 30.
   Of these, only 15, 21, and 27 are odd.
   Since the perfect squares between 10 and 30 are 16 and 25, of the three numbers, only 27 is 2 more than a perfect square (25).

b) The even multiples of 3 between 10 and 30 are 12, 18, 24, 30.
   Of these, only 18 is 2 more than a perfect square (16).

c) Answers will vary.

Something to Think About

An important strategy is to write down the possible answers at each stage of such problems, in order to reveal whether more than one solution is possible. If so, this should be noted in answers to part c).
Problem of the Week
Problem B
Uncharted Waters

The diagram below is a pattern on a 100s chart. (Only the numbers 1 and 100 are visible.)

100’s Chart

a) Describe the number pattern in as many ways as possible.

b) If the pattern is extended, will the number 76 be in the pattern set?

Check out other CEMC resources here:
http://cemc.uwaterloo.ca/resources/resources.html

STRAND: Number Sense, Pattern/Algebra
Problem of the Week
Problem B and Solution
Uncharted Waters

Problem

The diagram at the right is a pattern on a 100s chart. (Only the numbers 1 and 100 are visible.)

a) Describe the number pattern in as many ways as possible.

b) If the pattern is extended, will the number 76 be in the pattern set?

Solution

a) The pattern can be described in several ways.

   (i) The number pattern is increasing by 3 for each shaded square to the right, starting at 42 and ending at 69.

   (ii) The number pattern is decreasing by 3 to the left, starting at 69 and ending at 42.

   (iii) Every third square horizontally is shaded in the chart, with no blank, one blank, or two blank squares on the left edge.

b) The number 76 will not be shaded if the pattern is extended (cross-hatched squares), since $76 - 69 = 7$ is not a multiple of 3.

Something to Think About

Find a number that will be shaded in the second last row from the bottom of the 100s chart.
Problem of the Week
Problem B
Pitter Platter

The staff at Grey Valley Private School were preparing for their Holiday Social. Mr. Van Der Pas found a caterer who would provide a variety of platters for the event, priced as follows:

Cheese - $60,  Mediterranean - $50,  Antipasto - $60,  Fresh Fruit - $50,  Vegetable Tray - $40.

He orders seven platters costing exactly $360. If he orders at most two of any platter, what are the possible combinations of platters Mr. Van Der Pas could order?

Check out other CEMC resources here:
http://cemc.uwaterloo.ca/resources/resources.html

STRAND: Data Management, Number Sense
Problem of the Week
Problem B and Solution
Pitter Platter

Problem
The staff at Grey Valley Private School were preparing for their Holiday Social. Mr. Van Der Pas found a caterer who would provide a variety of platters, priced as follows:

- Cheese - $60
- Mediterranean - $50
- Antipasto - $60
- Fresh Fruit - $50
- Vegetable Tray - $40

He orders seven platters costing exactly $360. If he orders at most two of any platter, what are the possible combinations of platters Mr. Van Der Pas could order?

Solution
There are nine possible combinations of platters which cost a total of $360 such that there is at most two of any platter type. The combinations are shown in the charts below. Six combinations are shown in the first chart and the other three are shown in the second chart.

<table>
<thead>
<tr>
<th>Type</th>
<th>Combinations and Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antipasto</td>
<td>2 $120 2 $120 1 $60</td>
</tr>
<tr>
<td>Cheese</td>
<td>1 $60 1 $60 2 $120</td>
</tr>
<tr>
<td>Fresh Fruit</td>
<td>2 $100 1 $50 2 $100</td>
</tr>
<tr>
<td>Mediterranean</td>
<td>1 $50 2 $100 1 $50</td>
</tr>
<tr>
<td>Vegetable Tray</td>
<td>2 $80 2 $80 2 $80</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>7 $360 7 $360 7 $360</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type</th>
<th>Combinations and Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antipasto</td>
<td>2 $120 1 $60</td>
</tr>
<tr>
<td>Cheese</td>
<td>2 $120 1 $60</td>
</tr>
<tr>
<td>Fresh Fruit</td>
<td>2 $100 2 $100 2 $100</td>
</tr>
<tr>
<td>Mediterranean</td>
<td>2 $100 2 $100 2 $100</td>
</tr>
<tr>
<td>Vegetable Tray</td>
<td>1 $40 1 $40 1 $40</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>7 $360 7 $360 7 $360</td>
</tr>
</tbody>
</table>
Problem of the Week
Problem B

Cycling for Sweets

The time was 21:58:00 and Mrs. Arjmand was preparing her children’s lunches for school the next day. Suddenly she realized that she was supposed to send cupcakes to school for the bake sale the next day, for which she needed to buy icing sugar.

Mrs. Arjmand will have to ride her bicycle to one of the two convenience stores near her home, as shown in the diagram. Both stores are open until 10:00 p.m., but McShane’s clock is 5 minutes slow. If she can cycle at 6 metres per second, to which convenience store should she go? Explain your thinking.

Check out other CEMC resources here:
http://cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense, Measurement
Problem of the Week
Problem B and Solution

Cycling for Sweets

Problem

The time was 21:58:00 and Mrs. Arjmand was preparing her children’s lunches for school the next day. Suddenly she realized that she was supposed to send cupcakes to school for the bake sale the next day, for which she needed to buy icing sugar.

Mrs. Arjmand will have to ride her bicycle to one of the two convenience stores near her home; McShane’s is 1.44 kilometres away, and Andersen’s is 798 metres in another direction. Both stores are open until 10:00 p.m., but McShane’s clock is 5 minutes slow.

If she can cycle at 6 metres per second, to which convenience store should she go? Explain your thinking.

Solution

To ride 1.44 km (or 1440 metres) to McShane’s store at a speed of 6 metres per second will take Mrs. Arjmand

\[ \frac{1440}{6} = 240 \text{ seconds, or 4 minutes}. \]

To ride 798 metres to Andersen’s store will take her

\[ \frac{798}{6} = 133 \text{ seconds, or 2 minutes and 13 seconds}. \]

Since the time is 21:58:00, or 2 minutes before 10:00, she cannot make it to Andersen’s in time, since the clock there is accurate, so he will close 13 seconds before she gets there.

But McShane’s clock is 5 minutes slow, so it shows 7 minutes to 10:00 when she leaves home. Thus she can make it to McShane’s with 3 minutes to spare.
Problem of the Week
Problem B

Find the ‘Primadromes’

A palindrome is a number which is the same if its digits are reversed. For example, 131 is a palindrome, 133 is not.

A prime number is a natural number greater than one which has only two factors, the number 1 and itself. For example, 17 is a prime number because it has factors 1 and 17; 21 is not a prime number because it has factors 1, 3, 7, and 21.

a) What number other than 1 is a factor of every two-digit palindrome?

b) How many two-digit palindromes can be written as the product of two prime numbers?

Check out other CEMC resources at:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Number Sense
Problem of the Week  
Problem B and Solution  

Find the ‘Primadromes’

A palindrome is a number which is the same if its digits are reversed. For example, 131 is a palindrome, 133 is not.

A prime number is a natural number greater than one which has only two factors, the number 1 and itself. For example, 17 is a prime number because it has factors 1 and 17; 21 is not a prime number because it has factors 1, 3, 7, and 21.

a) What number other than 1 is a factor of every two-digit palindrome?

b) How many two-digit palindromes can be written as the product of two prime numbers?

Solution

a) The two-digit palindromes are 11, 22, 33, 44, ... , 88, 99. Each of these numbers has 11 as a factor.

b) Since 11 is a prime number, we need to pick only the two-digit palindromes that are $11 \times$ a prime number. They are  
\[ 22 = 2 \times 11, \quad 33 = 3 \times 11, \quad 55 = 5 \times 11, \quad 77 = 7 \times 11. \]

Thus there are 4 two-digit palindromes which can be written as a product of two prime numbers.

Note that since 11 is a prime number itself (having only the factors 1 and 11), it cannot be written as the product of two prime numbers because 1 is not a prime number.
Problem of the Week
Problem B

This Is No ‘Joak’

A team of scientists surveying a woodlot found that there were 4 oak trees for every 10 pine trees. If they counted 24 more pine trees than oak trees in total, how many oak trees were there in the woodlot?

HINTS:

1. What is the ratio of pine trees to oak trees?

2. The number of pine trees must be a whole number. Could there be an odd number of oak trees?

Check out other CEMC resources here:
http://cemc.uwaterloo.ca/resources/resources.html

STRAND: Number Sense, Pattern/Algebra
Problem of the Week
Problem B and Solutions

This Is No ‘Joak’

Problem

A team of scientists surveying a woodlot found that there were 4 oak trees for every 10 pine trees. If they counted 24 more pine trees than oak trees in total, how many oak trees were there in the woodlot?

Solution 1

Since the ratio of oak to pine trees is 4:10, there are \(\frac{10}{4} = 2.5\) times as many pine trees as oak trees in the woodlot. Thus we seek two numbers which satisfy this ratio, but also have a difference of 24. We make a table of possible combinations.

Using HINT 2, we note that the number of pines must be a whole number. Thus we cannot have an odd number of oaks, since an odd number times 2.5 would not be a whole number (e.g., \(9 \times 2.5 = 22.5\)). Also, since the difference, 24, is an even number, the number of pines cannot be odd either. This means the number of oaks cannot be 6, 10, 14,... since multiplying these by 2.5 gives 15, 25, 35,... . Thus the number of oaks must be a multiple of 4.

The table shows that there must be 16 oaks and 40 pines, a difference of 24.

Solution 2

Think of the trees in sets of 14 trees (4 oaks and 10 pines). Then, in each set, there are 6 fewer oaks than pines. Since there are 24 fewer oaks in total \((6 \times 4)\), there must be four such sets of 14 trees, or \(4 \times 14 = 56\) trees in total. Thus there are \(4 \times 4 = 16\) oak trees, since there are 4 oaks in each set.

Solution 3 (An algebraic solution for those interested.)

If \(m\) is the number of oaks, and \(n\) is the number of pines, then \(n - m = 24\). \([1]\)

But we know \(n = 2.5 \times m\). Substituting this into equation \([1]\) gives \(2.5m - m = 24\), or \(1.5m = 24\). Solving for \(m\) gives \(m = 24 \div 1.5\), or \(m = 16\).
Problem of the Week
Problem B
It All Adds Up!

What is the least number of consecutive whole numbers that have a sum of 1 000? What are these numbers? (Recall that consecutive numbers follow one after the other, such as 239, 240, 241, or 333, 334, 335, 336, 337.)

For example, $499 + 500 = 999$, and $500 + 501 = 1001$, so two consecutive numbers do not work. Do you think three consecutive numbers will work?

Check out other CEMC resources here:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Number Sense
Problem of the Week
Problem B and Solution
It All Adds Up!

Problem

What is the least number of consecutive whole numbers that have a sum of 1000? What are these numbers?

For example, $499 + 500 = 999$, and $500 + 501 = 1001$, so two consecutive numbers does not work. Do you think three consecutive numbers will work?

Solution

To check whether three consecutive numbers will work, we note that 1000 divided by 3 is approximately 333. But

$$332 + 333 + 334 = 999 \text{ (too low)}, \text{ and } 333 + 334 + 335 = 1002 \text{ (too high)},$$

so it appears three consecutive numbers with a sum of 1000 cannot be found either. Similarly, if we try four consecutive numbers, we expect terms around 250. But

$$248 + 249 + 250 + 251 = 998, \text{ and } 249 + 250 + 251 + 252 = 1002,$$

so four consecutive numbers won’t work either.

Finally, since 1000 divided by 5 is 200, we try for five consecutive numbers:

$$198 + 199 + 200 + 201 + 202 = 1000.$$

Thus we see that 5 consecutive numbers will sum to 1000.

Something to Think About

Do you think four consecutive numbers could ever have an odd sum? Why, or why not?
Problem of the Week
Problem B
Down on the Farm

The Grade 1 students at Upper Drive Elementary School recently went to visit a local farm.

While they were there, they saw cows, horses, dogs, and baby chicks (at least one of each). They counted a total of 27 animals. The farmer told them that the animals have 76 legs altogether, and that there are 16 chicks, and 4 more dogs than cows.

How many were there of each animal? (More than one solution is possible.)

<table>
<thead>
<tr>
<th>Animals</th>
<th>No. of Animals</th>
<th>No. of legs per Animal</th>
<th>Total No. of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cows</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horses</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dogs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chicks</td>
<td></td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td>27</td>
<td></td>
<td>76</td>
</tr>
</tbody>
</table>

Check out other CEMC resources here:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Number Sense, Pattern/Algebra
Problem of the Week
Problem B and Solution

Down on the Farm

Problem
The Grade 1 students at Upper Drive Elementary School recently went to visit a local farm. While they were there, they saw cows, horses, dogs, and baby chicks (at least one of each). They counted a total of 27 animals. The farmer told them that the animals have 76 legs altogether, and that there are 16 chicks, and 4 more dogs than cows.

How many were there of each animal? (More than one solution is possible.)

Solution
Since there are 16 chicks, there are \(27 - 16 = 11\) four-legged animals.
But there are 4 more dogs than cows. Thus there could be 1, 2, or 3 cows, giving 5, 6, or 7 dogs, and 5, 3, or 1 horses, respectively. The table below gives the data.

<table>
<thead>
<tr>
<th>Animals</th>
<th>No. of Animals</th>
<th>No. of legs per Animal</th>
<th>Total No. of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cows</td>
<td>1 or 2 or 3</td>
<td>4</td>
<td>4 or 8 or 12</td>
</tr>
<tr>
<td>Horses</td>
<td>5 or 3 or 1</td>
<td>4</td>
<td>20 or 12 or 4</td>
</tr>
<tr>
<td>Dogs</td>
<td>5 or 6 or 7</td>
<td>4</td>
<td>20 or 24 or 28</td>
</tr>
<tr>
<td>Chicks</td>
<td>16</td>
<td>2</td>
<td>32</td>
</tr>
<tr>
<td>Totals</td>
<td>27</td>
<td></td>
<td>76</td>
</tr>
</tbody>
</table>

There are three possible solutions:

i) 1 cow, 5 horses, 5 dogs and 16 chicks; or

ii) 2 cows, 3 horses, 6 dogs and 16 chicks; or

iii) 3 cows, 1 horse, 7 dogs and 16 chicks.

Something to Think About
Can you still solve the problem if the farmer doesn’t reveal the number of chicks? Here’s a start: Suppose instead that all 27 animals have 4 legs. Then there would be \(27 \times 4 = 108\) legs in total. But really there are only 76 legs, so there are \(108 - 76 = 32\) ‘extra’ legs. How many chicks must there be?
Problem of the Week
Problem B

Short End of the Stick

A stick is cut into three pieces, each having a different length. Each piece (except the shortest) is twice as long as another piece.

What fraction of the whole stick is each piece?

Check out other CEMC resources here:
http://cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense
Problem of the Week
Problem B and Solution
Short End of the Stick

Problem

A stick is cut into three pieces, each having a different length. Each piece (except the shortest) is twice as long as another piece.

What fraction of the whole stick is each piece?

Solution

Think of the three pieces laid out in a row, from shortest to longest. Label the shortest piece $length = 1$. Then the middle piece must have $length = 2$, since it must be twice as long as the shortest piece. Finally, the longest piece must have $length = 4$, since it must be twice as long as the middle piece.

In these units, the length of the whole stick is 7. Thus the shortest piece is $\frac{1}{7}$ of the length of the whole stick. The middle piece is $\frac{2}{7}$ of the length of the whole stick. The longest piece is $\frac{4}{7}$ of the length of the whole stick.
Problem of the Week
Problem B
Spaced Out Posts

Farmer Jack wants to fence off one side of his driveway. He has 20 fenceposts, each 15 cm in diameter, and the driveway is 117 m long.

If the posts are to be evenly spaced, how far apart should they be (i.e., what is the distance \( L \))?  

\[ \text{Check out other CEMC resources at:} \]
http://cemc.uwaterloo.ca/resources/resources.html

**Strand:** Measurement
Problem of the Week  
Problem B and Solution  
Spaced Out Posts

Problem

Farmer Jack wants to fence off one side of his driveway. He has 20 fenceposts, each 15 cm in diameter, and the driveway is 117 m long.

If the posts are to be evenly spaced, how far apart should they be (i.e., what is the distance L)?

Solution

Think of the posts in a row along the 117 m driveway.

Between the 20 posts there will be 19 spaces of length L. However, the posts themselves will take some space; since there are 20 posts of width 15 cm = 0.15 m, they will occupy 20 × 0.15 m = 3 m.

Thus the total distance remaining for the space between the posts is 117 m − 3 m = 114 m.

Since there are 19 spaces of length L m, we know that 19 × L m = 114 m. Thus the length of each space must be  \[ L = 114 \div 19 = 6 \text{ m}. \]
Problem of the Week
Problem B

What, No 6?

Fill in the place value holders below to create a specific question and its answer. Use each of the digits 0, 1, 2, 3, 4, 5, 7, 8, 9 once, but do not use 0 as the first digit of any number.

\[
\begin{array}{c}
\boxed{} \\
+ \\
\boxed{}
\end{array}
\]

Can you find more than one question that works?

The number tiles below can be cut out and used to help solve this problem.

\[
\begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 7 & 8 & 9 \\
\end{array}
\]

Check out other CEMC resources here:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Number Sense
Problem of the Week
Problem B and Solution

What, No 6?

Problem
Fill in the place value holders at the right to create a specific question and its answer. Use each of the digits 0, 1, 2, 3, 4, 5, 7, 8, 9 once, but do not use 0 as the first digit of any number.

Can you find more than one question that works?

Solution
To start, consider the possibilities for the sum of a 3-digit and a 2-digit number. The only way to obtain a 4-digit number is to place a 9 in the hundreds digit of the first addend. Then the two tens digits of the addends must sum to a number greater than 10 so that there is a 1 to carry, giving the first two digits of the sum as 1 0. Thus you have a start to the solution as shown at the right.

Now consider the possible choices of tens digits for the addends, which must sum to at least 10. Summing 7 + 3 or 8 + 2 gives 10, but we have already used the 0 so these won’t work. Also, no pair of 2, 3, 4, 5 has a sum of 10, so such pairs are not candidates for the tens column. The useful pairs for the tens column turn out to be 4 and 8, 5 and 7, or 4 and 7, as is revealed by a bit of ‘guess and check’.

Here are the four possible solutions obtained using 4 and 8 in the tens column:

\[
\begin{array}{c}
9 & 4 & 5 \\
+ & 8 & 7 \\
\hline
1 & 0 & 3 & 2
\end{array}
\]
\[
\begin{array}{c}
9 & 8 & 5 \\
+ & 4 & 7 \\
\hline
1 & 0 & 3 & 2
\end{array}
\]
\[
\begin{array}{c}
9 & 8 & 7 \\
+ & 4 & 5 \\
\hline
1 & 0 & 3 & 2
\end{array}
\]
\[
\begin{array}{c}
9 & 4 & 7 \\
+ & 8 & 5 \\
\hline
1 & 0 & 3 & 2
\end{array}
\]

Eight further solutions can be obtained by using 5 and 7 in the tens column or using 4 and 7 in the tens column.
Patterning & Algebra
Problem of the Week
Problem B
Double Trouble

Riley Waters has decided to join Facebook™. A week after posting his profile, he discovers he has two friends! After another week, his number of friends has doubled to four. At the end of the third week, his total number of friends has doubled yet again to 8.

a) If his number of friends continue to double each week, how many friends will Riley have at the end of 6 weeks? Create a T-chart to show your calculations.

b) Continue your chart to show how many friends Riley would have after 10 weeks.

c) How long would it be until Riley has more than 15 000 friends?

d) There are about 34 million people in Canada. How long will it be until Riley is friends with at least that many people?

Check out other CEMC resources at:
http://cemc.uwaterloo.ca/resources/resources.html

**Strand:** Pattern/Algebra
Problem of the Week
Problem B and Solution

Double Trouble

Problem

Riley Waters has decided to join Facebook™. A week after posting his profile, he discovers he has two friends! After another week, his number of friends has doubled to four. At the end of the third week, his total number of friends has doubled yet again to 8.

a) If his number of friends continue to double each week, how many friends will Riley have at the end of 6 weeks? Create a T-chart to show your calculations.

b) Continue your chart to show how many friends Riley would have after 10 weeks.

c) How long would it be until Riley has more than 15 000 friends?

d) There are about 34 million people in Canada. How long will it be until Riley is friends with at least that many people?

Solution

a)

<table>
<thead>
<tr>
<th>Weeks</th>
<th>Friends</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>32</td>
</tr>
<tr>
<td>6</td>
<td>64</td>
</tr>
</tbody>
</table>

b)

<table>
<thead>
<tr>
<th>Weeks</th>
<th>Friends</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>128</td>
</tr>
<tr>
<td>8</td>
<td>256</td>
</tr>
<tr>
<td>9</td>
<td>512</td>
</tr>
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<td>10</td>
<td>1024</td>
</tr>
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<td>11</td>
<td>2048</td>
</tr>
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<td>13</td>
<td>8192</td>
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<td>14</td>
<td>16384</td>
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c)

<table>
<thead>
<tr>
<th>Weeks</th>
<th>Friends</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>2048</td>
</tr>
<tr>
<td>12</td>
<td>4096</td>
</tr>
<tr>
<td>13</td>
<td>8192</td>
</tr>
<tr>
<td>14</td>
<td>16384</td>
</tr>
</tbody>
</table>

From the tables, a) Ryan will have 64 friends after 6 weeks, b) 1 024 friends after 10 weeks, and c) he will have more than 15 000 friends after 14 weeks.

d) Using a calculator reveals that Ryan has 33 554 432 friends after 25 weeks, and 67 108 864 after 26 weeks. (Entering $2 \times 2$ and repeatedly pressing the “=” sign on some calculators may help.)
Problem of the Week

Problem B

Uncharted Waters

The diagram below is a pattern on a 100s chart. (Only the numbers 1 and 100 are visible.)

100’s Chart

a) Describe the number pattern in as many ways as possible.

b) If the pattern is extended, will the number 76 be in the pattern set?

Check out other CEMC resources here:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Number Sense, Pattern/Algebra
Problem of the Week
Problem B and Solution

Uncharted Waters

Problem

The diagram at the right is a pattern on a 100s chart. (Only the numbers 1 and 100 are visible.)

a) Describe the number pattern in as many ways as possible.

b) If the pattern is extended, will the number 76 be in the pattern set?

Solution

a) The pattern can be described in several ways.

(i) The number pattern is increasing by 3 for each shaded square to the right, starting at 42 and ending at 69.

(ii) The number pattern is decreasing by 3 to the left, starting at 69 and ending at 42.

(iii) Every third square horizontally is shaded in the chart, with no blank, one blank, or two blank squares on the left edge.

b) The number 76 will not be shaded if the pattern is extended (cross-hatched squares), since $76 - 69 = 7$ is not a multiple of 3.

Something to Think About

Find a number that will be shaded in the second last row from the bottom of the 100s chart.
Problem of the Week
Problem B
This Is No ‘Joak’

A team of scientists surveying a woodlot found that there were 4 oak trees for every 10 pine trees. If they counted 24 more pine trees than oak trees in total, how many oak trees were there in the woodlot?

HINTS:

1. What is the ratio of pine trees to oak trees?

2. The number of pine trees must be a whole number. Could there be an odd number of oak trees?

Check out other CEMC resources here: http://cemc.uwaterloo.ca/resources/resources.html

STRAND: Number Sense, Pattern/Algebra
Problem of the Week  
Problem B and Solutions  
This Is No ‘Joak’

Problem

A team of scientists surveying a woodlot found that there were 4 oak trees for every 10 pine trees. If they counted 24 more pine trees than oak trees in total, how many oak trees were there in the woodlot?

Solution 1

Since the ratio of oak to pine trees is 4:10, there are \( \frac{10}{4} = 2.5 \) times as many pine trees as oak trees in the woodlot. Thus we seek two numbers which satisfy this ratio, but also have a difference of 24. We make a table of possible combinations.

Using HINT 2, we note that the number of pines must be a whole number. Thus we cannot have an odd number of oaks, since an odd number times 2.5 would not be a whole number (e.g., \( 9 \times 2.5 = 22.5 \)). Also, since the difference, 24, is an even number, the number of pines cannot be odd either. This means the number of oaks cannot be 6, 10, 14,... since multiplying these by 2.5 gives 15, 25, 35,... . Thus the number of oaks must be a multiple of 4.

The table shows that there must be 16 oaks and 40 pines, a difference of 24.

Solution 2

Think of the trees in sets of 14 trees (4 oaks and 10 pines). Then, in each set, there are 6 fewer oaks than pines. Since there are 24 fewer oaks in total \((6 \times 4)\), there must be four such sets of 14 trees, or \(4 \times 14 = 56\) trees in total. Thus there are \(4 \times 4 = 16\) oak trees, since there are 4 oaks in each set.

Solution 3 (An algebraic solution for those interested.)

If \(m\) is the number of oaks, and \(n\) is the number of pines, then \(n - m = 24\). [1]
But we know \(n = 2.5 \times m\). Substituting this into equation [1] gives \(2.5m - m = 24\), or \(1.5m = 24\). Solving for \(m\) gives \(m = 24 \div 1.5\), or \(m = 16\).
Problem of the Week  
Problem B  
Down on the Farm

The Grade 1 students at Upper Drive Elementary School recently went to visit a local farm.

While they were there, they saw cows, horses, dogs, and baby chicks (at least one of each). They counted a total of 27 animals. The farmer told them that the animals have 76 legs altogether, and that there are 16 chicks, and 4 more dogs than cows.

How many were there of each animal? (More than one solution is possible.)

<table>
<thead>
<tr>
<th>Animals</th>
<th>No. of Animals</th>
<th>No. of legs per Animal</th>
<th>Total No. of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cows</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horses</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dogs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chicks</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td>27</td>
<td></td>
<td>76</td>
</tr>
</tbody>
</table>

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Strand: Number Sense, Pattern/Algebra
Problem of the Week
Problem B and Solution
Down on the Farm

Problem
The Grade 1 students at Upper Drive Elementary School recently went to visit a local farm. While they were there, they saw cows, horses, dogs, and baby chicks (at least one of each). They counted a total of 27 animals. The farmer told them that the animals have 76 legs altogether, and that there are 16 chicks, and 4 more dogs than cows.

How many were there of each animal? (More than one solution is possible.)

Solution
Since there are 16 chicks, there are \(27 - 16 = 11\) four-legged animals.

But there are 4 more dogs than cows. Thus there could be 1, 2, or 3 cows, giving 5, 6, or 7 dogs, and 5, 3, or 1 horses, respectively. The table below gives the data.

<table>
<thead>
<tr>
<th>Animals</th>
<th>No. of Animals</th>
<th>No. of legs per Animal</th>
<th>Total No. of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cows</td>
<td>1 or 2 or 3</td>
<td>4</td>
<td>4 or 8 or 12</td>
</tr>
<tr>
<td>Horses</td>
<td>5 or 3 or 1</td>
<td>4</td>
<td>20 or 12 or 4</td>
</tr>
<tr>
<td>Dogs</td>
<td>5 or 6 or 7</td>
<td>4</td>
<td>20 or 24 or 28</td>
</tr>
<tr>
<td>Chicks</td>
<td>16</td>
<td>2</td>
<td>32</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td><strong>27</strong></td>
<td><strong>76</strong></td>
<td></td>
</tr>
</tbody>
</table>

There are three possible solutions:

i) 1 cow, 5 horses, 5 dogs and 16 chicks; or

ii) 2 cows, 3 horses, 6 dogs and 16 chicks; or

iii) 3 cows, 1 horse, 7 dogs and 16 chicks.

Something to Think About
Can you still solve the problem if the farmer doesn’t reveal the number of chicks? Here’s a start: Suppose instead that all 27 animals have 4 legs. Then there would be \(27 \times 4 = 108\) legs in total. But really there are only 76 legs, so there are \(108 - 76 = 32\) ‘extra’ legs. How many chicks must there be?