The problems in this booklet are organized into strands. A problem often appears in multiple strands. The problems are suitable for most students in Grade 5 or higher.
Data Management & Probability
Problem of the Week
Problem B
A Tangle of Rectangles

Let’s see how the length and width of a rectangle change if we keep the area the same.

a) On graph paper, draw all rectangles which have an area of 12 square units and have sides which are whole numbers. Fill in the side lengths in the table.

<table>
<thead>
<tr>
<th>Dimensions of Rectangles with Area 12 square units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

b) On the graph below, plot the points representing the width and length of each of the rectangles from part a). Join the points as smoothly as possible. Describe the shape of the resulting graph.

Check out other CEMC resources for Grades 4 to 6 here:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Measurement, Data Management
Problem and Solution

Let’s see how the length and width of a rectangle change if we keep the area the same.

a) On graph paper, draw all rectangles which have an area of 12 square units and have sides which are whole numbers.

The permissible side lengths are shown in the table below. Check that you have drawn all six rectangles.

| Dimensions of Rectangles with Area 12 square units |
|------------------|-------|-------|-------|-------|-------|-------|
| **Width**        | 1     | 2     | 3     | 4     | 6     | 12    |
| **Length**       | 12    | 6     | 4     | 3     | 2     | 1     |

b) On the graph below, plot the points representing the width and length of each of the rectangles from part a). Join the points as smoothly as possible. Describe the shape of the resulting graph.

As the width increases the length decreases, giving a graph that curves downward; the points’ coordinates always satisfy $Width \times Length = 12$.
Problem of the Week
Problem B
The Munsch Off

In November, a ‘Munsch Off’ was held at Mountainside Public School, during which five Robert Munsch books were read to all the students in each classroom. On the last day, the staff and students each voted for their favourite of the five books. The results were as follows:

<table>
<thead>
<tr>
<th>Book</th>
<th>Number of Votes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Primary</td>
</tr>
<tr>
<td>Millicent and the Wind</td>
<td>10</td>
</tr>
<tr>
<td>Moose</td>
<td>160</td>
</tr>
<tr>
<td>The Dark</td>
<td>80</td>
</tr>
<tr>
<td>Get Out of Bed</td>
<td>23</td>
</tr>
<tr>
<td>From Far Away</td>
<td>5</td>
</tr>
</tbody>
</table>

a) It seems clear that Moose was the favourite book. Which book was second favourite? Third favourite?

b) John claims that about $\frac{1}{3}$ of the votes were for Get Out of Bed. Is that a reasonable estimate?

c) If all 40 staff members voted, and there are 604 students at Mountainside, how many students did not vote in the Munsch Off?

Check out other CEMC resources for Grades 4 to 6 here:
http://cemc.uwaterloo.ca/resources/resources.html

**STRAND:** Data Management, Number Sense
Problem of the Week
Problem B and Solution
The Munsch Off

Problem
In November, a ‘Munsch Off’ was held at Mountainside Public School, during which five Robert Munsch books were read to all the students in each classroom. On the last day, the staff and students each voted for their favourite of the five books. The results were as follows, added to give totals in the last column.

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</table>

a) It seems clear that Moose was the favourite book. Which book was second favourite? Third favourite?

b) John claims that about \( \frac{1}{9} \) of the votes were for Get Out of Bed. Is that a reasonable estimate?

c) If all 40 staff members voted, and there are 604 students at Mountainside, how many students did not vote in the Munsch Off?

Solution

a) From the Total Votes column in the chart above, the second favourite book was The Dark, and the third favourite book was From Far Away.

b) The number of votes for Get Out of Bed was 63; the total votes cast was \( 30 + 275 + 137 + 63 + 78 = 583 \). So the fraction of people who voted for Get Out of Bed was \( \frac{63}{583} \approx 0.108 \) and \( \frac{1}{9} \approx 0.111 \). So, \( \frac{1}{9} \) is a reasonable estimate for the fraction of total votes cast for Get Out of Bed.

c) We know that every staff member voted. So there are 40 votes out of 583 that were cast by staff. Therefore, \( 583 - 40 = 543 \) students voted. Since there are 604 students, we know that \( 604 - 543 = 61 \) students didn’t vote.
Problem of the Week
Problem B
What’s Your Game?

The following sports data is for the year 2009.

- The average salary for an NBA (National Basketball Association) player was $4.9 million.
- NFL (National Football League) players averaged $1.9 million.
- NHL (National Hockey League) players made about $2.6 million.
- MLB (Major League Baseball) players earned $3.1 million.

Based on this data, answer the following questions.

a) There are 82 regular season games in the NBA and the NHL, 16 in the NFL, and 162 in MLB. What is the average salary per game for the players in each of these four sports?

b) In 2009, one individual, Kevin Garnett, earned $24.75 million playing for the NBA’s Boston Celtics in 71 games. If he played an average of 32.8 minutes per game, how much money did he make per minute played?

c) Suppose his salary was based on the number of rebounds he made, an average of 9.2 rebounds per game. How much money did he make per rebound?

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**Strand:** Data Management
Problem of the Week
Problem B and Solution
What’s Your Game?

Problem

The following sports data is for the year 2009.

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c) Suppose his salary was based on the number of rebounds he made, an average of 9.2 rebounds per game. How much money did he make per rebound?

Solution

a) Since there are 82 regular season games in the NBA and the average salary for an NBA player was $4.9 million, then the average salary per game was $4,900,000 ÷ 82 = $59,756.10.

In the NHL there are also 82 regular season games and the average salary was $2.6 million. Thus, the average salary per game was $2,600,000 ÷ 82 = $31,707.32.

Continued...
In the NFL there are 16 regular season games and the average salary per player was $1.9 million. Then, the average salary per game was $1\,900\,000 \div 16 = $118\,750.

Lastly, since there are 162 regular season games in MLB and the average salary was $3.1 million, then the average salary per game was $3\,100\,000 \div 162 = $19\,135.80.

b) Since Kevin Garnett earned $24.75 million playing 71 games, he earned $24\,750\,000 \div 71 = $348\,591.55 per game. In each game, he played an average of 32.8 minutes, so he earned $348\,591.55 \div 32.8 = $10\,627.79 per minute played.

c) In part b) we discovered that Kevin Garnett earned on average $348\,591.55 per game. If he had an average of 9.2 rebounds per game, then he earned $348\,591.55 \div 9.2 = $37\,890.39 per rebound.
Problem of the Week

Problem B

Pair-a-dice

A pair of dice have sides labelled as follows: one has the numbers 1, 2, ... , 6, and the other the numbers 7, 8, ... , 12. Muhammed wins the toss if he rolls both dice and gets two numbers such that one is a multiple of the other. Ally wins if she gets one even and one odd number.

Is this a fair game?

NOTE: A game is fair if all players have an equal chance of winning.

Check out other CEMC resources for Grades 4 to 6 here:
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STRAND: Probability
Problem of the Week
Problem B and Solution

Pair-a-dice

Problem
A pair of dice have sides labelled as follows: one has the numbers 1, 2, ... , 6, and the other the numbers 7, 8, ... , 12. Muhammed wins the toss if he rolls both cubes and gets two numbers such that one is a multiple of the other. Ally wins if she gets one even and one odd number.

Is this a fair game?

Note: A game is fair if all players have an equal chance of winning.

Solution
To determine whether the game is fair, we must see whether both Ally and Muhammed have an equal chance of winning.
Recall that the theoretical probability, \( P \), of an event is a fraction, namely:

\[
P = \frac{\text{Number of desired outcomes}}{\text{Total number of outcomes possible}}
\]

A simple way to count these two numbers is by using a tree to write down the possibilities.
Ally wins if she gets one even and one odd number. Since there are 3 even and 3 odd numbers on each die, she has an equal chance (\( \frac{3}{6} \)) of tossing an even (or odd) number on each die. Thus we can depict the possible events as follows, with \( E \) meaning an even number, and \( O \) an odd number:

First Die

Second Die

Of the four possible outcomes, two are the desired combinations of one even and one odd number. Thus, Ally’s probability of winning is

\[
P_A = \frac{2}{4} = \frac{1}{2}
\]

continued · · ·
Muhammed wins the game if the two numbers he tosses are such that one is a multiple of the other. We can depict the possible events as follows:

First Die
Second Die
1
2
3
4
5
6
1
2
3
4
5
6
1
2
3
4
5
6

Now we can determine how many times the first die toss gives a multiple of the second die (the desired outcomes).

- Since 7 and 11 are primes, they are multiples of only 1 (two desired outcomes);
- 8 is a multiple of 1, 2, and 4 (three desired outcomes);
- 9 is a multiple of 1 and 3 (two desired outcomes);
- 10 is a multiple of 1, 2, and 5 (three desired outcomes);
- 12 is a multiple of 1, 2, 3, 4, and 6 (five desired outcomes).

Thus the number of desired outcomes is $2 + 3 + 2 + 3 + 5 = 15$, out of a total of 36 possible outcomes. So Muhammed’s probability of winning is

$$P_M = \frac{15}{36} = \frac{5}{12} < \frac{1}{2}$$

Since Ally has a greater chance of winning, the game is not fair.

**Suggestion:** In the above solution, we have assumed that the first die is the one with the digits 7 to 12. Try to construct the trees for the case when the first die has the digits 1 to 6. Hence show that it doesn’t matter which is assumed to be first or second.

**Comment:** Ally’s chance of winning can also be portrayed by writing out all 36 possibilities, as we have done for Muhammed’s chances, and then counting the total number of combinations of one even and one odd number (18 out of 36) to give the probability $\frac{1}{2}$. 
Geometry
&
Spatial Sense

TAKE ME TO THE COVER
Problem of the Week
Problem B
ELAS DC KNUP

A CD sale is advertised by a sign in the store window, as shown below.

PUNK CD SALE

a) How many letters of the alphabet (in this font) would be the same viewed through the front of the window (outside the store) as through the back of the window (inside the store)? Which letters are they?

b) Would the letters you found in part a) look the same if you viewed them in a mirror? What axis of symmetry do these letters have?

c) Which letters, if any, look the same if you invert them vertically (i.e., turn them upside down)? For example, A doesn’t work \( \text{A} \), but C does \( \text{C} \). If so, what axis of symmetry do these letters have?

Here is the alphabet for the font of the sign in the window:

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Check out other CEMC resources for Grades 4 to 6 here:
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Strand: Geometry
Problem of the Week
Problem B and Solution

ELAS DC KNUP

Problem

A CD sale is advertised by a sign in the store window, as shown below.

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a) How many letters of the alphabet (in this font) would be the same viewed through the front of the window (outside the store) as through the back of the window (inside the store)? Which letters are they?

b) Would the letters you found in part a) look the same if you viewed them in a mirror? What axis of symmetry do these letters have?

c) Which letters, if any, look the same if you invert them vertically (i.e., turn them upside down)? For example, A doesn’t work $\overline{A}$, but C does $\overline{C}$. If so, what axis of symmetry do these letters have?

Here is the alphabet for the font of the sign in the window:

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Solution

a) There are 11 of the letters in the alphabet that would be the same viewed through the front of the window as through the back of the window. They are A, H, I, M, O, T, U, V, W, X, and Y.

b) Yes, each of the letters found in part a) look the same if you viewed them in a mirror since they all have a vertical axis of symmetry.

c) The letters B, C, D, E, H, I, O, and X all look the same if you invert them vertically since they have a horizontal axis of symmetry midway through their height.
Problem of the Week
Problem B

Polygons: How Many Diagonals?

A diagonal of a regular polygon is a straight line joining two vertices which are not beside each other (adjacent). Thus a square has two diagonals, and a regular pentagon has five, as shown at the right.

a) How many diagonals does a regular hexagon have? (Draw them.)
b) How many diagonals does a regular heptagon have? (Draw them.)
c) How many diagonals does a regular octagon have?

d) Complete the table at right, up to 8 sides, using the information from a), b), c). Then predict the number of diagonals for a regular dodecagon (a 12-sided polygon), using the pattern in the table. Draw the diagonals on the figure below to see whether your prediction is correct.

<table>
<thead>
<tr>
<th>Number of sides</th>
<th>Number of diagonals</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

Extension

Suppose the polygons do not have to be regular, i.e., they could have sides of different lengths. Would your answers to the above problems change?

Check out other CEMC resources for Grades 4 to 6 here:
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Strand: Geometry, Pattern/Algebra
Problem of the Week
Problem B and Solution

Polygons: How Many Diagonals?

Problem and Solution
A diagonal of a regular polygon is a straight line joining two vertices which are not beside each other (adjacent). Thus a square has two diagonals, and a regular pentagon has five, as shown at the right.

a) How many diagonals does a regular hexagon have? (Draw them.)
b) How many diagonals does a regular septagon have? (Draw them.)
c) How many diagonals does a regular octagon have?

The hexagon has \(3 + 3 + 2 + 1 = 9\) diagonals; the septagon has \(4 + 4 + 3 + 2 + 1 = 14\) diagonals; and the octagon has \(5 + 5 + 4 + 3 + 2 + 1 = 20\) diagonals, as illustrated in the diagrams below.

Here is a detailed breakdown for the octagon.
d) Complete the table at right, up to 8 sides, using the information from a), b), c). Then predict the number of diagonals for a regular dodecagon (a 12-sided polygon), using the pattern in the table. Draw the diagonals on the figure below to see whether your prediction is correct.

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<td>0</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
</tr>
<tr>
<td>12</td>
<td>54</td>
</tr>
</tbody>
</table>

We can predict that the dodecagon has 54 diagonals by noting that the difference between successive numbers of diagonals increases by one at each step. Thus from 4 to 5 sides is an increase of 3 diagonals, from 5 to 6 is an increase of 4, from 6 to 7 is an increase of 5, and from 7 to 8 is an increase of 6. Thus a polygon with 9 sides will have $20 + 7 = 27$ diagonals, with 10 sides, $27 + 8 = 35$ diagonals, with 11 sides $35 + 9 = 44$ diagonals, and with 12 sides $44 + 10 = 54$ diagonals.

Following the pattern established for the octagon, we see that the dodecagon will have $12 - 3 = 9$ diagonals from each of the first two vertices (red and blue on the diagrams below), 8 from vertex 3 (black on the diagrams), 7 from vertex 4 (dashed red on the last diagram), 6 from vertex 5 (dashed blue on the last diagram), 5 from vertex 6 (dashed black on the last diagram), etcetera, giving a total of $9 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 54$ distinct diagonals.
Problem of the Week
Problem B
Connect the Dots

Each circle on the following page has a number of dots on the circumference. Starting with one dot and moving clockwise, connect every second dot with a straight line. If, when you return to your starting point, there are still ‘unconnected’ dots, pick one and repeat the construction until all the dots are connected. Shade the shape in the middle of the circle when you are done, and complete the table below based on your circle diagrams. Then think about the question:

If you had a circle with 12 dots, what would the middle shape be called?

<table>
<thead>
<tr>
<th>Number of Dots</th>
<th>Diagram of the centre shape created by all the lines</th>
<th>Name of the shape</th>
<th>Could you connect all the dots starting from just one point?</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td><img src="image" alt="Pentagon" /></td>
<td>pentagon</td>
<td>yes</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
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STRAND: Geometry
Problem of the Week
Problem B and Solution

Connect the Dots

Problem and Solution

Each circle on the following page has a number of dots on the circumference. Starting with one dot and moving clockwise, connect every other dot with a straight line. If, when you return to your starting point, there are still ‘unconnected’ dots, pick one and repeat the construction until all the dots are connected. Shade the shape in the middle of the circle when you are done, then complete the table below based on your circle diagrams.

The completed table is below; the figures are on the next page.

<table>
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<td><img src="image" alt="Pentagon Diagram" /></td>
<td>pentagon</td>
<td>yes</td>
</tr>
<tr>
<td>6</td>
<td><img src="image" alt="Hexagon Diagram" /></td>
<td>hexagon</td>
<td>no</td>
</tr>
<tr>
<td>7</td>
<td><img src="image" alt="Heptagon Diagram" /></td>
<td>heptagon</td>
<td>yes</td>
</tr>
<tr>
<td>8</td>
<td><img src="image" alt="Octagon Diagram" /></td>
<td>octagon</td>
<td>no</td>
</tr>
<tr>
<td>9</td>
<td><img src="image" alt="Nonagon Diagram" /></td>
<td>nonagon</td>
<td>yes</td>
</tr>
<tr>
<td>10</td>
<td><img src="image" alt="Decagon Diagram" /></td>
<td>decagon</td>
<td>no</td>
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</table>
If you had a circle with 12 dots, what would the middle shape be called?

The shape would be a 12-sided polygon, called a **dodecagon**.
Measurement
Problem of the Week
Problem B
A Tangle of Rectangles

Let’s see how the length and width of a rectangle change if we keep the area the same.

a) On graph paper, draw all rectangles which have an area of 12 square units and have sides which are whole numbers. Fill in the side lengths in the table.

**Dimensions of Rectangles with Area 12 square units**

<table>
<thead>
<tr>
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<th></th>
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<tr>
<td>Length</td>
<td></td>
<td></td>
<td></td>
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b) On the graph below, plot the points representing the width and length of each of the rectangles from part a). Join the points as smoothly as possible. Describe the shape of the resulting graph.

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**Strand:** Measurement, Data Management
Problem of the Week
Problem B and Solution

A Tangle of Rectangles

Problem and Solution

Let’s see how the length and width of a rectangle change if we keep the area the same.

a) On graph paper, draw all rectangles which have an area of 12 square units and have sides which are whole numbers.

The permissible side lengths are shown in the table below. Check that you have drawn all six rectangles.

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b) On the graph below, plot the points representing the width and length of each of the rectangles from part a). Join the points as smoothly as possible. Describe the shape of the resulting graph.

As the width increases the length decreases, giving a graph that curves downward; the points’ coordinates always satisfy \( \text{Width} \times \text{Length} = 12 \).
Problem of the Week
Problem B
Spinning Your Wheels!

Sally Belinda has a brand new tricycle. She has measured around each wheel with a piece of string and a ruler, and discovered that each of the two back wheels has a circumference of 50 cm, while the circumference of the larger front wheel is 125 cm.

**Note:** The circumference is the length of the perimeter (i.e., the distance around the edge) of a circle.

---

a) If she rides her tricycle for one short block (110 m), which wheel(s) make(s) a whole number of complete rotations (revolutions)?

b) Would your bicycle wheel make more or fewer rotations than Sally’s front wheel? Measure the circumference of one wheel carefully to confirm your answer.

c) As a tricycle travels along, which wheel(s) always make(s) fewer rotations, the larger front wheel or the smaller back wheels? Explain your reasoning.

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_Check out other CEMC resources for Grades 4 to 6 here:_
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**Strand:** Measurement
Problem of the Week
Problem B and Solution
Spinning Your Wheels!

Problem

Sally Belinda has a brand new tricycle. She has measured around each wheel with a piece of string and a ruler, and discovered that each of the two back wheels has a circumference of 50 cm, while the circumference of the larger front wheel is 125 cm.

NOTE: The circumference is the length of the perimeter of a circle.

a) If she rides her tricycle for one short block (110 m), which wheel(s) make a whole number of complete rotations (revolutions)?

b) Would your bicycle wheel make more or fewer rotations than Sally’s front wheel? Measure the circumference of one wheel carefully to confirm your answer.

c) As a tricycle travels along, which wheel(s) always make(s) fewer rotations, the larger front wheel or the smaller back wheels? Explain your reasoning.

Solution

a) Since there are 100 cm in a metre, we know that 110 m = 11 000 cm. Then we know that for every 125 cm, the larger wheel makes a complete revolution; and in every 50 cm, the smaller wheels make a complete revolution. So, the large wheel makes 88 complete revolutions since $11000 \div 125 = 88$ with no remainder.

We can also determine that each of the 2 smaller wheels makes 220 complete revolutions since $11000 \div 50 = 220$ with no remainder.

Therefore, all 3 wheels make a whole number of complete revolutions.

b) Don’t forget to try this for yourself!

c) Since the circumference of the front wheel is greater than that of the back wheels, the front wheel needs fewer rotations than the back wheels to travel the same distance along the ground.
Problem of the Week
Problem B

We Won’t “Prism” You Know This!

A triangular prism is 2.5 cm tall, with base a right-angle triangle having sides 3 cm, 4 cm, and 5 cm. A square prism is 4 cm tall, with base a square with side length 2 cm.

Which prism has the greater total surface area (the total area of all sides)?

Check out other CEMC resources for Grades 4 to 6 here:
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Strand: Measurement
Problem of the Week
Problem B and Solution

We Won’t “Prism” You Know This!

Problem

A triangular prism is 2.5 cm tall, with base a right-angle triangle having sides 3 cm, 4 cm, and 5 cm. A square prism is 4 cm tall, with base a square with side length 2 cm.

Which prism has greater total surface area (the total area of all sides)?

Solution

We can determine the surface area of the triangular prism by determining the surface area of each side and adding them together.

- Area of each triangular face: $3 \times 4 \times \frac{1}{2} = 6 \text{ cm}^2$
- Area of the vertical sides:
  
  $2.5 \times 5 + 2.5 \times 3 + 2.5 \times 4 = 12.5 + 7.5 + 10.0 = 30 \text{ cm}^2$
- Total Area: $2 \times 6 + 30 = 42 \text{ cm}^2$

We can determine the surface area of the square prism in a similar way.

- Area of top and bottom: $2 \times 2 + 2 \times 2 = 8 \text{ cm}^2$
- Area of the four vertical sides: $4 \times (2 \times 4) = 32 \text{ cm}^2$
- Total Area: $8 + 32 = 40 \text{ cm}^2$

Therefore, the triangular prism has greater total surface area.
Problem of the Week
Problem B

In the Dog House!

Victoria wants a big dog. She and her Dad have plans for the outdoor dog house they are going to build. The plan is a basic rectangular design with an A-shaped peaked roof and a floor. It has the following dimensions (in centimetres):

- the base (floor) of the dog house is 150 cm by 72 cm;
- the height from the floor panel to the peak of the roof is 120 cm;
- the end pieces (from the floor to the bottom of the roof) are 72 cm by 72 cm;
- the slanted edge of the roof is 60 cm in length.

Add the given dimensions to the diagram. Then make a list of the nine pieces of wood required to construct the dog house, and find the area of each. What is the total area (in square centimeters) of all nine pieces of wood?

Next week we’ll use your results, so keep them handy!

Check out other CEMC resources for Grades 4 to 6 here:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Measurement
Problem of the Week
Problem B and Solution
In the Dog House!

Problem

Victoria wants a big dog. She and her Dad have plans for the outdoor dog house they are going to build. The plan is a basic rectangular design with an A-shaped peaked roof and a floor. It has the following dimensions (in centimeters, abbreviated ‘cm’):

- the base (floor) of the dog house is 150 cm by 72 cm;
- the height from the floor panel to the peak of the roof is 120 cm;
- the end pieces (from the floor to the bottom of the roof) are 72 cm by 72 cm;
- the slanted edge of the roof is 60 cm in length.

Add the given dimensions to the diagram. Then make a list of the nine pieces of wood required to construct the dog house, and find the area of each. What is the total area (in square centimeters) of all nine pieces of wood?

Next week we’ll use your results, so keep them handy!

Solution

The above diagram has the dimensions of the dog house labelled. Refer to the chart below for the assembly diagram and areas.

Note that the height of the triangular end piece is the total height minus the height of the square end piece, or $h = 120 \text{ cm} - 72 \text{ cm} = 48 \text{ cm}$. 
<table>
<thead>
<tr>
<th>Piece name</th>
<th>Diagram</th>
<th># pieces</th>
<th>Area of pieces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof</td>
<td><img src="image1.png" alt="Diagram" /></td>
<td>2</td>
<td>$2(60 \times 150) = 18 000 \text{ cm}^2$</td>
</tr>
<tr>
<td>Triangle Piece</td>
<td><img src="image2.png" alt="Diagram" /></td>
<td>2</td>
<td>$2 \times \frac{1}{2}(48 \times 72) = 3 456 \text{ cm}^2$</td>
</tr>
<tr>
<td>End Wall</td>
<td><img src="image3.png" alt="Diagram" /></td>
<td>2</td>
<td>$2(72 \times 72) = 10 368 \text{ cm}^2$</td>
</tr>
<tr>
<td>Side Wall</td>
<td><img src="image4.png" alt="Diagram" /></td>
<td>2</td>
<td>$2(72 \times 150) = 21 600 \text{ cm}^2$</td>
</tr>
<tr>
<td>Floor</td>
<td><img src="image5.png" alt="Diagram" /></td>
<td>1</td>
<td>$72 \times 150 = 10 800 \text{ cm}^2$</td>
</tr>
<tr>
<td><strong>Total Area:</strong></td>
<td></td>
<td></td>
<td><strong>64 224 cm$^2$</strong></td>
</tr>
</tbody>
</table>

For each rectangular piece, the area is calculated using the formula

\[
\text{Area} = \text{length} \times \text{width}.
\]

The area of the triangular piece is calculated using the formula

\[
\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}.
\]
Problem of the Week
Problem B
In the Dog House Again!

When Victoria and her Dad go to The Wood Depot to buy materials for their dog house, they discover that the plywood they need comes only in sheets which are 4 feet by 8 feet (i.e., 1.2 m by 2.4 m) and cost $16.50 each.

NOTE: 1 m = 100 cm.

a) If each piece must be cut whole from a sheet of plywood, what is the minimum number of sheets they will need?
   (Make a diagram showing the layout of pieces on each sheet.)

b) How much will the dog house cost?

c) How much wood will be wasted?

Check out other CEMC resources for Grades 4 to 6 here:
http://cemc.uwaterloo.ca/resources/resources.html

STRAND: Measurement
Problem of the Week
Problem B and Solution
In the Dog House Again!

Problem

When Victoria and her Dad go to The Wood Depot to buy materials for their dog house, they discover that the plywood they need comes only in sheets which are 4 ft by 8 ft (i.e., 1.2 m by 2.4 m), and cost $16.50 each. (NOTE: 1 m = 100 cm)

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(Make a diagram showing the layout of pieces on each sheet.)

b) How much will the dog house cost?

c) How much wood will be wasted?

Solution

a) They will need 4 sheets at minimum, and they will need to be very precise when cutting pieces which share a common side. Refer to the diagrams below.

![Diagram showing layout of pieces on each sheet.](image-url)
b) The wood for the dog house will cost $16.50 \times 4 = $66.00.

c) The waste is as follows:

- From the first piece: the lower area to the left, between, and to the right of the two triangles (up to the first dotted line) has area equal to the two triangles, plus a rectangle 78 cm wide by 48 cm high (up to the second dotted line), plus the tall thin rectangle 18 cm wide by 120 cm tall, giving
  \[
  \frac{1}{2} \times 48 \times 72 \times 2 + 48 \times 78 + 18 \times 120 = 3456 + 3744 + 2160 = 9360 \text{ cm}^2.
  \]
- From the second piece (lower left): two areas of waste, a rectangle 72 cm wide by 48 cm high, and a thin rectangle 18 cm wide by 120 cm high, giving
  \[
  48 \times 72 + 18 \times 120 = 3456 + 2160 = 5616 \text{ cm}^2.
  \]
- From the third piece (upper right): the waste rectangle on the right is 90 cm by 120 cm, plus the lower waste rectangle, which is 48 cm by 150 cm, giving
  \[
  90 \times 120 + 48 \times 150 = 10800 + 7200 = 18000 \text{ cm}^2.
  \]
- From the fourth piece: (Same as third piece) 18 000 cm².

Total Waste: 9360 + 5616 + 18000 + 18000 = 50976 cm² of plywood.

An Alternative Solution for c):

Last week, we calculated the total area of the required pieces to be 64 224 cm². Since the area of four sheets of plywood is \(4 \times 240 \times 120 = 115200 \text{ cm}^2\), the total waste will be 115 200 − 64 224 = 50 976 cm² of plywood, as we found above.
Problem of the Week

Problem B

Stash Your Cash

If you had $250,000 in $10 bills, how many bills would you have? Would they all fit into your backpack?

NOTE: Each $10 bill is 7 cm wide by 15 cm long, and is about the same thickness as a sheet of photocopy paper. A typical backpack has a volume of about 30 L.

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STRAND: Measurement
Problem of the Week
Problem B and Solution
Stash Your Cash

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NOTE: Each $10 bill is 7 cm wide by 15 cm long, and is about the same thickness as a sheet of photocopy paper. A typical backpack has a volume of about 30 L.

Solution
To determine the total number of bills, we divide the total amount of money by the amount of each bill. So we have $250,000 ÷ $10 = 25,000 bills.

The diagram given in the question shows that 500 sheets of paper has a height of 5 cm. Assuming the bills are the same thickness, each 500 bills makes a stack 5 cm thick. Since there are 25,000 bills, there must be 25,000 ÷ 500 = 50 such stacks. Thus the total height of the money would be 50 × 5 = 250 cm, if all the bills were stacked one on top of the other.

Each $10 bill is 7 cm wide by 15 cm long, and the total volume of a stack 250 cm tall is the product of the length, width, and height. Thus the total volume of the money is

\[ 15 \text{ cm} \times 7 \text{ cm} \times 250 \text{ cm} = 26,250 \text{ cm}^3. \]

We can convert this volume from cm\(^3\) to ml by using the fact that 1 ml = 1 cm\(^3\). Thus 26,250 cm\(^3\) = 26,250 ml = 26.25 L.

Since a typical backpack has a volume of about 30 L, the money would all fit.
(The bills would have to be grouped according to the shape of the backpack.)
Problem of the Week
Problem B

Track It Down

In September, Mrs. Weston’s class participated in the Terry Fox run at their school. They ran laps on the two tracks in the school yard, a 400 metre track and a 750 metre track, keeping a tally of each complete lap. The combined tallies of laps run by the whole class gave a total of 303 laps.

One proud class member claimed that the class ran a total of 151.72 km. Is this claim true, or false? Explain your reasoning.

Check out other CEMC resources for Grades 4 to 6 here:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Measurement, Number Sense
Problem of the Week
Problem B and Solution
Track It Down

Problem

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One proud class member claimed that the class ran a total of 151.72 km. Is this claim true, or false? Explain your reasoning.

Solution

Since a total of 303 laps were run by the whole class on the two tracks of 750 metres and 400 metres, the total distance run must have the form:

a multiple of 400 metres + a multiple of 750 metres = total distance in metres.

The claim is that a total of 151.72 km = 151720 m was run. But any multiple of 400 will end in 00, and any multiple of 750 will end in 50 or 00.

Thus this claim is incorrect, since the sum of numbers ending in 50 or 00 cannot end in 20.
Number Sense & Numeration
Problem of the Week
Problem B

What’s My Number?

I am a 5-digit number. My ones digit is 6 more than my tens digit. My tens digit is three less than my thousands digit. My hundreds digit is 3. My ten thousands digit is 3 times my tens digit.

What numbers could I be? __ __ __ __ __

Check out other CEMC resources for Grades 4 to 6 here:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Number Sense
Problem of the Week
Problem B and Solution

What’s My Number?

Problem

I am a 5-digit number. My ones digit is 6 more than my tens digit. My tens digit is three less than my thousands digit. My hundreds digit is 3. My ten thousands digit is 3 times my tens digit.

What number(s) could I be? ___ ___ ___ ___ ___

Solution

Note that my tens digit cannot be 0, since that would make my ten-thousands digit 0, in which case I am no longer a 5-digit number.

Since my ones digit is 6 more than my tens digit, my ones digit could be 7, 8, or 9, corresponding to a tens digit 1, 2, or 3, respectively.

My thousands digit is 3 more than my tens digit, so it could be 4, 5, or 6.

Lastly, my ten-thousands digit, which is 3 times my tens digit, could be 3, 6, or 9. Therefore, I can be 34317, or 65328, or 96339.
Problem of the Week

Problem B

Angry Nerds App

Mei Li wants to get the game app “Angry Nerds” for her old myPhone. She is very frugal with her allowance money, so she wants to pay the least amount per level.

a) Which of the following versions should she buy?

<table>
<thead>
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<th>Levels</th>
</tr>
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<tbody>
<tr>
<td>Ultralight</td>
<td>$1.00</td>
<td>2</td>
</tr>
<tr>
<td>Light</td>
<td>$2.00</td>
<td>5</td>
</tr>
<tr>
<td>Complete</td>
<td>$3.00</td>
<td>8</td>
</tr>
<tr>
<td>Professional</td>
<td>$4.00</td>
<td>11</td>
</tr>
</tbody>
</table>

b) If the Complete version is on sale for $0.12 less, does your answer to part a) change? Explain.

Check out other CEMC resources for Grades 4 to 6 here:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Number Sense
Problem of the Week
Problem B and Solution

Angry Nerds App

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<td>$4.00</td>
<td>11</td>
</tr>
</tbody>
</table>

b) If the Complete version is on sale for $0.12 less, does your answer to part a) change? Explain.

Solution

a) We can determine the amount she would pay per level by dividing the total cost of the version of “Angry Nerds” by the number of levels in the version. The chart below shows the cost per level for each version.

<table>
<thead>
<tr>
<th>Version</th>
<th>Cost</th>
<th>Levels</th>
<th>Cost per Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultralight</td>
<td>$1.00</td>
<td>2</td>
<td>$0.50</td>
</tr>
<tr>
<td>Light</td>
<td>$2.00</td>
<td>5</td>
<td>$0.40</td>
</tr>
<tr>
<td>Complete</td>
<td>$3.00</td>
<td>8</td>
<td>$0.38</td>
</tr>
<tr>
<td>Professional</td>
<td>$4.00</td>
<td>11</td>
<td>$0.36</td>
</tr>
</tbody>
</table>

Thus, in order to pay the least amount per level, Mei Li should buy the Professional version.

b) If the Complete version goes on sale for $0.12 less, the total cost would be $2.88 and the cost per level would be $0.36. Since the Complete version and the Professional version would then have the same price per level, Mei Li must decide whether she is happy with 8 levels, or wants to spend $4.00 for 11 levels.
Problem of the Week
Problem B
Pyramid - No Camels!

Complete the pyramid of numbers below so that each number is the sum of the two numbers directly below it.

```
  58
 /|
/  |
12 16
```

Check out other CEMC resources for Grades 4 to 6 here:
http://cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense,
Pattern/Algebra
Problem of the Week
Problem B and Solution
Pyramid - No Camels!

Problem
Complete the pyramid of numbers below so that each number is the sum of the two numbers directly below it.

\[
\begin{array}{c}
58 \\
12 & 16
\end{array}
\]

Solution
We know that the number in each block in the middle row of the pyramid is the sum of the two numbers directly below it and that 58 is the sum of the two numbers in the boxes in the middle row of the pyramid. Since one of those numbers is the sum of 12 plus the middle number in the bottom row, and the other is the sum of 16 plus the middle number, we see that in obtaining the sum 58, the middle number will occur twice. But 12 + 16 already gives 28, so twice the middle number must be 30. Hence the middle number must be 15, giving the solution shown at the right.

The solution can also be determined using a bit of algebra. Let the number in the middle box on the bottom row be represented by \(x\). Then the numbers in the middle row of the pyramid are \(12 + x\) and \(16 + x\). Since

\[
58 = 12 + x + 16 + x \\
58 = 28 + 2x \\
30 = 2x \\
x = 15
\]

we see that the middle number is 15, as above.
Problem of the Week
Problem B
Will This Hamper Your Thinking?

Matt, the Coordinator at the local emergency food hamper program, noticed the following trends:

- typically 200 hampers were distributed on Mondays;
- on Tuesdays 40 fewer hampers were distributed than on Mondays;
- on Wednesdays there was another peak, with 1.3 times Tuesday’s volume;
- on Thursdays the number of hampers distributed was usually $\frac{3}{4}$ of Monday’s volume;
- on Fridays 50% of Thursday’s volume was distributed.

In a typical week, how many food hampers were distributed?

Extension

The above were typical numbers for a week in early spring, when food needs were heavier. What would you expect for a week in early fall, when the demand is reduced by $\frac{1}{3}$?

Check out other CEMC resources for Grades 4 to 6 here:
http://cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense
Problem of the Week
Problem B and Solution

Will This Hamper Your Thinking?

Problem

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- on Fridays 50% of Thursday’s volume was distributed.

In a typical week, how many food hampers were distributed?

Solution

Here are the daily distributions:

- on Monday 200 hampers were distributed,
- on Tuesday $200 - 40 = 160$ hampers,
- on Wednesday $160 \times 1.3 = 208$ hampers,
- on Thursday $200 \times \frac{3}{4} = 150$ hampers,
- and on Friday $150 \times 0.50 = 75$ hampers were distributed.

This gives a total of $200 + 160 + 208 + 150 + 75 = 793$ hampers distributed in a typical week.

Solution to Extension

With demand reduced by $\frac{1}{3}$, the number of hampers distributed on each day will be $\frac{2}{3}$ of what they were in early spring. With rounding, we have 133 hampers distributed on Monday, 107 on Tuesday, 139 on Wednesday, 100 on Thursday and 50 on Friday for a total of 529 (i.e., $\frac{2}{3}$ of 793) hampers per week.
Problem of the Week
Problem B

Some Real Characters!

Here is a way to make 1000 using only the digit 1 and the operation +, with a total of 37 characters (digits and + signs):

\[ 1000 = 111 + 111 + 111 + 111 + 111 + 111 + 111 + 111 + 111 + 1 \]

Try making 1000 in the following ways, using as few characters as possible:

a) using only the digit 8 and the operation +;
b) using only the digit 5 and the operation +;
c) using only the digit 4 and the operation +.

Check out other CEMC resources for Grades 4 to 6 here:
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**STRAND:** Number Sense
Problem of the Week
Problem B and Solution
Some Real Characters!

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Try making 1000 in the following ways, using as few characters as possible:

a) using only the digit 8 and the operation +;

b) using only the digit 5 and the operation +;

c) using only the digit 4 and the operation +.

Solution

In order to use the fewest number of characters, we want to sum terms which are as large as possible until we get to 1000, but never exceed 1000. Here are the sums with fewest characters for each case.

a) \[ 1000 = 888 + 88 + 8 + 8 + 8, \] using 12 characters.

b) \[ 1000 = 555 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 5, \] using 29 characters.

c) \[ 1000 = 444 + 444 + 44 + 44 + 4 + 4 + 4 + 4 + 4 + 4, \] using 25 characters.
Problem of the Week
Problem B
Pushing Your Buttons!

For some time, Johnny has relied on a calculator to multiply. He has just realized
the 8 button (key) on his calculator is broken.

a) Come up with a method so that Johnny could still use his calculator to solve
the problem $82 \times 816$.

b) Suppose both the 8 key and the 4 key were broken. Could Johnny still use
his calculator to find the product in part a)?

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Strand: Number Sense
Problem of the Week
Problem B and Solution
Pushing Your Buttons!

Problem

For some time, Johnny has relied on a calculator to multiply. He has just realized the 8 button (key) on his calculator is broken.

a) Come up with a method so that Johnny could still use his calculator to solve the problem $82 \times 816$.

b) Suppose both the 8 key and the 4 key were broken. Could Johnny still use his calculator to find the product in part a)?

Solution

a) To solve the problem, Johnny can write each number as a product of numbers that do not involve an 8. He can write 82 as $2 \times 41$ and 816 as $4 \times 204$. Then the problem becomes $82 \times 816 = 2 \times 41 \times 4 \times 204$ which Johnny can solve using his calculator.

b) If the 4 key were broken as well, Johnny would not be able to solve this problem using the same technique because the only way to obtain a product of 82 is $1 \times 82$, or $2 \times 41$, since the factors of 82 are 1, 2, 41, and 82. This means that we cannot write 82 as a product without one of the numbers involving a 4 or an 8.

However, Johnny could write 82 as a sum of numbers without using 4 or 8, e.g., $30 + 52 = 82$. Since $816 = 2 \times 2 \times 2 \times 102$, the product could be found by doing $(30 + 52) \times (2 \times 2 \times 2 \times 102)$.

Comment: If your calculator has brackets, part a) could also be done using sums; for example,

$$82 \times 816 = (32 + 50) \times (500 + 316).$$
Problem of the Week
Problem B

The Price is Right!

“Mathematics is the door and key to the sciences.”  Roger Bacon

a) If $a = 1$ cent, $b = 2$ cents, $c = 3$ cents, ..., and $z = 26$ cents, how much is the word “mathematics” worth?

b) What is your name worth? Comparing with other students’ names in your classroom, whose name is worth closest to a loonie (a one-dollar coin)?

c) Try to find a mathematical term that is worth exactly a loonie (i.e., 100 cents). For example, “factor” is worth $6 + 1 + 3 + 20 + 15 + 18 = 63$ cents.
Problem of the Week
Problem B and Solution
The Price is Right!

Problem

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Solution

a) To determine how much “mathematics” is worth, we need to add up the values of each letter in the word. So, “mathematics” is worth $13 + 1 + 20 + 8 + 5 + 13 + 1 + 20 + 9 + 3 + 19 = 112$ cents.

b) Did the name of anyone in your class come close to being worth a loonie (i.e., 100 cents)?

c) “Squares” is worth $19 + 17 + 21 + 1 + 18 + 5 + 19 = 100$ cents = a loonie. Can you find any others?
Problem of the Week
Problem B

Rank the Runners

In preparation for their upcoming track meet, the coaches at Grand River Academy wanted to assess their top six runners. So they timed each of their top six runners for one minute and recorded the distance in laps they had run, as shown in the table below. Rank the runners in order from fastest to slowest.

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<th>Runner</th>
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</tr>
</thead>
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</tr>
<tr>
<td>Rebecca</td>
<td>4/6</td>
</tr>
<tr>
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<td>1/2</td>
</tr>
<tr>
<td>Rajan</td>
<td>2/3</td>
</tr>
<tr>
<td>Aleah</td>
<td>2/7</td>
</tr>
<tr>
<td>Nick</td>
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**Strand:** Number Sense
**Problem of the Week**

**Problem B and Solution**

**Rank the Runners**

**Problem**

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</table>

**Solution**

In order to compare them, we convert the distances run to fractions with a common denominator. Since the Lowest Common Multiple of the four denominators is LCM(6,2,3,7)=42, 42 will be the common denominator. Alternatively, convert the fractions to decimals (rounded).

The following table lists the runners’ speeds from fastest to slowest:

<table>
<thead>
<tr>
<th>Runner</th>
<th>Distance run in one minute (a distance of 1 = 1 lap)</th>
<th>Rounded decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arman</td>
<td>49/42</td>
<td>1.17</td>
</tr>
<tr>
<td>Rebecca</td>
<td>28/42</td>
<td>0.67</td>
</tr>
<tr>
<td>Rajan</td>
<td>28/42</td>
<td>0.67</td>
</tr>
<tr>
<td>Nick</td>
<td>24/42</td>
<td>0.57</td>
</tr>
<tr>
<td>Kevin</td>
<td>21/42</td>
<td>0.50</td>
</tr>
<tr>
<td>Aleah</td>
<td>12/42</td>
<td>0.29</td>
</tr>
</tbody>
</table>
Problem of the Week
Problem B

No Business Like Snow Business!

One of the responsibilities of Ms. Reahl’s property management company in Blue Mountain is to clear the snow from the parking lot and walkways of the River Grass Condominiums. She is trying to decide which of the programs offered by Bry’s Lawn Care and Snow Removal (as shown below) would best suit her needs.

<table>
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</thead>
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<tr>
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</tr>
<tr>
<td>B</td>
<td>Two payments of $3 652 for unlimited clearing after a snowfall</td>
</tr>
<tr>
<td>C</td>
<td>Three payments of $2 561 for unlimited clearing after a snowfall</td>
</tr>
<tr>
<td>D</td>
<td>One payment of $7 200 for unlimited clearing after a snowfall</td>
</tr>
</tbody>
</table>

Based on last year’s snowfall, Ms. Reahl estimates that she will need to have the plowing done about 35 times. Which option will cost her the least amount of money?

Check out other CEMC resources for Grades 4 to 6 here:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Number Sense
Problem of the Week
Problem B and Solution

No Business Like Snow Business!

Problem

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Based on last year’s snowfall, Ms. Reahl estimates that she will need to have the plowing done about 35 times. Which option will cost her the least amount of money?

Solution

If Ms. Reahl chose option A, it would cost her a total of $225 \times 35 = $7 875. If she chose option B, it would cost her a total of $3 652 \times 2 = $7 304. If she chose option C, it would cost her a total of $2 561 \times 3 = $7 683. And, if she chose option D, it would cost her a total of $7 200. Therefore, option D costs the least amount of money.
Problem of the Week
Problem B

It’s News to Me!

Students at Mario Lemieux Public School are planning to publish a special edition newspaper commemorating the 25th anniversary of their school. They have decided on the following details:

- each newspaper will have 16 pages;
- 8700 copies will need to be printed, using rolls of special newsprint;
- each roll of newsprint can be cut into about 1600 pages;
- the special newsprint costs $75.00 per roll.

If the students’ goal is to just recover their costs, how much should they charge for each newspaper?

Check out other CEMC resources for Grades 4 to 6 here:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Number Sense
Problem of the Week
Problem B and Solution

It’s News to Me!

Problem

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- each roll of newsprint can be cut into about 1600 pages;
- the special newsprint costs $75.00 per roll.

If the students’ goal is to just recover their costs, how much should they charge for each newspaper?

Solution

Since each newspaper will have 16 pages and they need to print 8700 copies, the total number of pages to be printed is $16 \times 8700 = 139200$ pages.

Since each roll can be cut into about 1600 pages, they will need to order $139200 \div 1600 = 87$ rolls of paper.

Since each roll costs $75.00, the total cost to print the newspapers is $87 \times 75.00 = 6525.00$.

Thus if they just want to recover their costs, they should sell the special editions for $6525.00 \div 8700 = 0.75$ each.
Problem of the Week
Problem B
Be There or Be Square!

A few of the whole numbers 1, 2, 3, ..., 25 have an odd number of factors. What are these numbers, and why do they have an odd number of factors?

Check out other CEMC resources for Grades 4 to 6 here:
http://cemc.uwaterloo.ca/resources/resources.html

**Strand:** Number Sense
Problem of the Week
Problem B and Solution

Be There or Be Square!

Problem

A few of the whole numbers 1, 2, 3, ... , 25 have an odd number of factors. What are these numbers, and why do they have an odd number of factors?

Solution

From the table of factors on the following page, we see that there are five numbers from 1 to 25 that have an odd number of factors. They are 1, 4, 9, 16, and 25.

What is special about these numbers is that each is a perfect square. This means that each has at least one factor, say \( k \), such that \( k \times k \) is equal to the number. (The number \( k \) is called the square root.) For example, \( 4 = 2 \times 2 \), and has 3 factors, 1, 2, and 4. Similarly, \( 9 = 3 \times 3 \), and has 3 factors 1, 3, and 9. And 25 also has 3 factors, 1, 5, and 25.

On the other hand, \( 16 = 4 \times 4 \), but 4 itself has factors. We can write \( 16 = 2 \times 2 \times 2 \times 2 \), revealing that it has 5 factors, the expected 1, 4, and 16, plus 2 additional factors, 2 itself, and \( 2 \times 2 \times 2 = 8 \).

It can be shown that any perfect square number has an odd number of factors.
# Table of Factors

<table>
<thead>
<tr>
<th>Number</th>
<th>Factors</th>
<th>No. of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1, 2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1, 3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1, 2, 4</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>1, 5</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>1, 2, 3, 6</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>1, 7</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>1, 2, 4, 8</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>1, 3, 9</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>1, 2, 5, 10</td>
<td>4</td>
</tr>
<tr>
<td>11</td>
<td>1, 11</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>1, 2, 3, 4, 6, 12</td>
<td>6</td>
</tr>
<tr>
<td>13</td>
<td>1, 13</td>
<td>2</td>
</tr>
<tr>
<td>14</td>
<td>1, 2, 7, 14</td>
<td>4</td>
</tr>
<tr>
<td>15</td>
<td>1, 3, 5, 15</td>
<td>4</td>
</tr>
<tr>
<td>16</td>
<td>1, 2, 4, 8, 16</td>
<td>5</td>
</tr>
<tr>
<td>17</td>
<td>1, 17</td>
<td>2</td>
</tr>
<tr>
<td>18</td>
<td>1, 2, 3, 6, 9, 18</td>
<td>6</td>
</tr>
<tr>
<td>19</td>
<td>1, 19</td>
<td>2</td>
</tr>
<tr>
<td>20</td>
<td>1, 2, 4, 5, 10, 20</td>
<td>6</td>
</tr>
<tr>
<td>21</td>
<td>1, 3, 7, 21</td>
<td>4</td>
</tr>
<tr>
<td>22</td>
<td>1, 2, 11, 22</td>
<td>4</td>
</tr>
<tr>
<td>23</td>
<td>1, 23</td>
<td>2</td>
</tr>
<tr>
<td>24</td>
<td>1, 2, 3, 4, 6, 8, 12, 24</td>
<td>8</td>
</tr>
<tr>
<td>25</td>
<td>1, 5, 25</td>
<td>3</td>
</tr>
</tbody>
</table>
Problem of the Week

Problem B

It’s All in the Game

Four years ago, the Kimura family’s DX3 gaming system cost $320. Now it’s worth $140. If the system depreciates (decreases in value) by the same amount each year, how much will it be worth when they plan to sell it, two years from now?

Check out other CEMC resources for Grades 4 to 6 here:
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Strand: Number Sense
Problem of the Week
Problem B and Solution
It’s All in the Game

Problem
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Solution
First, we need to determine the annual depreciation.

• Four years ago the console was $320, now it’s only worth $140.
• In 4 years, the console depreciated $320 − $140 = $180 in total.
• Thus, each year the console depreciated $180 ÷ 4 = $45.

Now we can determine the value of the console when the family sells it.

• In 2 years the console will depreciate another 45 × 2 = $90.
• Therefore, 2 years from now when the family sells the console it will be worth $140 − $90 = $50.
Problem of the Week
Problem B
Txt Me!

Emeka is one of many Canadians who send texts to their friends. For example, in September 2009 about 3 billion texts were sent by the (approximately) 10 million Canadians who text.

a) If the other months of 2009 were similar, how many texts were sent in total in 2009?

b) About how many million texts were sent each day?

c) If Emeka texts like an average Canadian, how many texts does she send in a year?

d) If each text message takes about 30 seconds to compose and send, about how much time does Emeka spend texting in a year?

Check out other CEMC resources for Grades 4 to 6 here:
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STRAND: Number Sense
Problem of the Week
Problem B and Solution
Txt Me!

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d) If each text message takes about 30 seconds to compose and send, about how much time does Emeka spend texting in a year?

Solution

a) If about 3 billion texts were sent per month, then in total for the year 2009, about $3 \times 12 = 36$ billion texts were sent.

b) There are 365 days in a year, thus about $36\,000\,000\,000 \div 365 \approx 98\,630\,137$, or about 99 million texts per day were sent in 2009.

c) If Emeka is one of about 10 million Canadians who text, then Emeka sends about $36\,000\,000\,000 \div 10\,000\,000 = 3600$ texts.

d) The time that it takes Emeka to send 3600 texts is about $3600 \times 30 = 108\,000$ seconds, which is equivalent to about 1800 minutes, or 30 hours a year.
Problem of the Week
Problem B
The Munsch Off

In November, a ‘Munsch Off’ was held at Mountainside Public School, during which five Robert Munsch books were read to all the students in each classroom. On the last day, the staff and students each voted for their favourite of the five books. The results were as follows:

<table>
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<td></td>
<td>Primary</td>
</tr>
<tr>
<td>Millicent and the Wind</td>
<td>10</td>
</tr>
<tr>
<td>Moose</td>
<td>160</td>
</tr>
<tr>
<td>The Dark</td>
<td>80</td>
</tr>
<tr>
<td>Get Out of Bed</td>
<td>23</td>
</tr>
<tr>
<td>From Far Away</td>
<td>5</td>
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a) It seems clear that Moose was the favourite book. Which book was second favourite? Third favourite?

b) John claims that about $\frac{1}{3}$ of the votes were for Get Out of Bed. Is that a reasonable estimate?

c) If all 40 staff members voted, and there are 604 students at Mountainside, how many students did not vote in the Munsch Off?

Check out other CEMC resources for Grades 4 to 6 here:
http://cemc.uwaterloo.ca/resources/resources.html

**Strand:** Data Management, Number Sense
Problem of the Week
Problem B and Solution
The Munsch Off

Problem

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a) It seems clear that Moose was the favourite book. Which book was second favourite? Third favourite?

b) John claims that about \( \frac{1}{9} \) of the votes were for Get Out of Bed. Is that a reasonable estimate?

c) If all 40 staff members voted, and there are 604 students at Mountainside, how many students did not vote in the Munsch Off?

Solution

a) From the Total Votes column in the chart above, the second favourite book was The Dark, and the third favourite book was From Far Away.

b) The number of votes for Get Out of Bed was 63; the total votes cast was \( 30 + 275 + 137 + 63 + 78 = 583 \). So the fraction of people who voted for Get Out of Bed was \( \frac{63}{583} \approx 0.108 \) and \( \frac{1}{9} \approx 0.111 \). So, \( \frac{1}{9} \) is a reasonable estimate for the fraction of total votes cast for Get Out of Bed.

c) We know that every staff member voted. So there are 40 votes out of 583 that were cast by staff. Therefore, \( 583 - 40 = 543 \) students voted. Since there are 604 students, we know that \( 604 - 543 = 61 \) students didn’t vote.
Problem of the Week
Problem B
Track It Down

In September, Mrs. Weston’s class participated in the Terry Fox run at their school. They ran laps on the two tracks in the school yard, a 400 metre track and a 750 metre track, keeping a tally of each complete lap. The combined tallies of laps run by the whole class gave a total of 303 laps.

One proud class member claimed that the class ran a total of 151.72 km. Is this claim true, or false? Explain your reasoning.

Check out other CEMC resources for Grades 4 to 6 here:
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**Strand:** Measurement, Number Sense
Problem of the Week
Problem B and Solution

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One proud class member claimed that the class ran a total of 151.72 km. Is this claim true, or false? Explain your reasoning.

Solution
Since a total of 303 laps were run by the whole class on the two tracks of 750 metres and 400 metres, the total distance run must have the form:
a multiple of 400 metres + a multiple of 750 metres = total distance in metres.

The claim is that a total of 151.72 km = 151 720 m was run. But any multiple of 400 will end in 00, and any multiple of 750 will end in 50 or 00.

Thus this claim is incorrect, since the sum of numbers ending in 50 or 00 cannot end in 20.
Problem of the Week
Problem B
Sum Fun!

Picture ten number cards, as shown below (or cut them out for hands-on use).

0 1 2 3 4 5 6 7 8 9

Consider an addition problem of the form shown below.

+ + +

Use all ten number cards in this sum, using each card once, to create sums which help you to answer the following questions. You may not use 0 as the lead digit in any of the four numbers.

a) What is the greatest possible sum?
b) What is the least possible sum?
c) Try to obtain a sum of 2000. How close can you get?
d) Try to obtain a sum of 9000. How close can you get?

Check out other CEMC resources for Grades 4 to 6 here:
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Strand: Number Sense
Problem of the Week
Problem B and Solution

Sum Fun!

Problem

Picture ten number cards, as shown below (or cut them out for hands-on use).

Consider an addition problem of the form shown at the right.

Use all ten number cards in this sum, each card once, to answer the following questions.

a) What is the greatest possible sum?
b) What is the least possible sum?
c) Try to obtain a sum of 2000. How close can you get?
d) Try to obtain a sum of 9000. How close can you get?

Solution

a) The greatest sum occurs when the greater numbers are placed in higher place value positions. Two ways, for example, to create the greatest sum are:

\[ 1 + 40 + 752 + 9863 = 10656 = 3 + 51 + 860 + 9742 \]

Did you find another way?

b) Similarly, the least sum occurs when the lesser numbers are placed in higher place value positions. However, note that 0 cannot be in the highest place value position here. For example, two ways to get the least sum are:

\[ 9 + 58 + 247 + 1036 = 1350 = 7 + 46 + 238 + 1059 \]

\[ 7 + 43 + 658 + 1290 = 1998 = 8 + 50 + 697 + 1243 \]

\[ 5 + 74 + 290 + 8631 = 9000 = 4 + 91 + 635 + 8270 \]
Problem of the Week
Problem B
Sweet Sixteen Plus One!

Arrange the ten number cards from the previous problem in the boxes so that the sums both across and down are all equal to 17. (2, 6, 7 and 9 have already been placed.)

Try to find more than one solution.

Check out other CEMC resources for Grades 4 to 6 here:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Number Sense
Problem of the Week
Problem B and Solution
Sweet Sixteen Plus One!

Problem

Arrange the ten number cards from the previous problem in the boxes so that the sums both across and down are all equal to 17. (2, 6, 7 and 9 have already been placed.)

\[
\begin{array}{c@{}c@{}c@{}c@{}c@{}c}
9 & + & & + & 7 & = 17 \\
\text{II} & + & 2 & + & \text{II} & = 17 \\
17 & & & 17 \\
\end{array}
\]

Try to find more than one solution.

Solution

One possible solution is:

\[
\begin{array}{c@{}c@{}c@{}c@{}c@{}c}
9 & + & 1 & + & 0 & + & 7 & = & 17 \\
+ & + & 5 & & & + & 6 \\
+ & + & 3 & + & 2 & + & 8 & + & 4 & = & 17 \\
= & = & & & & & & & & & & = \\
17 & & & 17 \\
\end{array}
\]

Note that the symmetry of the squares predicts other solutions: the numbers 1 and 0 can be switched because they only count for the top row (2 and 8 could also be switched on the bottom row). If none of the numbers were placed initially, then the left and right columns, or the top and bottom rows, could also be switched without affecting the sum of 17 in each case.
Problem of the Week
Problem B
Cuatro Amigos

Juan, Maria, Anna, and Carlos each choose a different whole number between 0 and 20 (but not equal to either 0 or 20). Use the given clues to find each person’s number.

1. Juan’s number is a multiple of 5.
2. Maria’s number is greater than Juan’s number.
3. Anna’s number is exactly two times Maria’s number.
4. Carlos’ number is \( \frac{1}{3} \) of Anna’s number, but it is not the smallest number.

Check out other CEMC resources for Grades 4 to 6 here:
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Strand: Number Sense
Problem of the Week
Problem B and Solution

Cuatro Amigos

Problem
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3. Anna’s number is exactly two times Maria’s number.
4. Carlos’ number is \( \frac{1}{3} \) of Anna’s number, but it is not the smallest number.

Solution
Since Juan’s number is a multiple of 5, his number can be 5, 10, or 15.
However, clue 2 says that Maria’s number is more than Juan’s number and clue 3 says Anna’s number is exactly two times Maria’s number. Thus if Juan’s number were 10 or 15, Maria’s number would be greater than 10, and Anna’s number would be greater than 20. Hence Juan’s number has to be 5.

Since clue 4 says that Carlos’ number is \( \frac{1}{3} \) Anna’s number, the greatest possible value for Carlos’ number is 6; otherwise, Anna’s number would be over 20. But clue 4 also says that Carlos’ number is not the smallest number, so we know it cannot be less than 5, because Juan’s number is 5. Hence, Carlos’ number has to be 6, and Anna’s number is 18.

Lastly, Maria’s number is half of Anna’s number, which is half of 18, or 9.

Therefore Juan’s number is 5, Maria’s number is 9, Anna’s number is 18, and Carlos’ number is 6.
Patterning
&
Algebra

TAKE ME TO THE COVER
Problem of the Week
Problem B
Pyramid - No Camels!

Complete the pyramid of numbers below so that each number is the sum of the two numbers directly below it.

Check out other CEMC resources for Grades 4 to 6 here:
http://cemc.uwaterloo.ca/resources/resources.html

**STRAND:** Number Sense, Pattern/Algebra
Problem of the Week
Problem B and Solution
Pyramid - No Camels!

Problem
Complete the pyramid of numbers below so that each number is the sum of the two numbers directly below it.

```
      58
     /   \
   12    16
```

Solution
We know that the number in each block in the middle row of the pyramid is the sum of the two numbers directly below it and that 58 is the sum of the two numbers in the boxes in the middle row of the pyramid. Since one of those numbers is the sum of 12 plus the middle number in the bottom row, and the other is the sum of 16 plus the middle number, we see that in obtaining the sum 58, the middle number will occur twice. But 12 + 16 already gives 28, so twice the middle number must be 30. Hence the middle number must be 15, giving the solution shown at the right.

The solution can also be determined using a bit of algebra. Let the number in the middle box on the bottom row be represented by $x$. Then the numbers in the middle row of the pyramid are $12 + x$ and $16 + x$. Since

\[ 58 = 12 + x + 16 + x \]
\[ 58 = 28 + 2x \]
\[ 30 = 2x \]
\[ x = 15 \]

we see that the middle number is 15, as above.
Problem of the Week
Problem B

Chips But No Dip

a) Colour the circles in the triangular stack below using three different colours, with no two circles beside each other (adjacent) having the same colour.

b) How many circles are there of each of the three colours in the stack a)?

c) Suppose the stack had only 3 rows of circles instead of 4. How many circles would there be of each colour? What if the stack had 5 rows?

d) Fill in the table below, and hence decide when there will be the same number of circles of each colour. Predict the first number of rows greater than 7 for which there is NOT the same number of circles of each colour.

<table>
<thead>
<tr>
<th>No. of rows</th>
<th>No. of Circles</th>
<th>Colour 1</th>
<th>Colour 2</th>
<th>Colour 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Check out other CEMC resources for Grades 4 to 6 here:*

[http://cemc.uwaterloo.ca/resources/resources.html](http://cemc.uwaterloo.ca/resources/resources.html)

**Strand:** Pattern/Algebra
Pyramid Templates
Problem of the Week
Problem B and Solution

Chips But No Dip

a) Colour the circles in the triangular stack below using three different colours, with no two circles beside each other (adjacent) of the same colour.

b) How many circles are there of each of the three colours in the stack a)?

c) Suppose the stack had only 3 rows of circles instead of 4. How many circles would there be of each colour? What if the stack had 5 rows?

d) Fill in the table below, and hence decide when there will be the same number of circles of each colour. Predict the first number of rows greater than 7 for which there is NOT the same number of circles of each colour.

a) Using R = red, B = black, W = white as the choice of colours, one solution is shown at the right. Others can be found by interchanging the colours (e.g., one way would be to replace R with B, B with W, and W with R).

b), c), d) Using the diagram at the right, the completed table is:

<table>
<thead>
<tr>
<th>No. of Rows</th>
<th>No. of Circles</th>
<th>Red</th>
<th>Black</th>
<th>White</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>28</td>
<td>10</td>
<td>9</td>
<td>9</td>
</tr>
</tbody>
</table>

Thus we see that there will be the same number of circles of each colour provided that the total number of circles is divisible by 3. The next time this will NOT occur is when there are 10 rows, with a total of $28 + 8 + 9 + 10 = 55$ circles.
Problem of the Week
Problem B

Polygons: How Many Diagonals?

A diagonal of a regular polygon is a straight line joining two vertices which are not beside each other (adjacent). Thus a square has two diagonals, and a regular pentagon has five, as shown at the right.

a) How many diagonals does a regular hexagon have? (Draw them.)
b) How many diagonals does a regular heptagon have? (Draw them.)
c) How many diagonals does a regular octagon have?

d) Complete the table at right, up to 8 sides, using the information from a), b), c). Then predict the number of diagonals for a regular dodecagon (a 12-sided polygon), using the pattern in the table. Draw the diagonals on the figure below to see whether your prediction is correct.

<table>
<thead>
<tr>
<th>Number of sides</th>
<th>Number of diagonals</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

Extension

Suppose the polygons do not have to be regular, i.e., they could have sides of different lengths. Would your answers to the above problems change?

Check out other CEMC resources for Grades 4 to 6 here:
http://cemc.uwaterloo.ca/resources/resources.html

Strand: Geometry, Pattern/Algebra
Problem of the Week
Problem B and Solution

Polygons: How Many Diagonals?

Problem and Solution
A diagonal of a regular polygon is a straight line joining two vertices which are not beside each other (adjacent). Thus a square has two diagonals, and a regular pentagon has five, as shown at the right.

a) How many diagonals does a regular hexagon have? (Draw them.)

b) How many diagonals does a regular septagon have? (Draw them.)

c) How many diagonals does a regular octagon have?

The hexagon has $3 + 3 + 2 + 1 = 9$ diagonals; the septagon has $4 + 4 + 3 + 2 + 1 = 14$ diagonals; and the octagon has $5 + 5 + 4 + 3 + 2 + 1 = 20$ diagonals, as illustrated in the diagrams below.

Here is a detailed breakdown for the octagon.
d) Complete the table at right, up to 8 sides, using the information from a), b), c). Then predict the number of diagonals for a regular dodecagon (a 12-sided polygon), using the pattern in the table. Draw the diagonals on the figure below to see whether your prediction is correct.

<table>
<thead>
<tr>
<th>Number of sides</th>
<th>Number of diagonals</th>
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</thead>
<tbody>
<tr>
<td>3</td>
<td>0</td>
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<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
</tr>
<tr>
<td>12</td>
<td>54</td>
</tr>
</tbody>
</table>

We can predict that the dodecagon has 54 diagonals by noting that the difference between successive numbers of diagonals increases by one at each step. Thus from 4 to 5 sides is an increase of 3 diagonals, from 5 to 6 is an increase of 4, from 6 to 7 is an increase of 5, and from 7 to 8 is an increase of 6. Thus a polygon with 9 sides will have \(20 + 7 = 27\) diagonals, with 10 sides, \(27 + 8 = 35\) diagonals, with 11 sides \(35 + 9 = 44\) diagonals, and with 12 sides \(44 + 10 = 54\) diagonals.

Following the pattern established for the octagon, we see that the dodecagon will have \(12 - 3 = 9\) diagonals from each of the first two vertices (red and blue on the diagrams below), 8 from vertex 3 (black on the diagrams), 7 from vertex 4 (dashed red on the last diagram), 6 from vertex 5 (dashed blue on the last diagram), 5 from vertex 6 (dashed black on the last diagram), etcetera, giving a total of \(9 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 54\) distinct diagonals.