The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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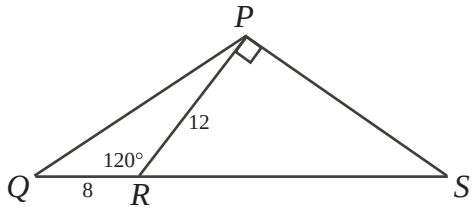
In honour of the 50th anniversary of the Faculty of Mathematics, at the beginning of each month of 2017, a set of five problems from the 54 years of CEMC contests will be posted. Solutions to the problems will be posted at the beginning of the next month. Hopefully, these problems will intrigue and inspire your mathematical mind. For more problem solving resources, please visit `cemc.uwaterloo.ca`.

1. *2008 Fermat Contest, Question 19*
   In the diagram, R is on QS and $QR = 8$. Also, $PR = 12$, $\angle PRQ = 120°$, and $\angle RPS = 90°$. What is the area of $\triangle QPS$?

   (A) $72\sqrt{3}$ **(B)** 72 **(C)** 36
   (D) $60\sqrt{3}$ **(E)** $96\sqrt{3}$

2. *1988 Gauss Contest, Grade 7, Question 24*
   A set of positive integers, each of which is different, has a sum of 329 and an average of 47. If one of the integers is 97, then the largest possible integer that could appear in the set is

   (A) 97 **(B)** 211 **(C)** 217 **(D)** 227 **(E)** 228

3. *2000 Gauss Contest, Grade 7, Question 6*
   In the addition shown, a digit, either the same or different, can be placed in each of the two boxes. What is the sum of the two missing digits?

   $$\begin{array}{rcccc} & 8 & 6 & 3 \\ & \square & 9 & 1 \\ + & & 7 & \square & 8 \\ \hline & 2 & 1 & 8 & 2 \end{array}$$

   (A) 9 **(B)** 11 **(C)** 13
   (D) 3 **(E)** 7

4. *2005 Cayley Contest, Question 25*
   The positive integers a, b, and c satisfy $\dfrac{1}{a^2} + \dfrac{1}{b^2} = \dfrac{1}{c^2}$. The sum of all possible values of $a \leq 100$ is

   (A) 315 **(B)** 615 **(C)** 680 **(D)** 555 **(E)** 620

5. *2017 Cayley Contest, Question 19*
   Two lines with slopes $\frac{1}{4}$ and $\frac{5}{4}$ intersect at (1,1). What is the area of the triangle formed by these two lines and the vertical line $x = 5$?

   (A) 5 **(B)** 10 **(C)** 8 **(D)** 12 **(E)** 15