The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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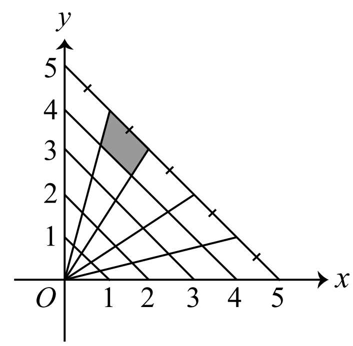
In honour of the 50th anniversary of the Faculty of Mathematics, at the beginning of each month of 2017, a set of five problems from the 54 years of CEMC contests will be posted. Solutions to the problems will be posted at the beginning of the next month. Hopefully, these problems will intrigue and inspire your mathematical mind. For more problem solving resources, please visit `cemc.uwaterloo.ca`.

1. *1990 Cayley Contest, Question 22*
   The five marked segments are equal in length. The area of the shaded region is

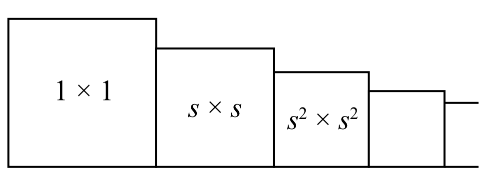
   (A) 0.5 **(B)** 0.9 **(C)** 1.0
   (D) 1.1 **(E)** 1.8



2. *2002 Descartes Contest, Question A5*
   If $0^\circ < x < 90^\circ$ and $\tan(2x) = -\frac{24}{7}$, determine the value of $\sin x$.

3. *2007 Cayley Contest, Question 20*
   What is the largest integer n for which $3(n^{2007}) < 3^{4015}$?
   (A) 2 **(B)** 3 **(C)** 6 **(D)** 8 **(E)** 9

4. *1994 Descartes Contest, Question 7*
   As shown below, a figure consists of an infinite sequence of squares which have sides of length 1, s, s^2, s^3, ..., where $0 < s < 1$.



   (a) Let A be the area of the figure. Express A in terms of s.
   (b) Let P be the outside perimeter of the figure. Express P in terms of s.
   (c) Determine all s such that $\dfrac{A}{P} = \dfrac{8}{35}$.

5. *2016 Euclid Contest, Question 8a*
   In the diagram, $ABCD$ is a parallelogram. Point E is on DC with AE perpendicular to DC, and point F is on CB with AF perpendicular to CB. If $AE = 20$, $AF = 32$, and $\cos(\angle EAF) = \frac{1}{3}$, determine the exact value of the area of quadrilateral $AECF$.