Grade 6 Math Circles
February 9th, 2022
Percentages and Applications

Introduction

As you have likely seen in your math classes, we can represent numbers using fractions and decimals. We can also represent numbers as percentages in a similar way.

A **percent** is a ratio that compares a number to 100 (per-*cent* is like per-*hundred*). We use the symbol % to denote percentages.

Representing Percentages

We can represent percentages as fractions out of 100, since they are parts of 100. For example, 45% is $\frac{45}{100}$.

When we deal with decimals that have two decimal places, they are also parts of 100 (which is why we call the second decimal place the "hundredth" place). For example, 0.45 is $\frac{45}{100}$.

Therefore, we can also represent percentages as decimals. To do this, we divide the percent by 100. For example, to write 32% as a decimal, we calculate $\frac{32}{100}$ to get 0.32. Alternatively, we can look at moving the decimal point two places to the left. For example, if we want to represent 76% as a decimal, we start with 76.0 and move the decimal two places to the left to give us 0.76.

$$76\% \longrightarrow 76.0 \longrightarrow 0.76$$

Reversing this, we can also change decimals into percentages. To do this, we multiply the decimal by 100. For example, if we have 0.76, multiplying by 100 gives 76, and therefore we know that 0.76 is 76%. Alternatively, we can also use the moving decimal method, except this time we move the decimal two places to the right.

$$0.76 \longrightarrow 0.76 \longrightarrow 76\%$$

Exercise 1

Change the following percentages to decimals or decimals to percentages.

(a) 55% (b) 9% (c) 0.4 (d) 0.02

Bigger Than 100 or Smaller Than 1?

We are not limited to numbers between 1 and 100 to represent percentages. When we look at 100%, we think of this as one whole or 1. So, if we have 200% of something, we know that it's $2 \times 100\%$ and therefore is 2 whole things. Furthermore, if we have less than 1% of something, for example if we have 0.6%, we have to think smaller and go into the thousands, and the more decimal places, the smaller we go (for example, to ten thousands, one-hundred thousands, etc.). As we saw in the previous section, we can divide 0.6% by 100 to find out how to represent it as a decimal. Notice that $\frac{0.6}{100} = 0.006$, and we can further represent this using a fraction out of 1000: $\frac{6}{1000}$.

Percentage	1000%	100%	10%	1%	0.1%	0.01%	0.001%
Fraction	$\frac{1000}{100}$	$\frac{100}{100}$	$\frac{10}{100}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10\,000}$	$\frac{1}{100\,000}$
Decimal	10.0	1.0	0.1	0.01	0.001	0.0001	0.00001

Different Sizes of Percentages and Their Representations

Exercise 2

Represent each of the following percentages as decimals and fractions.

(a) 0.5% (b) 300% (c) 1.07% (d) 0.084%

Percentages of Amounts

Since percentages represent parts of 100, which we saw also represents parts of a whole, sometimes we want to know exactly how much of a whole is represented by a certain percentage. For example, if we had 4 apples, how many apples would be needed if someone said they wanted 50% of our apples?

To do this, we can multiply the whole amount we have by the decimal representation of the percent. Since 50% is 0.5, multiplying 4×0.5 will give 50% of 4. The answer here is 2.

In other words, *(Amount from percent)* = *(Whole amount)* × *(Percent as a decimal)*.

Exercise 3

Practice finding amounts from percentages by calculating the following.

(a) What is 70% of 34?

(b) What is 0.5% of 500?

(c) If Chen bought a pizza with 8 slices, how many slices is 37.5% of the pizza?

(d) If James has \$20 and his brother takes 15% of his money, how much money did James lose?

On the other hand, we may want to figure out what percentage of a whole we have. For example, suppose your quiz score is $\frac{7}{8}$ and you wanted to change this to your grade as a percent. We can do this by dividing what you earned by the whole amount, then multiplying by 100. So, since you got 7 marks out of 8 possible marks on a quiz, we calculate $\frac{7}{8} = 0.875$, and then calculate $0.875 \times 100 = 87.5$, which tells you that you got 87.5% on the quiz.

In other words, *(Percentage)* = *(Part of amount)* ÷ *(Whole amount)* × 100.

Exercise 4

Practice finding percentages from amounts by calculating the following.

(a) What percentage is $\frac{7}{35}$?

(b) If Julia got 33 marks on her math test that was out of 40 marks, what percent grade did she get?

(c) If Dave and Liu bought 18 of the 120 chocolate bars at the store, what percent of the store's stock did they purchase?

(d) If Anaisha has 16 rings and sells 6 of them, what percent of her rings did she keep?

Real Life Applications

Discounts

At the store, you might see a sale that says something like "30% off". This is an example of how percentages are used in everyday life and how knowing them is important!

When we see that something is 30% off, that means that we subtract 30% of the price. There are a couple of ways that we can calculate this for ourselves.

1) You can calculate what 30% of the price is, then subtract that from the price.
2) You can instead calculate what 70% of the price is, and this will represent how much money you will still be paying.

Stop and Think

Why will these two different methods always give the same answer? Try to come up with your own explanation before reading the explanation given in the example.

Example 1

Let's calculate how much we'll pay for an item that's normally priced at \$12.00 and is listed for 30% off.

Using method one, we know that 30% is 0.3 as a decimal, and so 30% of \$12.00 will be $\$12 \times 0.3 = \$3.60$. Therefore, we will save \$3.60 and end up spending $\$12.00 - \$3.60 = \$8.40$.

Using method two, we know that 70% is 0.7 as a decimal, and so 70% of \$12.00 will be $\$12 \times 0.7 = \$8.40$. Therefore, we will end up spending \$8.40 with the 30% discount, and as you can see, this answer is the same as method one.

We get the same answer because 30% off is the same as 70% "on", i.e. $100\% - 30\% = 70\%$.

Exercise 5

Practice using discounts by calculating the following.

(a) Hasan's store is having a 15% off sale. What would be the discounted price for an item that is regularly \$70.00?

(b) Ashley has been wanting to purchase a new laptop but waits until it's on sale. The laptop she wants is regularly \$999.00 and the store has discounted it by 32% so she purchases it with the discount. How much money did Ashley save by waiting to buy the laptop?

(c) Mathew's favourite restaurant gives a 20% off discount to students. If he wants a sandwich and soup combo that's regularly priced at \$11.00, what price will the restaurant charge him if he is a University student?

(d) Trisha is buying coffee for her co-workers. The company gets 8% off orders over \$30.00. If Trisha orders \$34.50 worth of coffee, how much money will she save on her order?

Sales Tax

Still at the store, we can use percentages to help us at the cash register. In Ontario, we add taxes such as HST to our total spending amount, which is extra money we pay for our goods that goes to the government. The current amount of HST is 13%. This means that when you add together all the prices for your items, 13% of that total amount will be added to your bill.

Example 2

If you spent \$10.00, you would have to pay an additional amount of \$1.30 in tax (since $10 \times 0.13 = 1.3$). As you can see, we can easily use the same method as finding amounts from percentages to calculate the amount of tax. Then, you can simply add together the price of your items and the tax to find the total amount you are paying: $\$10.00 + \$1.30 = \$11.30$.

Again, there is another way to calculate how much you will be spending, which you might find even easier. You can take the total price of your items and simply multiply it by 1.13.

Stop and Think

Why does the multiplication by 1.13 work? Try to come up with your own explanation before reading the explanation on the next page.

This works because when you multiply by 1.13, it's the same as 113% which is 100%+13%. The price of the item is the 100%, and then 13% will be the amount of taxes. You are essentially multiplying the whole price by 1, then also including the additional 13%.

We can use this method for our example to solidify it: $\$10.00 \times 1.13 = \$11.30$.

Exercise 6

Practice using sales tax by calculating the following.

(a) Jacqueline wants to buy a shirt that is \$15.00. How much money will she actually spend at the register when HST is added to her bill?

(b) Elena is buying ice cream for her and her three friends. If ice cream cones are \$3 each and HST is added to her bill, how much money will Elena end up spending?

(c) Miguel is buying a couch for his new apartment that is priced \$1250.00 at the store. How much more will he have to pay for HST?

Simple Interest

Interest is similar to sales tax, where if you borrow money from a company, they will likely want an additional percent of the total amount to be paid when you return the money. This is their way to profit from letting you borrow the money. Similarly, you can receive interest for your money, like when you have a savings account at a bank, where the bank gives you an extra percentage of the money you store. This is usually done to encourage you to keep your money with that bank.

Simple interest is a specific type of interest that is based on simple multiplication of the amount of money borrowed (or stored), the interest rate, and how long you take to repay (or store) the money.

The formula for simple interest is $I = Prt$, where I stands for interest, P stands for principal (which is the amount of money borrowed or stored), r stands for the rate of interest, and t stands for the time to repay or store the money, in years.

Note that the rate of interest, or interest rate, is usually given as a percentage, but for our calculations, we use the decimal representation of the interest rate for r.

Additionally, note that when we write our variables beside each other, such as Prt, it means that we are multiplying those variables together. So, $Prt = P \times r \times t$.

Example 3

Janice is going to borrow \$10 000 from a company at an interest rate of 6% and will take 5 years to repay them. In this case, $P = \$10\,000$, and then we convert the rate from percent to decimal, giving us $r = 0.06$, and lastly we have that $t = 5$. Now, we can calculate

$$\begin{aligned} I &= Prt \\ &= \$10\,000 \times 0.06 \times 5 \\ &= \$3000 \end{aligned}$$

So, Janice would end up having to pay an additional \$3000 in interest. In other words, Janice would return the \$10 000 she initially borrowed, and also have to give them an extra \$3000 of interest for borrowing their money. Therefore, overall, Janice would end up paying \$13 000.

Similar to sales tax, there's also a way to find the total amount you pay in a different way.

Accumulated value is a financial term we can use to refer to the final amount including interest. So, when finding the total amount you pay, we would call it calculating the accumulated value. We can represent accumulated value with the letter A.

To find the accumulated value for simple interest, we can use the following formula:

$$A = P + Prt = P(1 + rt)$$

Note that like before, writing variables beside each other means multiplication, so $P(1 + rt) = P \times (1 + r \times t)$.

We need to know a couple of things to understand why our formula for A works.

First, we should understand order of operations. If you have not seen order of operations before, or if you would like a review, please watch this YouTube video: https://youtu.be/jhVpMsjnhZc.

Second, we need to understand why $P + Prt = P(1 + rt)$. Here is a link to a YouTube video to explain this: https://youtu.be/RiThQP7_MU0.

Now that you've watched those two videos, let's do an example, and hopefully you understand the reasoning for the formula, why it works, and how to use it.

Example 4

We can use the same P, r, and t from the last example with Janice to show that the accumulated value is the same. Using our new formula, we calculate that

$$\begin{aligned} A &= P(1 + rt) \\ &= \$10\,000 \times (1 + 0.06 \times 5) \\ &= \$10\,000 \times (1 + 0.3) \\ &= \$10\,000 \times 1.3 \\ &= \$13\,000 \end{aligned}$$

Note how we had to use order of operations to solve this, and also that we did get the same answer as before.

Still dealing with interest equations, something that we might encounter is when money is borrowed or stored for an amount of time that isn't a whole number of years.

For example, Lee might borrow money for only 9 months, or Ahmed might borrow money for 20 months (i.e. 1 year and 8 months). In these cases, we have to be careful with how we represent the time with the variable t, since t is meant to be the number of years.

Specifically, we can use a fraction of months out of 12 to represent how many years the money is borrowed, since there are 12 months in a year. For example, for Lee, we would use $t = \frac{9}{12}$, and similarly for Ahmed, we would use $t = \frac{20}{12}$.

Exercise 7

Practice finding interest and accumulated value by calculating the following.

(a) Fran arranges to borrow \$15 200 from a company with an interest rate of 7%. If she takes 18 months to pay back the money, how much interest will she end up paying?

(b) Celine saves \$32 000 in her savings account at the bank with an interest rate of 3%. What will be the accumulated value of her bank account in 10 years?

Lastly, we might be given the accumulated value and asked to find the principal amount.

Present value is a financial term we can use to refer to the principal amount when we are calculating this type of problem. We call it this because given the interest rate, time, and accumulated value, finding the principal amount would be like finding the amount you have to borrow in the present in order to repay the given accumulated amount in the future.

To calculate present value for simple interest, we rearrange the equation that we used for accumulated value to solve instead for the principal amount, which is also the present value. Since the present value is equal to the principal amount, we can use the variable P to represent both.

The process of rearranging an equation to find the value of a different variable is called *isolating for variables*. This might be a new or unknown concept for many of you, so a breakdown of the rearranging process for this equation is included below to better your understanding of how we get the final formula.

$$A = P(1 + rt)$$

Dividing both sides by $(1 + rt)$:

$$\frac{A}{(1 + rt)} = \frac{P(1 + rt)}{(1 + rt)}$$

Since $\frac{(1 + rt)}{(1 + rt)} = 1$, this simplifies to

$$\frac{A}{(1 + rt)} = P \times 1$$

Then since $P \times 1 = P$, we have,

$$\frac{A}{(1 + rt)} = P$$

Or equivalently,

$$P = \frac{A}{(1 + rt)}$$

Note that this is equal to $P = A \div (1 + r \times t)$.

Now that we have figured out the formula, we can use it in an example to see it in action.

Example 5

Amy borrowed money at an interest rate of 5% and the company told her that she would have to repay \$7500 including interest in 4 years. How much money did Amy borrow?

In this case, we are given that $A = \$7500$, and then we change the interest rate to a decimal to get that $r = 0.05$, and lastly we have that $t = 4$. We are asked to find the principal amount that Amy borrowed, so we use our present value equation and calculate

$$\begin{aligned} P &= A \div (1 + rt) \\ &= \$7500 \div (1 + 0.05 \times 4) \\ &= \$7500 \div (1 + 0.2) \\ &= \$7500 \div 1.2 \\ &= \$6250 \end{aligned}$$

Notice how we used order of operations to calculate this, and we found that Amy must have borrowed \$6250 from the bank.

Exercise 8

Practice finding the present value by calculating the following.

(a) Jack borrowed money 2 years ago and has now repaid an accumulated value of \$9800. If the interest rate for his borrowing was 6%, how much money did Jack originally borrow?

(b) Kylie has \$68 000 in her savings account after 210 months of keeping her money there with an interest rate of 4%. How much money must Kylie have put into her savings account?

Conclusion

We use percentages in many different parts of life. We use them in simple calculations for finding amounts as well as representing grades or quantities. We can also use them to represent discounts, or to apply taxes. Finally, we use them in other aspects of math such as interest rates. Knowing percentages now can be very useful for you later!