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# Grade 6 Math Circles

20/21 March, 2018

***Circuits Solutions***

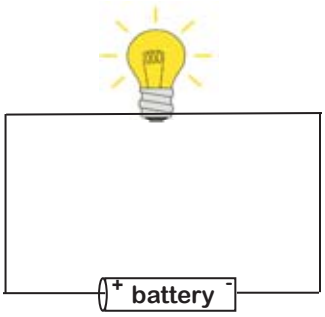
## Introduction

Electricity is an intricate part of our everyday lives and comes in two main forms, static electricity and current electricity. **Static electricity** is when electricity gathers at one spot, like when you rub a balloon against your head! **Current electricity** is when electricity moves from one spot to another such as the electrons moving through an extension cord.

Today we are going to look at current electricity and some of the mathematics involved in electric circuits. Let's begin with some terminology.

### Terminology

A **circuit** is a closed path that allows for an electrical current to flow through. An example would be the wires connecting a power source to a light bulb.



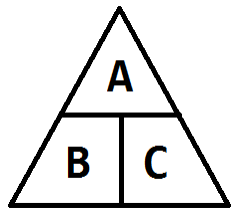

**Current** is a steady flow of electrons that moves throughout the circuit. It is measured in amperes (or amps) and represented by the symbol "I".

**Voltage** is the electrical force that makes the electric current move through the wire. We represent voltage using a "V" and it is measured in volts.

**Resistance** is a property by which the passage of current is opposed. Resistance causes some of the electric energy to be transformed into heat. Resistance is represented by "R" and measured in ohms ($\Omega$). A light bulb is an example of a **resistor**.

## Equation Pyramids

Equation pyramids are useful in describing the relationship between different variables, helping us solve different types of problems. Let's look at the following example with the variables A, B, and C:




How to use the pyramid:

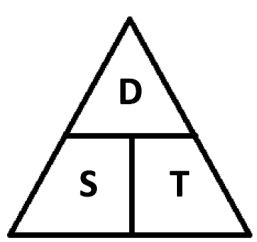
- Cover the variable representing the value you are looking for
- If the remaining two letters are beside each other, multiply them to get your answer
- If they are on top of each other, then divide them to get your answer.

**Examples**

- If we cover A, we have that B and C are beside each other, so $A = B \times C$
- If we cover B that leaves A on top of C, thus $B = \dfrac{A}{C}$
- If we cover C, we have A on top of B, therefore $C = \dfrac{A}{B}$

**Make Your Own**

Create an equation pyramid for the relationship between speed, distance, and time given that Speed $= \dfrac{\text{Distance}}{\text{Time}}$.

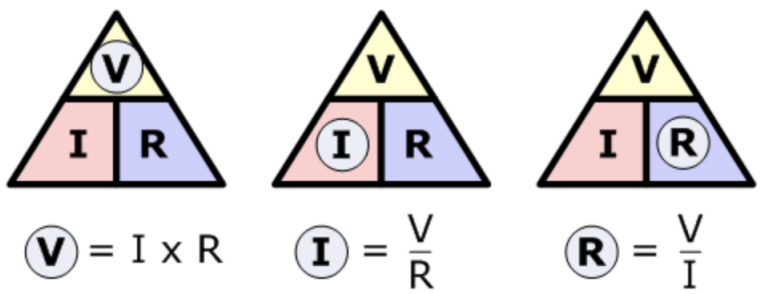


## Ohm's Law

Georg Ohm was a German mathematician and physicist who discovered an important relationship between voltage, current, and resistance. **Ohm's Law** states that the amount of voltage across a circuit is proportional to the amount of current running through it. This proportionality is called the resistance.

Recall that:

- Voltage (V) is measured in Volts (V).
- Resistance (R) is measured in Ohms (Ω).
- Current (I) is measured in amps (A).




Retrieved from: $http://www.electronics-tutorials.ws/dccircuits/dcp_2.html$

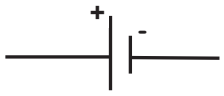
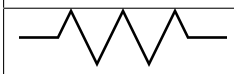
We can use Ohm's law to find the current, resistance, or voltage of any circuit if we are given the other two values.

**Example:**

In a simple circuit, there is a 20 volt battery and a light bulb with a resistance of 4 ohms. What is the electrical current running through this circuit?

Current, $I = \frac{V}{R}$
$= \frac{20}{4} = 5$

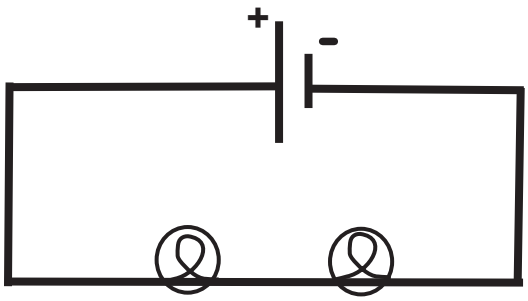
**Common Symbols:**

| | |
|---|---|
|  | Battery |
|  | Light Bulb |
|  | Switch |
|  | Resistor |

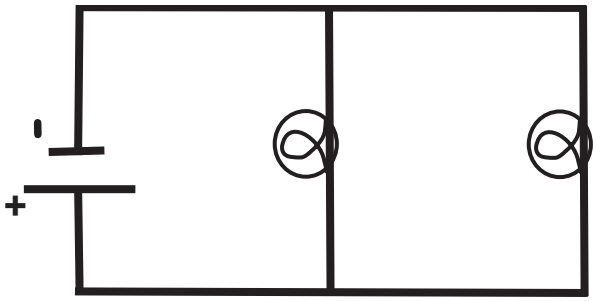
**Types of Circuits**

There are two ways in which light bulbs (or other resistors) can be arranged in one circuit. The two types are **Parallel** and **Series** circuits, each type having their own special properties.

Can you tell what type of circuits these are?



Series Circuit



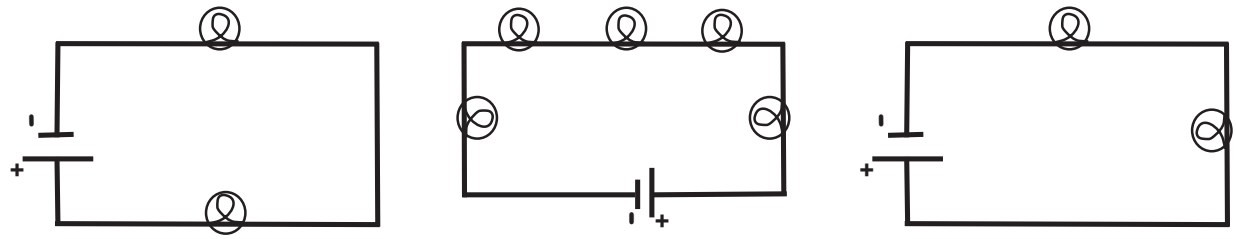
Parallel Circuit

In a Series circuit, the light bulbs run along one single path and are placed beside each other.

In a Parallel circuit, the light bulbs run along different paths and are Parallel to each other.

**Series Circuits**

Each of the following diagrams are examples of how light bulbs can be arranged in a series circuit.




**Properties**

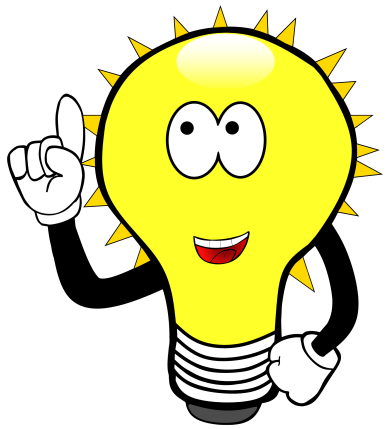
1. The total current in the circuit is equal to the current flowing through each resistor. (ie. $I_{total} = I_1 = I_2 = ...$)

2. The total voltage in the circuit is equal to the sum of the voltage at each resistor. (ie. $V_{total} = V_1 + V_2 + ...$)

3. The total resistance in the circuit is equal to the sum of the resistance at each resistor. (ie. $R_{total} = R_1 + R_2 + ...$)

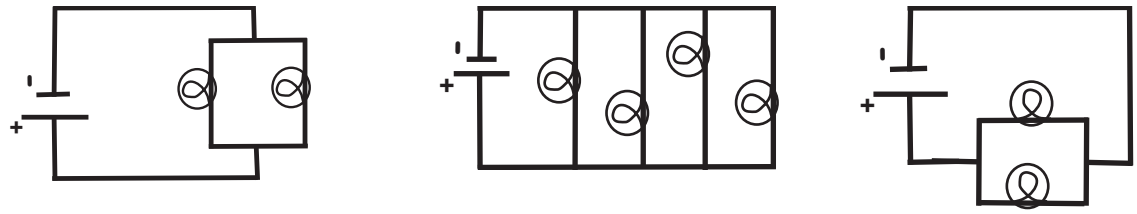
4. If the circuit is broken at anytime (ie. A light bulb burns out, a switch is opened, or a wire is cut) the entire circuit stops working.

5. If you add a light bulb to a series circuit the brightness of the bulbs will decrease. *Adding a light bulb increases the resistance and reduces the current causing the lights to be dimmer.*



**Parallel Circuits**

Each of the following diagrams are examples of how light bulbs can be arranged in a parallel circuit.




**Properties**

1. The total current in the circuit is equal to the sum of the current flowing through each of the resistors in the circuit.
   (ie. $I_{total} = I_1 + I_2 + ...$)

2. The total voltage in the circuit is equal to the voltage at each of the resistors in the circuit.
   (ie. $V_{total} = V_1 = V_2 = ...$)

3. The inverse of the total resistance in the circuit is equal to the sum of inverses of the resistance at each of the resistors.
   (ie. $\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + ...$)

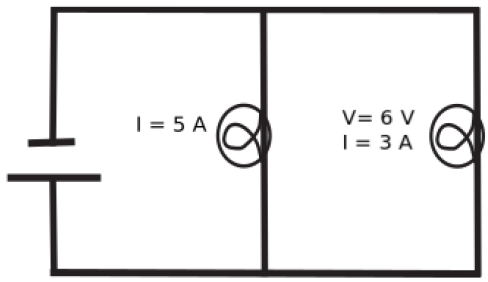
4. If one of the paths break (ie. a light bulb burns out, a switch is opened, or a wire is cut) the rest of the circuit will continue to work.

5. If you add a light bulb to a parallel circuit the brightness of the bulbs will remain the same. *Since each light bulb has it's own path the brightness remains the same since it does not share voltage*

**Examples**

Determine if the circuit is parallel or series, then find the total current, voltage, and resistance of the circuit.

1.




The light bulbs are arranged in a parallel circuit, which means the voltage will be the same for both light bulbs and for the total voltage. We also know that the total current is the sum of both currents, thus to find resistance we can just use Ohm's law.

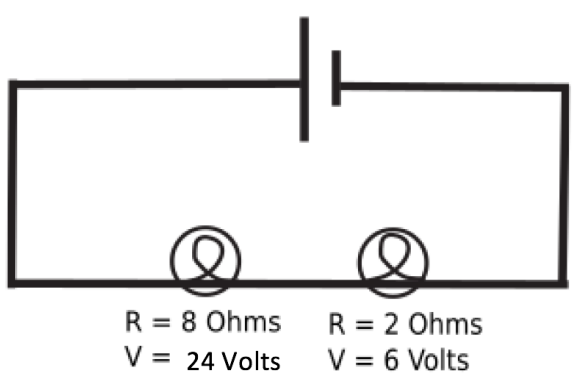
$V_{total} = V_1 = V_2 = 6V$

$I_{total} = I_1 + I_2 = 5 + 3 = 8A$

$R_{total} = \frac{V}{I}$

$= \frac{6}{8} = \frac{3}{4}\Omega$

2.




This is a series circuit, which means the total voltage is equal to the sum of the voltage at each resistor.

$V_{total} = V_1 + V_2 = 6V + 24V = 30V$

$R_{total} = 8 + 2 = 10$

$R_{total} = 10\Omega$

$I_{total} = \frac{V}{R} = \frac{30}{10} = 3A$

**Advantages and Disadvantages of Series and Parallel Circuits**

You may be wondering when it is more beneficial to have a series circuit over a parallel circuit - or vice versa, so here are some advantages and disadvantages:

**Advantages of Series**

- They do not overheat
- Easy to learn and make
- We can add more power devices (increase specific voltages)
- All resistors carry the same current

**Advantages of Parallel**

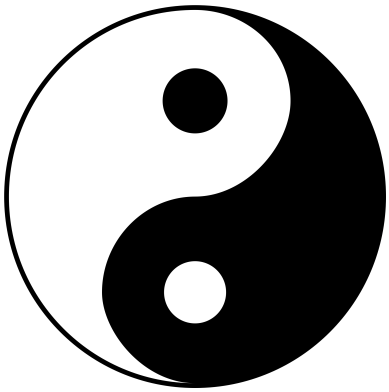
- Each resistor gets the same amount of voltage
- Easy to add or replace resistors without impacting the rest of the circuit
- The current is able to flow through multiple paths

**Disadvantages of Series**

- If there is one break, the entire circuit stops working
- The more resistors you add the greater the resistance will be

**Disadvantages of Parallel**

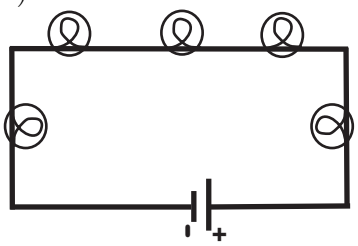
- Requires a lot of wires (expensive)
- Cannot increase the voltage of specific resistors



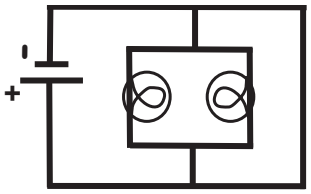
**Problem Set**

1. For each of the diagrams below, determine if the light bulbs are in a series or parallel circuit.

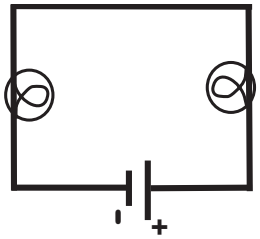
a) Series



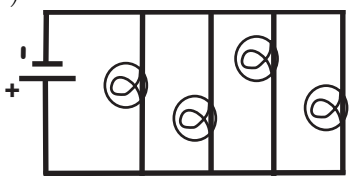
b) Parallel



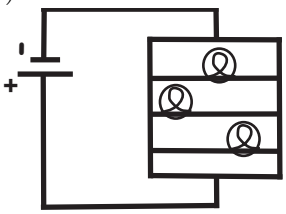
c) Series



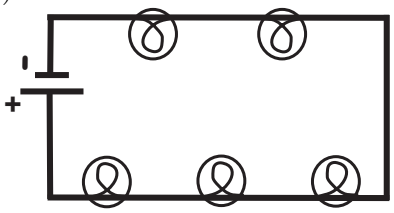
d) Parallel



e) Parallel



f) Series



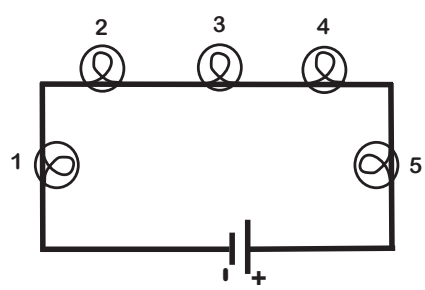
2. Ryan has a string of Christmas lights on his tree in his house, and wants to know if his lights are connected in a parallel or series circuit. How can he check this?
Ryan could unscrew one of his lights on the string to find out if they are connected in a series or parallel circuit. If the entire string of lights goes out when he takes out one light it is series, if all the other lights remain on they are connected in a parallel circuit.

3. Sally is making a series circuit with light bulbs and batteries. When she connects 1 light bulb with 2 batteries it comes on with normal brightness. Fill out the table below to see what happens when Sally uses a different amount of light bulbs and batteries.

| Number of Light Bulbs | Number of Batteries | Brightness (Dim, normal, bright) |
|---|---|---|
| 1 | 2 | Normal |
| 2 | 2 | Dim |
| 2 | 4 | Normal |
| 2 | 6 | Bright |
| 3 | 6 | Normal |

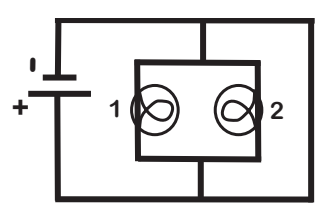
4. For each of the diagrams and corresponding tables below, fill in the missing values in the table using Ohms Law and circuit properties.

a)



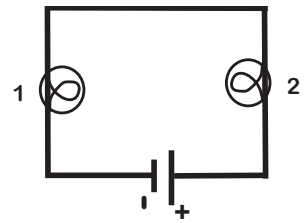

| | | | | | |
|---|---|---|---|---|---|
| $V_{total} = 66V$ | $V_1 = 12V$ | $V_2 = 36V$ | $V_3 = 12\text{V}$ | $V_4 = 2V$ | $V_5 = 4\text{V}$ |
| $I_{total} = 6A$ | $I_1 = 6A$ | $I_2 = 6A$ | $I_3 = 6\text{A}$ | $I_4 = 6A$ | $I_5 = 6A$ |
| $R_{total} = 11\Omega$ | $R_1 = 2\Omega$ | $R_2 = 6\Omega$ | $R_3 = 2\Omega$ | $R_4 = \frac{1}{3}\Omega$ | $R_5 = \frac{2}{3}\Omega$ |

b)



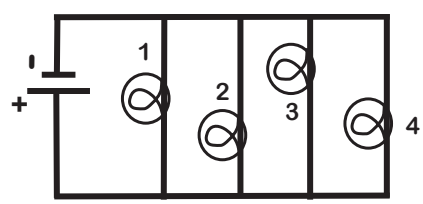

| | | |
|---|---|---|
| $V_{total} = 5V$ | $V_1 = 5V$ | $V_2 = 5\text{V}$ |
| $I_{total} = \frac{17}{4}A$ | $I_1 = 3\text{A}$ | $I_2 = \frac{5}{4}A$ |
| $R_{total} = \frac{20}{17}\Omega$ | $R_1 = \frac{5}{3}\Omega$ | $R_2 = 4\Omega$ |

c)



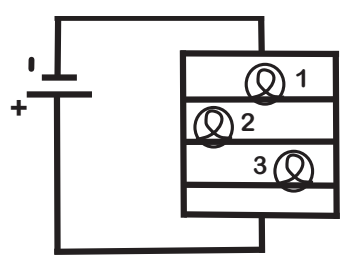

| | | |
|---|---|---|
| $V_{total} = 10\text{V}$ | $V_1 = 8\text{V}$ | $V_2 = 2\text{V}$ |
| $I_{total} = 0.5\text{A}$ | $I_1 = 0.5\text{A}$ | $I_2 = 0.5\text{A}$ |
| $R_{total} = 20\Omega$ | $R_1 = 16\Omega$ | $R_2 = 4\Omega$ |

d)



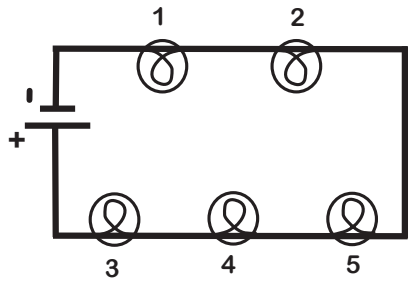

| | | | | |
|---|---|---|---|---|
| $V_{total} = 11\text{V}$ | $V_1 = 11\text{V}$ | $V_2 = 11\text{V}$ | $V_3 = 11\text{V}$ | $V_4 = 11\text{V}$ |
| $I_{total} = 22\text{A}$ | $I_1 = 11\text{A}$ | $I_2 = \frac{11}{2}A$ | $I_3 = \frac{11}{3}A$ | $I_4 = \frac{11}{6}A$ |
| $R_{total} = \frac{1}{2}\Omega$ | $R_1 = 1\Omega$ | $R_2 = 2\Omega$ | $R_3 = 3\Omega$ | $R_4 = 6\Omega$ |

e)




| | | | |
|---|---|---|---|
| $V_{total} = 128\text{V}$ | $V_1 = 128\text{V}$ | $V_2 = 128\text{V}$ | $V_3 = 128\text{V}$ |
| $I_{total} = 32\text{A}$ | $I_1 = 16\text{A}$ | $I_2 = 8\text{A}$ | $I_3 = 8\text{A}$ |
| $R_{total} = 4\Omega$ | $R_1 = 8\Omega$ | $R_2 = 16\Omega$ | $R_3 = 16\Omega$ |

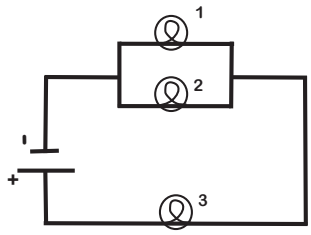
f)




| | | | | | |
|---|---|---|---|---|---|
| $V_{total} = 100\text{V}$ | $V_1 = 12\text{V}$ | $V_2 = 16\text{V}$ | $V_3 = 20\text{V}$ | $V_4 = 24\text{V}$ | $V_5 = 28\text{V}$ |
| $I_{total} = 4\text{A}$ | $I_1 = 4\text{A}$ | $I_2 = 4\text{A}$ | $I_3 = 4\text{A}$ | $I_4 = 4\text{A}$ | $I_5 = 4\text{A}$ |
| $R_{total} = 25\Omega$ | $R_1 = 3\Omega$ | $R_2 = 4\Omega$ | $R_3 = 5\Omega$ | $R_4 = 6\Omega$ | $R_5 = 7\Omega$ |

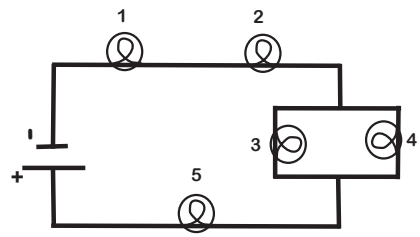
In some cases, a combination of both parallel and series circuits may be used. In order to solve the following questions, treat the parallel component as one single resistor in a series circuit. Solve each component individually. The values for current, voltage, and resistance of the single resistor will be the total values of the parallel component.

g)



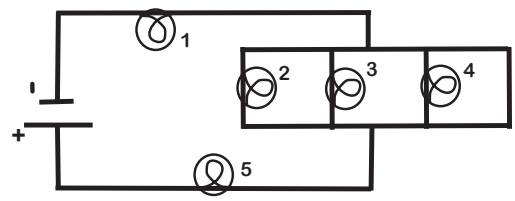

| | | | |
|---|---|---|---|
| $V_{total} = 24\text{V}$ | $V_1 = 12\text{V}$ | $V_2 = 12\text{V}$ | $V_3 = 12\text{V}$ |
| $I_{total} = 4\text{A}$ | $I_1 = 1\text{A}$ | $I_2 = 3\text{A}$ | $I_3 = 4\text{A}$ |
| $R_{total} = 6\Omega$ | $R_1 = 12\Omega$ | $R_2 = 4\Omega$ | $R_3 = 3\Omega$ |

h)



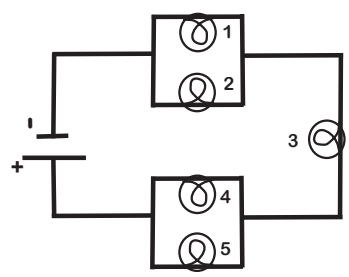

| | | | | | |
|---|---|---|---|---|---|
| $V_{total} = 18\text{V}$ | $V_1 = 6\text{V}$ | $V_2 = 3\text{V}$ | $V_3 = 6\text{V}$ | $V_4 = 6\text{V}$ | $V_5 = 3\text{V}$ |
| $I_{total} = \frac{3}{2}\text{A}$ | $I_1 = \frac{3}{2}\text{A}$ | $I_2 = \frac{3}{2}\text{A}$ | $I_3 = 1\text{A}$ | $I_4 = \frac{1}{2}\text{A}$ | $I_5 = \frac{3}{2}\text{A}$ |
| $R_{total} = 12\Omega$ | $R_1 = 4\Omega$ | $R_2 = 2\Omega$ | $R_3 = 6\Omega$ | $R_4 = 12\Omega$ | $R_5 = 2\Omega$ |

i)




| | | | | | |
|---|---|---|---|---|---|
| $V_{total} = 154\text{V}$ | $V_1 = 132\text{V}$ | $V_2 = 12\text{V}$ | $V_3 = 12\text{V}$ | $V_4 = 12\text{V}$ | $V_5 = 10\text{V}$ |
| $I_{total} = 22\text{A}$ | $I_1 = 22\text{A}$ | $I_2 = 4\Omega$ | $I_3 = 6\text{V}$ | $I_4 = 12\text{V}$ | $I_5 = 22\text{A}$ |
| $R_{total} = 7\Omega$ | $R_1 = 6\Omega$ | $R_2 = 3\Omega$ | $R_3 = 2\Omega$ | $R_4 = 1\Omega$ | $R_5 = \frac{5}{11}\Omega$ |

j)




| | | | | | |
|---|---|---|---|---|---|
| $V_{total} = 25\text{V}$ | $V_1 = 8\text{V}$ | $V_2 = 8\text{V}$ | $V_3 = 12\text{V}$ | $V_4 = 5\text{V}$ | $V_5 = 5\text{V}$ |
| $I_{total} = 10\text{A}$ | $I_1 = 8\text{A}$ | $I_2 = 2\text{A}$ | $I_3 = 10\text{A}$ | $I_4 = 8\text{A}$ | $I_5 = 2\text{A}$ |
| $R_{total} = \frac{5}{2}\Omega$ | $R_1 = 1\Omega$ | $R_2 = 4\Omega$ | $R_3 = \frac{6}{5}\Omega$ | $R_4 = \frac{5}{8}\Omega$ | $R_5 = \frac{5}{2}\Omega$ |