1. Use the Euclidean algorithm to compute the following GCDs:
   
   (a) \( \gcd(204, 99) \)
   
   (b) \( \gcd(1053, 993) \)
   
   (c) \( \gcd(7404, 7032) \)

2. Find a solution for each of the following LDEs, or explain why one does not exist.

   (a) \( 204x + 99y = 3 \)
   
   (b) \( 1053x + 993y = 7 \)
   
   (c) \( 7404x + 7032y = 36 \)

3. Can 10 000 be expressed as a sum of two integers, one of which is divisible by 126 and the other divisible by 81? If so, find examples of such integers. If not, explain why.

4. Can 10 000 be expressed as a sum of two integers, one of which is divisible by 614 and the other divisible by 72? If so, find examples of such integers. If not, explain why.

5. Find an integer \( n \), which, when divided by 78 leaves a remainder of 37; and when divided by 29 leaves a remainder of 17.

6. (a) Use the Euclidean algorithm to find a solution to \( 25x + 10y = 215 \), the LDE from example (III).

   (b) Does your answer in part (a) make sense in the context of the problem? If not, how can we find a solution that does make sense?
**Challenge Problems**

7. (a) Use the division algorithm to show that \( \gcd(k + 1, k) = 1 \) for any integer \( k \).
    (b) Use the division algorithm twice (i.e., the Euclidean algorithm) to show that
        \[
        \gcd(7k + 6, 6k + 1) = 1
        \]
        for any integer \( k \geq 1 \).

8. (a) Let \( a \) and \( b \) be integers. For which integers \( c \) does \( ax + by = c \) have a solution?
    (b) Let \( a, b, \) and \( c \) be integers. For which integers \( d \) does \( ax + by + cz = d \) have a solution?
    (c) Find a solution to the 3-variable LDE \( 18x + 14y + 63z = 5 \).