Problem Set 1

Intermediate Math Circles Winter 2018
Fun With Inequalities

Linear Inequalities- Single Variable

Solve each of the following.

1. \( x + 5 < \frac{7}{2} \) \( x < \frac{7}{2} - 5 \)
   \( x < \frac{7}{2} - \frac{10}{2} \)
   \( x < -3 \)

2. \( 3 - \frac{x}{2} \geq -8 \)

3. \( -1 - 3x \leq 4x + 10 \)

4. \( 2x + 5 > 4x - 7 \)

5. \( -\frac{2}{3}x + \frac{3}{7} \leq 5 - \frac{x}{2} \)

Absolute Values

Solve each of the following algebraically. Check your answer graphically.

1. \( |x + 6| = 5 \)

2. \( |x - 4| \geq 1 \)

3. \( |4 - x| \geq 1 \)

4. \( |2x + 1| < 7 \)

5. \( |x - 2| + |x + 5| = 8 \)

6. \( |x| + |2 - x| \leq 12 \)
Properties

1. Which of the eight properties of $\leq$ also hold for $<$?

2. Use whichever of the properties (1) to (8) that you need to prove the following
   
   (a) If $a \leq b$ and $c \leq d$, then $a + c \leq b + d$.
   
   (b) If $0 \leq a \leq b$ and $0 \leq c \leq d$, then $0 \leq ac \leq bd$.

3. (a) If $a \leq b$ and $c \leq d$, is it true that $ac \leq bd$?

   (b) If $a \leq b$, is it true that $\frac{1}{b} \leq \frac{1}{a}$?

4. Show that if $a < b$, then $a < \frac{1}{2}(a + b) < b$.

5. Show that the sum of a positive number and its reciprocal is at least 2.
   In other words show that 
   
   $$a + \frac{1}{a} \geq 2$$