

Math Circles  
 11 October 2017  
 Logic and Problem Solving, Part 1  
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**EXERCISES**

1. Prove the following:
  - (a)  $A - (B \cap C) = (A - B) \cup (A - C)$
  - (b)  $A \cup (A \cap B) = A$
  - (c)  $A - B = A \cap \overline{B}$
  - (d)  $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$
2. In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in art and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories?
3. Five teams, A, B, C, D, and E, play each other in a hockey tournament. Each team plays every team exactly once. B and D play each other first, and B wins. We mark wins with a W and losses with an L. The table of results is shown below:

	A	B	C	D	E
A	X				
B		X		W	
C			X		
D		L		X	
E					X

After that game, the tournament results are as follows:

Team	# of Wins	# of Losses
D	1	3
C	2	2
A	0	4
E	4	0

4. Zoey, Zoey's husband, their son, their daughter and Zoey's brother were involved in a theft. One of the five stole a credit card from the one of other four. The following facts are known:
  - (a) A man and a woman were eating together at a restaurant at the time of the theft.
  - (b) The thief and the victim were together at the bank at the time of the theft.
  - (c) One of the two children were alone at the time of the theft.

- (d) Zoey and her husband were not together at the time of the theft.
- (e) The victim's twin was innocent.
- (f) The thief was younger than the victim.

Who was the thief and who was the victim? Explain your reasoning.

5. Four people conspired to murder their accountant.

Based on information gathered from other reliable sources, the police know that each of the four suspects visited the accountant individually exactly once on the day of the murder.

As well, before the four suspects were interviewed, they conspired together and agreed that every statement they made to the police would be a lie. Each of the suspects made exactly two statements:

Avery said:

- (a) None of the four of us killed the accountant.
- (b) The accountant was alive when I left.

Blake said:

- (a) I was the second person to arrive.
- (b) The accountant was dead when I arrived.

Carl said:

- (a) I was the third person to arrive.
- (b) The accountant was alive when I left.

Dave said:

- (a) The killer did not arrive after I did.
- (b) The accountant was dead when I arrived.

Which one of the four killed the accountant? Show your reasoning.

6. Xena is 13 years old. Over Thanksgiving, 12 children, including Xena, gathered at her home: 4 children each from 3 families, A, B and C, one of which was Xena's family. Interestingly enough, each child was of a different age and together represented 12 of the numbers from 1 to 13. In other words, one of the ages was not represented by a child.

For the fun of it, Xena added up the ages of the children in each family and got the following results:

- Family A: Total of 41 years, including the 12 year old;
- Family B: Total of 22 years, including the 5 year old;
- Family C: Total of 21 years, including the 4 year old.

Only Family A had two children who were born one year apart.

- Which family is Xena from?
- How old is each child in each family?