Diophantus of Alexandria Problem Set

(i) Convert the following polynomials into the notation that the Greeks used. The table in the slides will help.

1. $3x^2 + 4x$
2. $-167x^3 + 24x$
3. $2x^5 - 1$
4. $x^6 - 2x^5 + 3x^4 + 4x^3 + 5x^2 - 6x + 7$

(ii) Convert the following Greek polynomials into standard notation. The table in the slides will help.

1. $K^\Upsilon \mu \zeta \pi \& \Delta^\Upsilon \phi$
2. $K^\Upsilon K^\tau \nu \theta \zeta \delta \& \Delta^\Upsilon \zeta$
3. $\& K^\Upsilon \zeta \Delta^\Upsilon \nu \gamma M \eta$
4. $\Delta^\Upsilon \Delta \delta M \omega$

(iii) Solve the following problems from Diophantus’ Arithmetica

1. Book I Problem 16. Find three numbers such that when any two of them are added, the sum is one of three given numbers. Say the given sums are 20, 30 and 40.
2. Book I Problem 27. Find two numbers such that their sum and product are given numbers; say their sum is 20 and their product is 96. [Hint: Call the numbers $10 + x$ and $10 - x$ so that one of the conditions is already satisfied.]
3. Book II Problem 10. Find two square numbers having a given difference; say the given difference is 60. [Hint: Take $x^2$ for one of the squares and $(x + a)^2$ for the other, where $a$ is an integer chosen so that $a^2$ is not greater than 60.
4. Book II Problem 12. Find a number whose subtraction from two given numbers (say 9 and 21) allows both remainders to be squares. [Hint: Call the required number $9 - x^2$ so that one condition holds automatically.]
5. Book II Problem 13. From the same required number to subtract two given numbers so as to make both remainders squares; say the given numbers are 6 and 7. [Hint: Seek a number which exceeds a square by 6, say $x^2 + 6$.]
6. Book II Problem 26. Find two [rational] numbers such that their product added to either gives a [rational] square and the sides of the two [rational] squares so formed (ie. the square roots of the two squares) added together produces a given number, say 6. Hint: Let $x$ and $4x - 1$ be the numbers.
7. Book III Problem 21. To divide a given number into two parts and to find a square which, when added to either of the parts, gives a square. We will give the number 20 and the square $x^2 + 2x + 1$. [Hint: What polynomials can be added to $x^2 + 2x + 1$ to give another square?]
8. Book IV Problem 6. Find two [rational] square numbers and a [rational] cube such that when the squares are added together, another square is formed and when one of the squares is added to the cube, another cube is formed. Hint: Let $x^3$ and $9x^2$ be the cube and the square. Find a square to be added to the given square to give a second square and then consider the cubic condition.