



Grade 6 Math Circles
November 22nd/23rd
Beaver Computing Competition

Computer science is a field that emerged from mathematics. Today these two fields continue to push the advancement of the other one forward. Theoretical aspects of computer science is often the playground for mathematicians to explore new and existing branches of math and computationally difficult problems in math required the use of computers to solve.

To provide an glimpse of what solving problems in computer science is like and how the fields of math and computer science are intertwined, we will be solving problems from the Beaver Computing Contest (BCC). No prior knowledge in computer science is required. The Beaver Computing Contest (BCC) is a problem solving contest with a focus on computational and logical thinking but only require comfort with concepts found in school.

Catch Up

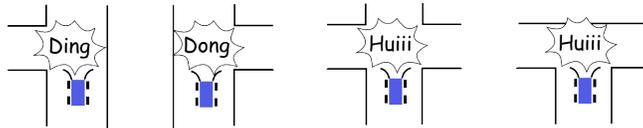
Allison Beaver has a pile of 10 trees. Beatrice Beaver has only 1 tree. Allison Beaver and Beatrice Beaver start chewing at exactly the same time. They add every tree they chew down to their own pile of trees. Alison Beaver chews down one tree per hour. Beatrice Beaver chews down trees at a different rate. In the first hour, she will chew down one tree. In the second hour, she will chew down two trees. In the third hour, she will chew down three trees, and so on.



After how many hours will Beatrice first have at least as many trees as Allison?

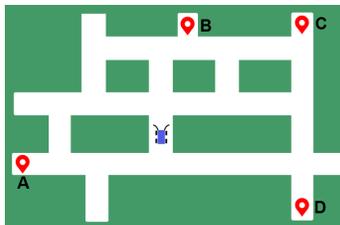
Robotic Car

Beavers have developed a robotic car. It has sensors that detect intersections. It produces the sounds shown below, when it is possible to turn left, right or both directions. The robotic car can go straight through an intersection (when possible), turn right (when possible) or turn left (when possible). The robotic car cannot make U-turns and cannot reverse.



It automatically stops when it senses an obstacle in front of it.

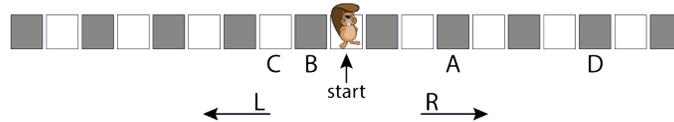
The car drives around the map shown below, starting at the indicated position. As it drives around the map, it produces the sound **Huii Ding Huii Dong**, in that order.



At which location of the four locations shown above did the car stop at?

Jumping

A beaver moves in strange ways. He starts at the middle, as shown below. He will make five moves, alternating between right (R) and left (L): he first moves right, then left, then right, then left, and finally right.

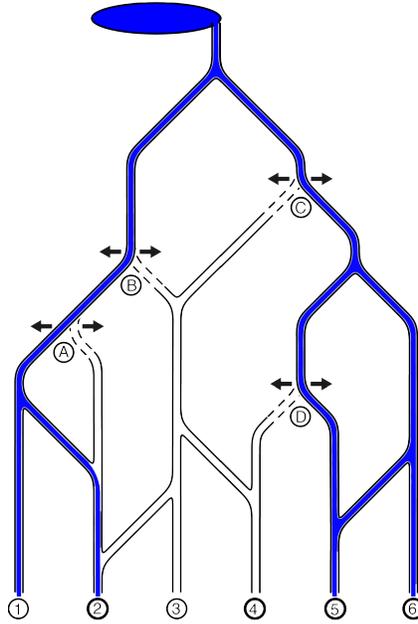


On each move, he can jump 1, 2, 3, 4 or 5 positions from his current position. He picks each distance exactly once. For example, he can move right by 2, left by 1, right by 5, left by 4 and right by 3, ending at $2 - 1 + 5 - 4 + 3 = 5$ positions to the right from where he started. For your convenience, every second position is shaded.

Out of the four positions marked by a letter, there is one he can not end up on. Which one?

Irrigation System:

Beavers have created a nifty irrigation system for their fields. The water flows from a lake at the top of the hill all the way down to the fields numbered 1 to 6 at the bottom. Along the water canals, the beavers have installed four water gates A to D, where the water can only flow either to the left \leftarrow or to the right \rightarrow . An example showing how these may be set to have the water flow to fields 1, 2, 5 and 6 is shown below.



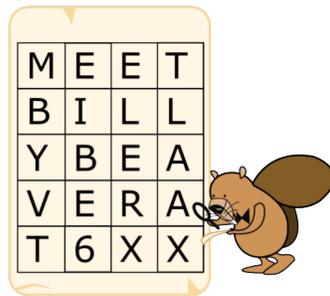
What is the correct configuration for the water gates to irrigate only fields 2, 4, 5 and 6?

Secret Message

Agents Boris and Bertha communicate using secret messages. For example, Boris wants to communicate the following message to Bertha.

MEETBILLYBEAVERAT6

He writes each character in a 4 column grid from left-to-right and row-by-row starting from the top. He puts an X in any unused spaces in the bottom row. The result is shown below.



Then he creates the encrypted message by reading the characters from top-to-bottom and column-by-column starting from the left:

MBYVTEIBE6ELERXTLAAX

Bertha then uses the exact same method to reply to Boris. The encrypted message she sends to him is:

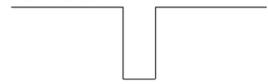
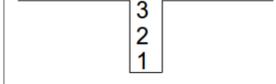
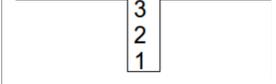
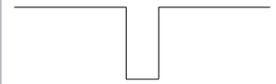
OIERKLTIELH!WBEX

What was the original message sent by Bertha? (It does not contain X)

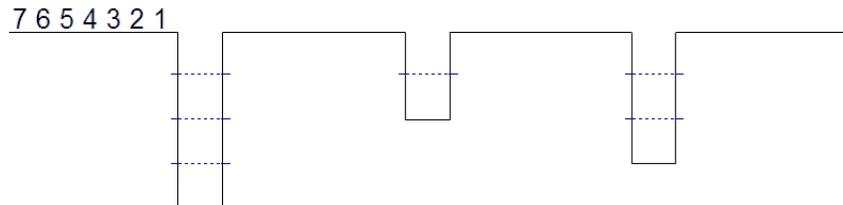
Rabbit Hole

Beavers are going for a stroll in the woods. They walk in a line, one beaver after another.

Nasty rabbits have dug holes along the beavers route. The holes are deep enough so that some number of beavers will fall in. Once the hole is full of beavers, all the beavers behind the hole walk on top of the beavers in the hole. Then the beavers climb out of the hole from the top to the bottom. The example below involves beavers 1, 2, 3, 4, 5; (1 being the first one in line, and 5 being the last in line) and one hole deep enough for three beavers:

Initially	First three beavers all in the hole	Walking on top of beavers in the hole	Every beaver out of the hole and back in line
5 4 3 2 1 	5 4 3 2 1 	5 4 3 2 1 	1 2 3 5 4 

If there are 7 beavers (with 1 being the first one in line, and 7 being the last in line), and the first hole encountered holds four beavers, the second hole encountered holds two beavers and the last hole encountered hold three beavers, what is the order of the beavers after all beavers have passed over these three holes?



Chain

Cindy wrote a computer program which can be used to paint a chain of squares and triangles. The following instructions make the program draw single shapes:

- S draws a big square
- s draws a small square
- T draws a big triangle
- t draws a small triangle

There is also a repeat instruction $N[I]$ where N is a number and I is a sequence of instructions. This command makes the program follow the instruction sequence I a total of N times. For example, the instruction sequence $s\ 2[T\ t]\ S$ makes the program paint this chain:

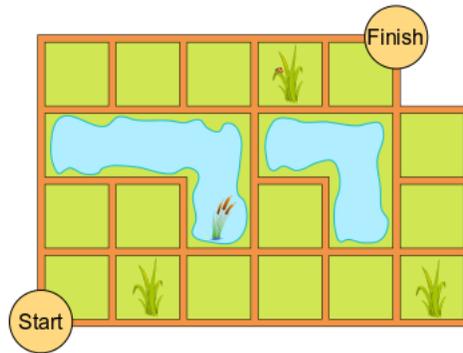


Write a program that will paint the following chain using the least amount of code possible?



You Must Turn

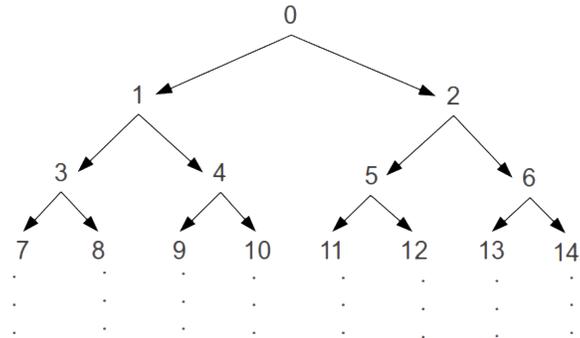
A king loves long travels in his coach, so he orders his coachman to never go straight when reaching a new road. That is, the coachman must turn either right or left if he comes to any intersection. This applies even for intersections with three roads.



Find a shortest path for the coachmen from start to the finish that does not break the rule that the coach can never go straight. What length does it have?

Descend the Tree

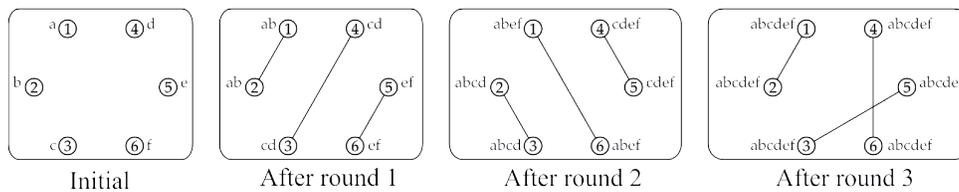
Your friend writes down all of the integers starting from 0 to 2046 in the following way:



Specifically, below every number there are two numbers: one on the left and one on the right. For example, below 3, the number 7 is on the left, and the number 8 is on the right. The numbers can be read in increasing order from the top row to bottom row and from left-to-right within a row. Notice that we can get from 0 to 11 by going right (R) (from 0 to 2), left (L) (from 2 to 5) then left (L) (from 5 to 11). Starting at 0, what sequence of left (L) and right (R) moves will end up at 100? what about 1000?

Spies

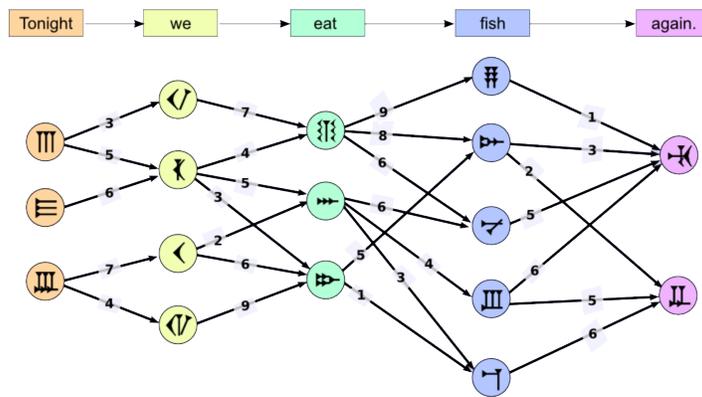
Every Friday, six spies share all the information they have gathered that week. A spy can never be seen with more than one other spy at the same time. They have to conduct several rounds of meetings where they meet up in pairs and share all information they have at that point. The group of 6 spies needs only three rounds to distribute all information. Before the meetings, each spy holds a single piece of information. (spy 1 knows a, spy 2 knows b, etc.). In the first round, spies 1 and 2 meet and share information so now both know ab. The diagram shows the initial information as well as the three rounds of meetings, with lines indicating which spies meet in each round. It also shows which pieces of information they all have. After three rounds all information has been distributed.



After an international incident one spy has stopped attending the meetings. What is the minimum number of rounds needed for the five remaining spies to share all information?

Translation Machine

Betty programs a machine that translates an English sentence to a Beavarian sentence one word at a time. However, there are several possible Beavarian words for each English word! Betty noticed that different words occur next to each other at different rates. For example, smart beaver is more common than intelligent beaver. She gives scores for word pairs: the higher the score, the more common the word pair is. An English sentence with five words must be translated into five Beavarian symbols. In the picture below, arrows labelled with scores connect all valid word pairs. The total score for a translation is the sum of the scores of the four arrows used.



What is the highest possible total score for a translation of this sentence.

Height Game

Young beavers Amy, Beavy, Cuttree, Diggy, and Eary, are all different heights. They line up, one after another, facing the same way, in some order. Then each beaver finds all the other beavers that are taller than himself/herself. He/she counts how many of these taller beavers are in front of him/her and how many are behind him/her.

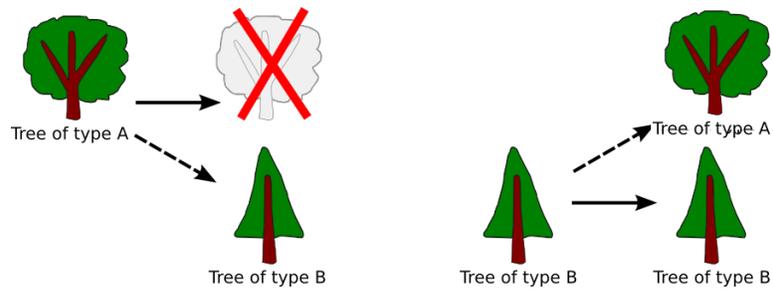
The results are shown in the following table:

Name	Number of taller beavers	
	in front	behind
Amy	1	2
Beavy	3	1
Cuttree	1	0
Diggy	0	0
Eary	2	0

In what order are they standing in, from the front of the line to the back of the line.

Trees in a Forest

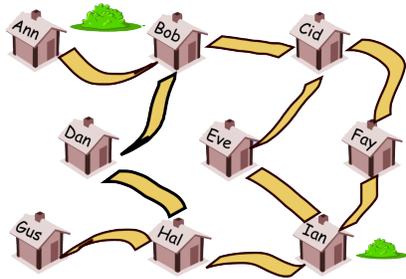
In a forest, there are two types of trees. Type A trees live for only one year, but after this year, they transform into a tree of type B. Type B trees live forever and produce a new tree of type A at the end of every year. These two scenarios can be illustrated as follows, with each arrow representing the transformation at the end of one year.



For example, if we start with one type A tree, after one year there will be one type B tree in the forest. Similarly, if we start with one B tree, there will be one type A tree and one type B tree in the forest after one year. If we start with just one type A tree in the forest, how many type A trees and type B trees will there be after 10 years?

Firefighter

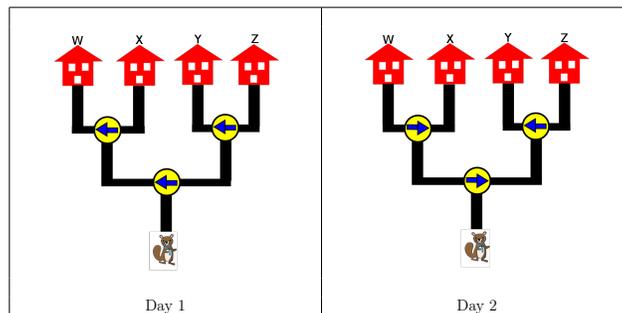
The mayor of Beaverville is looking for volunteer firefighters. A map showing the possible volunteers homes and how they are connected by roads is shown below. He wants to ensure that every home in the town is either the home of a volunteer or is connected by a single road to the home of a volunteer.



What is the minimum number of volunteers the mayor needs?

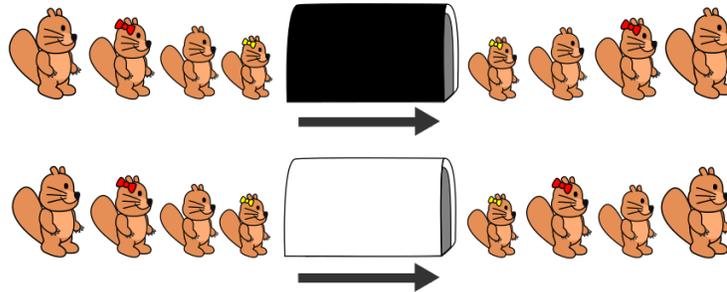
Visiting Friends

Mr. Beaver has 4 friends living in different villages, and he plans to visit one of these friends every afternoon. Mr. Beaver will follow the direction of the arrow on signs at each intersection. Initially, all arrows point to the left road. When passing an intersection, Mr. Beaver switches the arrow to the opposite direction. For example, on Day 1, Mr. Beaver takes the road on the left at the first intersection, takes the left road on the second intersection, and reaches Village W. On Day 2, Mr. Beaver turns right at the first intersection, then left at the second intersection, and arrives Village Y.



Problem Set

1. **Zebra Tunnel** There are two kinds of tunnels in BeaverLand. When a sequence of beavers enter a black tunnel one after the other, they come out in reverse order. When a sequence of beavers enter a white tunnel one after the other, they come out with the first and the last beaver swapped.

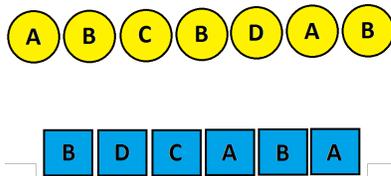


A Beaver family goes through three tunnels as shown.



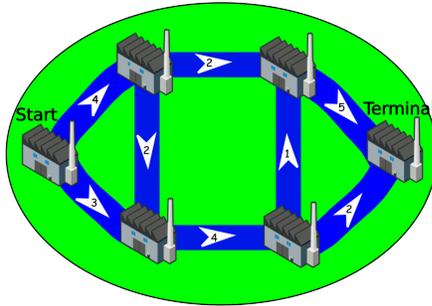
In what possible order are they arranged in when they come out of that last tunnel?

2. **Best Match** You can connect a circle and a square that both have the same letter in them using a straight line.



What is the maximum number of such connections you can make without crossing any lines?

3. **Log Mover** Leslie the Beaver must drag logs one at a time through the system of canals and stations shown below. The logs must be dragged through canals in the direction of the arrows. The number in each arrow is the maximum total number of logs that can ever be dragged through the corresponding canal.



What is the maximum number of logs that Leslie can drag from the start station to the terminal station?

4. Putting People in a Line

You are arranging people in order based on the numbers on their shirts. The order to start is:

732985146

You will arrange individuals using the following technique:

- If the person on the left has a number which is larger than that of the person on the right, switch the positions of those two people; otherwise, leave them in the order they are in.
- Move to the right one position, so that you are comparing one new person with one of the people just compared, and repeat the above comparison and potential swap. Once you have compared the right-most two people in the list, we call this one pass over the list. How many passes over the list are required until the list is in the order

123456789

5. **The Thief** The famous Blue Diamond was stolen from a museum. A thief swapped it for a cheap green imitation diamond. The day it was stolen, 2000 people entered the diamond room one by one. Inspector Bebro must find the thief by interviewing some of these people. He has a list of all 2000 people in the order they entered the room. The only question he can ask a person is:

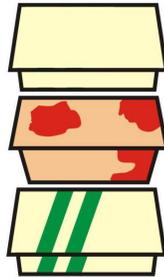
Was the diamond green or blue when you saw it?

The thief will lie and say green but everyone else will tell the truth.

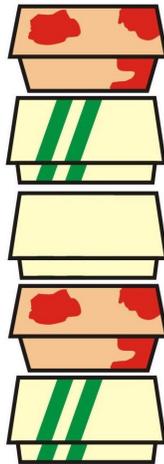
Inspector Bebro is very clever and will use a strategy where the number of people interviewed is as small as possible.

Is it possible to find the thief after interviewing under 200 people? If so how?

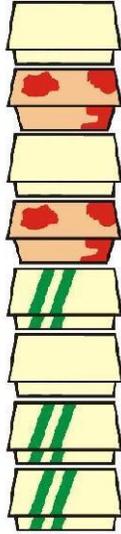
6. **Burgers** Tim and Tom are working at a burger restaurant. Tim cooks burgers one at a time. After cooking a burger, he places it into one of three different boxes: one with stripes, one with a pattern and one plain box. If he has cooked three burgers, he would have a stack as follows:



If he cooked two more burgers, he would have a stack like:



As Tim cooks a burger, he places that box on the top of the stack of not yet sold burgers, and continues to cycle through the three different boxes (stripe, pattern, plain, stripe, pattern, plain, ...) into which to place the burger. Tom is selling the burgers one at a time and always takes the uppermost box from the stack. Tim is cooking faster than Tom can sell the burgers. After some time, Tom has sold some burgers and Tim has cooked more burgers. Suppose the stack of unsold burgers looks like the following:



What is the fewest possible number of burgers sold by Tom?

7. **Magic Potions** Betaro Beaver discovered five types of magic potions with the following effects:

- makes ears longer
- makes teeth longer
- makes whiskers longer
- colours a nose white
- colours eyes white

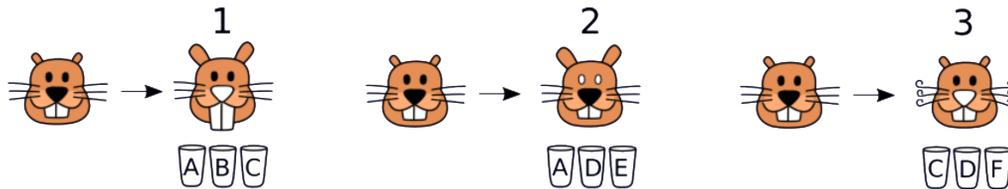
Betaro put each magic potion into a different cup and additionally put water into a sixth cup. Betaro labelled the cups A to F and forgot to record which cup contains which magic potion!



Betaro called Taki for help. She solved the problem by experimenting on three of their other friends:

- Using the contents of cups A, B and C together produced the effects shown in Figure 1.

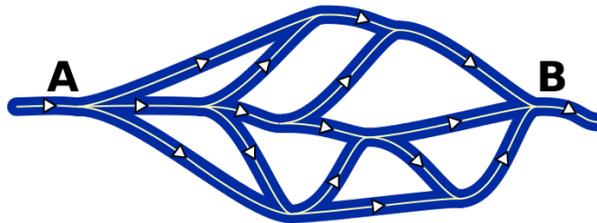
- Using the contents of cups A, D and E together produced the effects shown in Figure 2.
- Using the contents of cups C, D and F together produced the effects shown in Figure 3.



Which one of the cups contains pure water?

8. River Inspection

Beavers want to explore the system of rivers below. At least one beaver has to swim along each river. Due to the heavy current, beavers can only swim downstream and they can only do one trip from A to B. So the beavers start at A, and meet at B.



What is the minimum number of beavers needed to explore the system of rivers?

9. **Deleting Game** Bob the Beaver plays a game against Sam the Squirrel. At first Bob removes four numbers from the list 1, 2, 3, 4, 5, 6, 7, 8. Then Sam deletes two of the remaining numbers. Bob wants the positive difference between the remaining two numbers to be as large as possible. Sam wants this positive difference to be as small as possible. Both Bob and Sam know what the other player is trying to achieve. What is the positive difference between the last two numbers if both Bob and Sam play as well as possible?