



Grade 7/8 Math Circles

March 29 & 30 2016

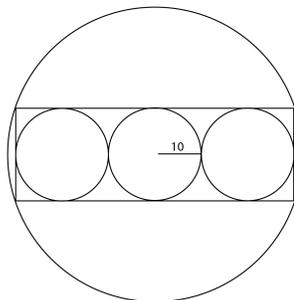
Review

Logic

1. The word MATHEMATICS is to be centered in a space allowing 37 letters. No spaces are permitted between the letters. How many blank spaces must the typist leave before starting to type the word?
2. Each letter in the subtraction below represents a single digit. What are $k, p, q,$ and r ?

$$\begin{array}{r}
 6 \quad p \quad q \quad r \\
 - \quad k \quad 3 \quad 5 \quad 9 \\
 \hline
 1 \quad 5 \quad 8 \quad 8
 \end{array}$$

3. At a party, exactly 15 people ate hot dogs, 12 ate hamburgers. 10 people ate both, and 3 ate neither. How many people were at the party?
4. In the diagram below, what is the area of the largest circle?



5. A positive integer is to be placed in each box. The product of any 4 adjacent integers is 120. What is the value of x ?



Modular Arithmetic

1. What is $604 \times 123 \pmod{60}$?
2. Convert 1110001 to decimal.
3. Convert 341 to binary.
4. Anna was facing East and rotated 3125° counterclockwise. Which direction is she facing now?
5. You have 7 goblets one of which is real gold. When you align them and count back and forth then the golden goblet would be the 1000th one that you count. Which one is the golden goblet?

Number Theory

1. How many positive integers less than 20 are co-prime with 20? What are those numbers?
2. What is the GCD of 861 and 984?
3. What is the remainder when 2^{2007} is divided by 15?
4. What is the remainder when 5^{119} is divided by 59?
5. The number of students in a school is an integer between 500 and 600. When grouped into groups of 12, 20 or 36, there are 7 students left over. How many students are in the school?

Word Problems

1. A set of 5 different positive integers has an average of 11. What is the largest possible number in this set? (Guass, 2000).
2. In a certain month, three of the Sundays have dates that are even numbers. Which day of the week does the tenth of this month fall on? (Gauss, 2000).
3. Water is poured from a full 1.5 L bottle into an empty glass until both the glass and the bottle are $\frac{3}{4}$ full. What is the volume of the glass? (Gauss, 2004)
4. What is $a + b$ if $a(x + b) = 3x + 12$ is true for all values of x ? (Cayley, 2013).

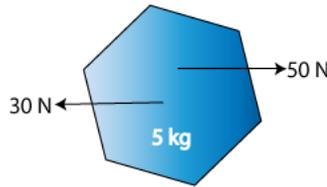
5. $109/x$ leaves a remainder of 4. What is the sum of all such two digit positive integers x ? (Cayley, 2013).

Physics

1. What is the difference between speed and velocity?
2. What quantity does the following formula represent and what are its units?

$$\frac{v_2 - v_1}{t}$$

3. What is the acceleration of the block? What is the acceleration if the left force is 50 N?

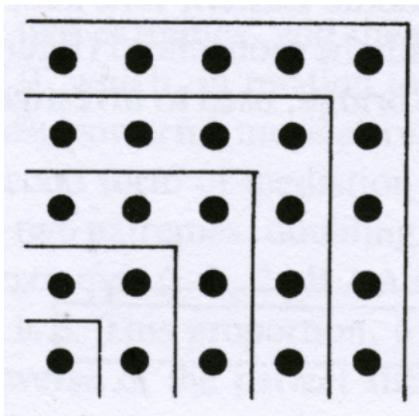


4. I travel 2 km [E] in 4 hrs and 700 m [N] in 45 mins. What is my velocity and speed? Give 2 decimal places.
5. When is size of velocity = speed?
6. How much would you weigh on Jupiter, where acceleration due to gravity is $2.65g$? g is the acceleration due to gravity on Earth. Assume your mass is 42 kg.
7. What is the weakest fundamental force?

Mathematical Thinking

1. How many primes are there?
2. What is the probability that I will not: roll a six and then something other than a six on a six-sided die? (in %).
3. Prove that the product of two odd numbers is odd.

4. This image shows that $1 + 3 + 5 + 7 + 9 = 5 \times 5$. What is $1 + 3 + 5 + \dots + 99$?



5. Prove that $\sqrt{2}$ cannot be written as a fraction (ie. it is irrational).