UNIVERSITY OF WATERLOO

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Centre for Education in
Mathematics and Computing

Grade 6 Math Circles

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Jeopardy Solutions

Jeopardy

Angles

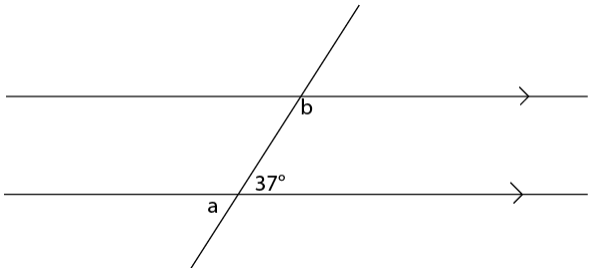
100 points: What type of angle is 73°? **acute**

200 points: What type of polygon is below? **concave**

300 points: What is the sum of the interior angles of a decagon? (10-sided polygon) **1440°** $\rightarrow 180° \times (n-2)$

400 points: How many diagonals are in a decagon? (10-sided polygon) **35** $\rightarrow \dfrac{n \times (n-3)}{2}$

500 points: What are the angles a and b below? $a = 37°$**,** $b = 143°$



Prime Time

100 points: Is 1 prime or composite? **neither**

200 points: Find the prime factorization of 1225. $5 \times 5 \times 7 \times 7$

300 points: Is 4587 prime? Why or why not? **No it is not prime. It is divisible by 3 since all of the digits add to a number divisible by 3.**

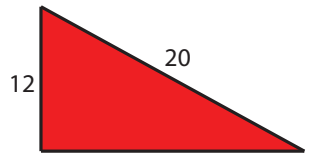
400 points: What is the smallest number that you must multiply 45 by so that the product is divisible by 14? **14**

500 points: Find the LCM and GCD of 50, 75, and 9. **LCM is 450, GCD is 1**

Pythagorean Theorem

100 points: What are the two shorter sides of a right-angled triangle called? **legs**

200 points: Find the missing side length below. **16**



300 points: Find a Pythagorean Triple with 21 as the first number. **21, 220, 221**

400 points: Find a Pythagorean Triple where m is 17 and n is 14. **93, 476, 485**

500 points: Find the area of a square with a diagonal of length $\sqrt{128}$. **64**

Proportions

100 points: Convert 150% into a fraction. $\frac{3}{2}$

200 points: Which is a better deal? 4 chocolates for \$7 or 15 chocolates for \$27?

4 chocolates for \$7 (unit price is 1.75 compared to 1.80)

300 points: Given a 40% discount on a \$514 suit, what is the discounted price? **\$308.40**

400 points: Gary makes a 100mL solution in chemistry. He adds 36mL of a 17% acidic solution and 64mL of a 52% acidic solution. How acidic is the final solution? **39.4%**

500 points: On a graduated commission of 10% of the first \$150, 12% of the next \$300 and 15% of all sales afterwards, how much must you sell in order to make \$1200? **\$8110**

Random

100 points: Which of the following are prime? 1, 3, 4, 6, 23, 231, 234, 23896 **3, 23**

200 points: What is the sequence containing the first 2 odd prime numbers? **{3,5}**

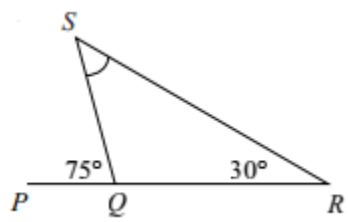
300 points: In a class, a total of 5 people like orange. A total of 5 people like purple. A total of 3 people like both. How many people are there in the class? **7**

400 points: Solve for x: $(x+2) \times 3 = 21$ $x = 5$

500 points: Free Points (No Question).

Gauss Prep (Full solutions to Gauss problems can be found online)

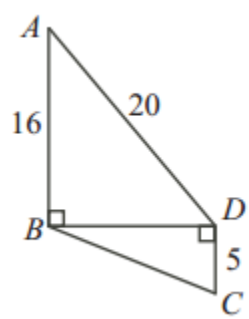
100 points: In the diagram shown, PQR is a straight line segment. What is angle QSR? **45°** (Grade 7, 2014, #17)



200 points: What number, when doubled and then increased by 13, equals 89? **38** (Grade 8, 2003, #6)

300 points: In a jar, the ratio of the number of oatmeal cookies to the number of chocolate chip cookies is 5:2. If there are 20 oatmeal cookies, how many chocolate chip cookies are there? **8** (Grade 8, 2002, #8)

400 points: In the diagram, what is the length of BC? **13** (Grade 8, 2006, #16)



500 points: Three tenths of our planet Earth is covered with land and the rest is covered with water. 97% of the water is salt water and the rest is fresh water. What percentage of the Earth is covered in fresh water? **2.1%** (Grade 8, 2000, #16)

Double Jeopardy

Sets

200 points: What is a member of a set called? **element**

400 points: What is the size of the following set? **1**

{number of questions on the Jeopardy board}

600 points: Given the universal set U = {all positive whole numbers less than 17} and P = {even numbers} find $\overline{P}$. **{1,3,5,7,9,11,13,15}**

800 points: Given α = {A,B,C,D,E} and β = {R,D,A,H,F} find $\overline{\alpha} \cup \overline{\beta}$ **{B,C,E,R,F}**

1000 points: Given α = {A,B,C,D,E}, β = {R,D,A,H,F}, σ = {G,B,D,E,H}, and μ = {F,C,R,A,G} find $(\alpha \cap \beta) \cup (\overline{\sigma \cup \mu})$. **{A,D}**

Counting

200 points: In a family of 10, everyone plays soccer or quidditch. 4 members play both sports and 1 member plays only soccer. How many family members only play quidditch? **5**

400 points: You are making pizza. There are 3 choices of crusts, 2 choices of sauce, and 10 choices of toppings. Assuming you can only choose one of each, how many different pizzas can you make? **60**

600 points: Harry, Ron, Hermione, George, Bill, Charlie, Molly, Arthur, Neville, Ginny, and Luna are going to watch the Quidditch World Cup. How many different ways can they be seated? $11! =$ **39 916 800**

800 points: Padma goes to the ice cream parlour. There are 3 choices of cones and 17 choices of ice cream flavours. If Padma is allowed to have 3 scoops of ice cream, how many different ways can she order ice cream? $3 \times 17 \times 16 \times 15 =$ **12 240**

1000 points: Hannah, Sarah, Josh, Ben, and Gayle are going to watch *Despicable Me*. How many different ways can they sit so that Hannah and Gayle are next to each other? $4! \times 2! = 48$

Sequences

200 points: What is the rule (in words) for the following sequence? {1,2,4,7,11...}

Starting at 1, add one, and then one more than you did the last time.

400 points: What type of sequence is this? {17,51,153,459...} **geometric**

600 points: Describe this sequence with a closed-form definition: {7,9,11,13...} $t_n = 2n + 5$

800 points: Describe this sequence with a recursive definition: {3,4,11,116...}

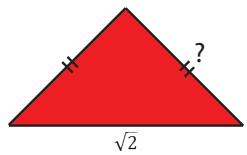
$t_1 = 3$, $t_n = (t_{n-1} \times t_{n-1}) - 5$

1000 points: Alfred likes to read. Every day that starts with a 'T' he reads 20 pages. Every day that starts with an 'S' he reads 30 pages. Every Wednesday he reads 25 pages. He continues to read for 10 weeks. Assuming he starts this routine on a Tuesday, how many pages has he read in total AFTER his 38th time reading? $(125 \times 7) + 20 + 25 + 20 = 940$

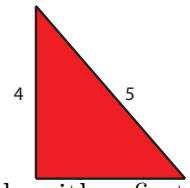
Pythagorean Theorem

200 points: State the Pythagorean Theorem. $a^2 + b^2 = c^2$

400 points: Find the missing length below: **1**



600 points: Find the area of the triangle below: **6**



800 points: Find a Pythagorean Triple with a first number 65. **65, 2112, 2113**

1000 points: If the area of an isosceles triangle is 18 cm^2, and the base and height of the triangle are the same, what are the lengths of the other two sides? $\sqrt{45}$

Random

200 points: Free Points (No Question).

400 points: Evaluate 2^3. **8**

600 points: Convert $\frac{6}{5}$ to a percentage. **120%**

800 points: Name the first 13 primes. **2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41**

1000 points: Non-Math Question.

Gauss Prep (Full solutions to Gauss problems can be found online)

200 points: Henri scored a total of 20 points in his basketball team's first three games. He scored $\frac{1}{2}$ of these points in the first game and $\frac{1}{10}$ of these points in the second game. How many points did he score in the third game? **8** (Grade 7, 2001, #10)

400 points: Harry charges \$4 to babysit for the first hour. For each additional hour, he charges 50% more than he did for the previous hour. How much money in total would Harry earn for 4 hours of babysitting? **\$32.50** (Grade 7, 2006, #15)

600 points: Kalyn is trying out a new recipe that calls for 5 cups of flour and 1 cup shortening. She only has $\frac{2}{3}$ cup of shortening, and uses all of it. How much flour should she use to keep the ingredients in the same ratio as called for in the recipe? $\frac{10}{3}$ (Grade 7, 2004, #17)

800 points: Five people are in a room for a meeting. When the meeting ends, each person shakes hands with each of the other people in the room exactly once. What is the total number of handshakes that occur? **10** (Grade 7, 2002, #21)

1000 points: Nathalie has some quarters, dimes and nickels. The ratio of the number of quarters to the number of dimes to the number of nickels that she has is 9:3:1. The total value of these coins is \$18.20. How many coins does Nathalie have? **91** (Grade 8, 2006, #21)

Final Jeopardy

Gauss Prep

A 3×3 grid is filled with the digits 1, 2, and 3 so that each number appears once in each row and column. How many different ways are there of filling the grid? See below for examples. (Grade 8, 2008, #22)

1	2	3
3	1	2
2	3	1

3	2	1
2	1	3
1	3	2

There are 6 possibilities for the first row of the grid:
1, 2, 3
1, 3, 2
2, 1, 3
2, 3, 1
3, 1, 2
3, 2, 1

Consider the first row of 1, 2, 3:

1	2	3

The first column could be 1, 2, 3 or 1, 3, 2:

1	2	3
2		
3		

or

1	2	3
3		
2		

Each of these grids can be finished with the given rules, but can only be finished in one way. (In the first grid, the middle number in the bottom row cannot be 2 or 3, so is 1, so the middle number in the middle row is 3, so the right column is 3, 1, 2. Similarly, in the second grid, the middle number in the middle row must be 1. Try completing this grid!)
Therefore, a first row of 1, 2, 3 gives two possible grids.
Similarly, each of the other 5 possible first rows will give two other grids.
(We can see this by trying each of these possibilities or by for example switching all of the 2s and 3s to get the grids with a first row of 1, 3, 2.)
Therefore, the total number of different ways of filling the grid is $6 \times 2 = \mathbf{12}$.