1. Prove by induction that the sum of the first \( n \) perfect squares is \( \frac{n(n+1)(2n+1)}{6} \). What form of induction did you use?

2. Find and prove a closed form for \( a_n = 2\sqrt{a_{n-1}} \) with \( a_0 = 2 \).

3. (a) Solve the recurrence \( a_n = a_{n-1} + 2a_{n-2} \) with \( a_0 = 2 \) and \( a_1 = 7 \).
    (b) Redo part (a), except this time, change \( a_0 = 10 \) and \( a_1 = 4 \).
    (c) Redo part (a), except this time, change \( a_0 = a \) and \( a_1 = b \).

4. Solve the recurrence \( a_n = -a_{n-1} + 4a_{n-2} + 4a_{n-3} \) with \( a_0 = 8 \), \( a_1 = 6 \) and \( a_2 = 26 \).

5. Solve the recurrence \( a_n = 6a_{n-1} - 9a_{n-2} \) with \( a_0 = 1 \) and \( a_1 = 6 \).

6. Solve the recurrence \( s_k = as_{k-1} \) for any value of \( a \). Using that solution, solve \( s_k = as_{k-1} + 1 \).

7. Solve \( a_n = 2a_{n-1} - a_{n-2} + 2^n \) with \( a_0 = 1 \) and \( a_1 = 2 \).

8. If \( \varphi = \frac{1+\sqrt{5}}{2} \), find \( -\frac{1}{\varphi} \). Then, find \( 1 - \varphi \). Be amazed.