Question 1

ToyCo, sells two different wooden products: trains and soldiers. They are seeking your help to optimize their weekly production plan. The production of each type of toy requires two types of specialized work (whose availability is in terms of man-hours): basic carpentry, finishing. E The following table lists the number of man-hours required for each product, the total available resources and the profit obtained by selling each toy.

	Train	Soldier	Availability
Basic carpentry	2	1	8
Finishing	1	3	9
Profit ($)	3	3	

(a) What decisions must the company make?

(b) If the company produces 3 trains, what is the total profit?

(c) If the company produces 2 soldiers, what is the total profit?

(d) What is the total profit if they produce 3 trains and 2 soldiers?

(e) If we let

x_1= the number of trains produced

x_2= the number of soldiers produced

Then write an expression to represent the total profit obtained by producing trains and soldiers.

x_1 and x_2 are called the *decision variables*. They represent the decisions that one wants to take in the problem.

The expression you wrote for this part is is called an *objective function*. It is a function of the decision variables that you want to maximize or minimize.

(f) Now x_1 and x_2 cannot assume any value we want. There are *Constraints* that must be imposed in their values in order to satisfy the statement of the problem.

Write down what are the constraints involved in this problem.

(g) Find the optimal solution.

Question 2

An oil refinery must blend two grades of gasoline to sell the resulting product. The first grade can be purchased from HyOctane, Inc. and has an octane number of 92. The second grade can be purchased from Allif Oil and has an octane number of 85.

The resulting blend must have an octane number of at least 89.

The refinery wants to produce exactly 120 barrels of the blend. They can purchase up to 90 barrels of each grade per week. The purchase price of the 92 octane grade is \$20 per barrel, and the 85 octane grade is \$15 per barrel.

The company must decide how many barrels of each grade should be used in the blend in order to minimize the production cost.

(a) What decisions must the company make?

(b) If we let

A= the number of barrels of 85 octane gasoline needed from Allif Oil for the blend

H= the number of barrels of 92 octane gasoline needed from HyOctane, Inc. for the blend

Then write an expression to represent the total gasoline purchase cost for the blend if he buys A barrels of gasoline from Allif Oil and H barrels of gasoline from HyOctane, Inc.

(c) Write two inequality statements representing the gasoline purchase restrictions.

(d) Write the constraint that restricts the number of barrels of the blend that will be produced.

(e) Now suppose you mix 5 barrel of Allif gasoline and 5 barrel of HyOctane gasoline. What is the average octane rating?

(f) Now suppose you mix 2 barrel of Allif gasoline and 6 barrel of HyOctane gasoline. What is the average octane rating?

(g) Using the same idea, write a constraint that represents the average octane rating requirement.

(h) Find the optimal solution to this problem.

Question 3

As machines grow older, their repair and maintenance costs increase. However, replacing them requires capital investment. An important issue here is how often to buy and replace machines. Assume that we have a 6 year planning horizon.

Consider the cost of purchasing the machines in years 1 through 6 as follows:

	Year					
	1	2	3	4	5	6
Purchase cost	20	19	16	21	18	22

Also, consider that, as machines get older, they require more maintenance, and thus their annual repair and maintenance costs increase. Consider the following repair and maintenance costs based on the age of the machine as follows:

	Age					
	0	1	2	3	4	5
Repair and maintenance cost	1	3	4	6	7	8

How can one determine the best repair policy using the shortest path problem?

Question 4

Farmer Jones must determine how many acres of corn and wheat to plant this year. An acre of wheat yields 25 bushels of wheat and requires 10 hours of labor per week. An acre of corn yields 10 bushels of corn and requires 4 hours of labor per week. All wheat can be sold at \$4 a bushel and all corn can be sold at \$3 a bushel. Seven acres of land and 40 hours per week of labor are available. Government regulations require that at least 30 bushels of corn be produced during the current year. He wants to plant corn and wheat to maximize his total revenue.

Let x_1 be the number of acres of corn planted, and x_2 be the number of acres of wheat planted. Using these decision variables, answer:

(a) Is $x_1 = 2$, $x_2 = 3$ in the feasible region? What is its objective function value?

(b) Is $x_1 = 4$, $x_2 = 3$ in the feasible region? What is its objective function value?

(c) Is $x_1 = 2$, $x_2 = -1$ in the feasible region? What is its objective function value?

(d) Is $x_1 = 3$, $x_2 = 2$ in the feasible region? What is its objective function value?

(e) What is the objective function?

(f) What are the constraints?

(g) Without finding the optimal solution to this problem, solve the following two questions (* marks a challenge question).

Give a lower bound on the optimal value, i.e., give a number that you can guarantee that the optimal solution will have value greater than or equal to that number. What is the largest lower bound that you can get?

(h) * Give an upper bound on the optimal value, i.e., give a number that you can guarantee that the optimal solution will have value less than or equal to that number. What is the smallest upper bound that you can get?

(i) Find the optimal solution to this problem.

Question 5

Gemstone Tool Company (GTC) is a privately held firm in the consumer and industrial market for construction tools. The Winnipeg plant only produces wrenches and pliers. Wrenches and pliers are made from steel, and the production process involves molding the tools on a molding machine and then assembling the tools on an assembly machine. To make one wrench requires 1.5 lbs of steel, 1 hour on the molding machine and 0.3 hours on the assembly machine. To make one plier requires 1 lb of steel, 1 hour on the molding machine and 0.5 hours on the assembly machine. There are only 27 lbs of steel, 21 hours of the molding machine and 21 hours of the assembly machine available on any particular day. Wrenches are sold for \$1.30 and pliers are sold for \$1.00.

(a) What are the decisions that GTC must take? (decision variables)

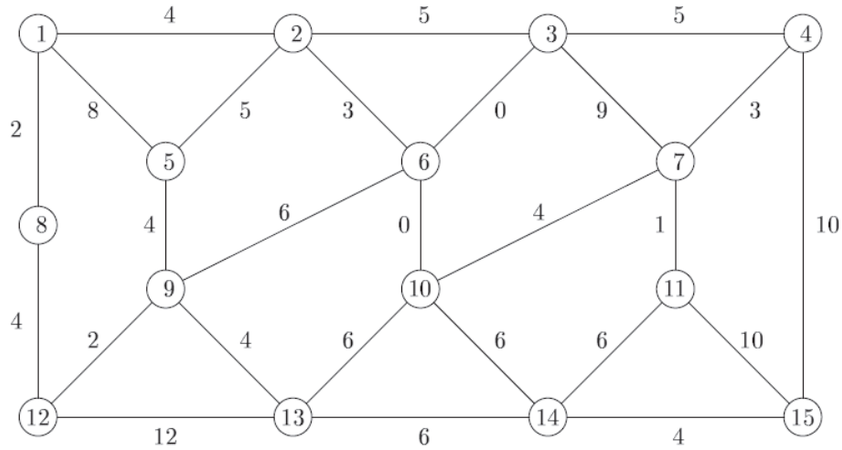
(b) What is the objective function?

(c) What are the constraints?

(d) Find the optimal solution to this problem.

Question 6

Consider the following graph:



Answer the following questions without using the shortest path algorithm (* represents a challenge question)

(a) Find a path from vertex 1 to vertex 15.

(b) What is the total cost of this path?

(c) What is the path of least total cost that you can find from vertex 1 to vertex 15?

(d) How many different paths can you find from vertex 1 to vertex 15?

(e) Give an upper bound on the cost of the shortest path, i.e., give a number that you can guarantee that the shortest path will have value less than or equal to that number. What is the smallest upper bound that you can get?

(f) * Give a lower bound on the cost of the shortest path, i.e., give a number that you can guarantee that the shortest path will have value greater than or equal to that number. What is the largest upper bound that you can get?

Now use the shortest path algorithm to find the actual shortest path from vertex 1 to vertex 15.