EXERCISES

1. Write a set of rewriting rules $S$ (using as few rules and symbols as possible) such that the word $S^n$ satisfies $|S^n| = 2^n$.

2. Write a set of rewriting rules $S$ (using as few rules and symbols as possible) such that the word $S^n$ satisfies $|S^n| = 3^n$.

3. Write a set of rewriting rules $S$ (using as few rules and symbols as possible) such that the word $S^n$ satisfies $|S^n| = 8^n$.

4. What shape is created with the following rewriting rule:

$$ F \rightarrow F + F - F - F + F $$

5. What does $\tau(h^{\omega}(c))$ represent, if we have $h$ defined as:

\[
\begin{align*}
a & \rightarrow a \\
b & \rightarrow ba \\
c & \rightarrow cdbb \\
d & \rightarrow d
\end{align*}
\]

and $\tau$ defined as

\[
\begin{align*}
a & \rightarrow 0 \\
b & \rightarrow 0 \\
c & \rightarrow 1 \\
d & \rightarrow 1
\end{align*}
\]

(Hint: look at the positions of the 1s)

6. (a) Consider the morphism $\varphi$ defined by $\varphi(0) = 01$ and $\varphi(1) = 0$. What can $\varphi^\omega(0)$ represent?

(b) Define $\Phi_1 = 1$, $\Phi_2 = 0$ and $\Phi_n = \Phi_{n-1}\Phi_{n-2}$. Show $\varphi^n(1) = \Phi_{n+1}$ and $\varphi^n(0) = \Phi_{n+2}$.

(c) For any string $x$ with $|x| \geq 2$, define the map $c(x)$ that interchanges the last two characters of $x$. Prove $\Phi_n\Phi_{n+1} = c(\Phi_{n+1}\Phi_n)$ for all $n \geq 1$. 

1