SOLUTION SET 1

Here are the solutions to exercises 2 to 5 from the first problem set. As explained before, Exercise 1 was meant to be a refresher on some basic concepts, and Exercises 6 and 7 were not reached. They will appear again on the second handout.

Solution 2.

1. 2, 1, 7, 10, 31, 61, 154, 337, 799, 1810...
2. 3, 2, 1, 0, −1, −2, −3, −4, −5, −6,...
3. 1, 1, 1, 1, 1, 1, 1, 1, 1,...
4. 1, 3, 7, 15, 31, 63, 127, 255, 511, 1023,...
5. 1, 2, 4, 8, 16, 32, 64, 128, 256, 512,...
6. 1, 1, 5, 1, 29, −23, 197, −335, 1517, −3527,...
7. 3, 2, −4, −8, 0, 16, 16, −48, −16,...
8. 4, 7, −11, 4, 7, −11, 4, 7,...
9. 1, 1, −1, −5, −7, 1, 23, 43, 17, −95,
10. 1, 1 + √2, 3 + 2√2, 7 + 5√2, 17 + 12√2, 41 + 29√2, 99 + 70√2, 239 + 169√2,... As for the bounds,
   n = 1 (1 + √2)^1 = 1 + √2
   n = 2 (1 + √2)^2 = 3 + 2√2
   n = 3 (1 + √2)^3 = (1 + √2)(1 + √2) = (3 + 2√2)(1 + √2) = 7 + 5√2
   n = 4 (1 + √2)^4 = (1 + √2)^3(1 + √2) = (7 + 5√2)(1 + √2) = 17 + 12√2

Look familiar?

Solution 3.

1. \(a_n = a_{n-1} + a_{n-2}\)

\[
\begin{array}{cccccccccc}
 a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\
 1 & 2 & 3 & 5 & 8 & 13 & 21 & 34 & 55 & 89 \\
 2 & 2 & 4 & 6 & 10 & 16 & 26 & 42 & 68 & 110 \\
 2 & 4 & 6 & 10 & 16 & 26 & 42 & 84 & 110 & 188 \\
 3 & 4 & 7 & 11 & 18 & 29 & 47 & 76 & 123 & 199 \\
 -1 & 0 & -1 & -1 & -2 & -3 & -5 & -8 & -13 & -21 \\
 4 & 6 & 10 & 16 & 26 & 42 & 68 & 110 & 178 & 288 \\
\end{array}
\]
(2) \( a_n = 3a_{n-1} - 2a_{n-2} \)

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<tr>
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<th>( a_0 )</th>
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**Solution 4.** We have already computed the sequence when \( a_0 = a_1 = 1 \) and \( a_0 = 1, a_1 = 2 \). They are 1, 1, 1, 1, \ldots and 1, 2, 4, 8, 16, \ldots (the powers of 2). Take these sequences as \( a_n \) and \( b_n \), and using the hint we have that \( c_n = 5a_n - 2b_n \). The first few terms:

\[
c_0 = 5a_0 - 2b_0 = 5 \cdot 1 - 2 \cdot 1 = 3
\]

\[
c_1 = 5a_1 - 2b_1 = 5 \cdot 1 - 2 \cdot 2 = 1
\]

\[
c_2 = 5a_2 - 2b_2 = 5 \cdot 1 - 2 \cdot 4 = -3
\]

\[
c_3 = 5a_3 - 2b_3 = 5 \cdot 1 - 2 \cdot 8 = -11
\]

\[
c_4 = 5a_4 - 2b_4 = 5 \cdot 1 - 2 \cdot 16 = -27
\]

**Solution 5.** Notice that \( a_n = 1 \) for all \( n \), and \( b_n = 2^n \) for all \( n \). Using that \( c_n = 5a_n - 2b_n \), we have that \( c_n = 5 \cdot 1 - 2 \cdot 2^n = 5 - 2^{n+1} \). Try this on your calculator, if you haven’t already.