

Grade 6 Math Circles Working with an Abacus

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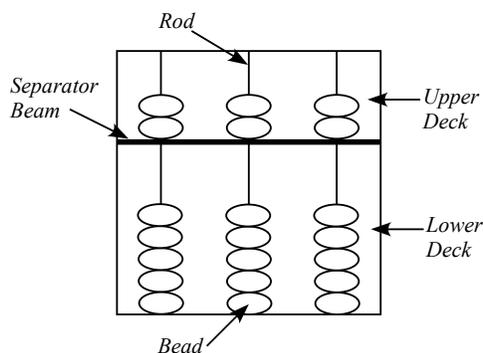
What is an Abacus?

An abacus is a mathematical tool used for computing large numbers quickly. The first abacuses were used over 4000 years ago by the Mesopotamians and are still used today by many merchants worldwide. Ancient Egyptians, Romans, Greeks, Chinese, Japanese, Russians and Native Americans also used abacuses, however each culture had a different name and its own unique version of the abacus.

The Chinese *Suanpan*, is an abacus which makes addition, subtraction, multiplication, and division by hand very quick. The *Suanpan* is the type of abacus we will focus on.

Making an Abacus

An abacus is a box frame which usually has four to fourteen *rods* with seven *beads* on each rod. There is a *separator beam* which divides the rods into the *upper deck* and the *lower deck*. Ensure there are two beads in the upper deck and five beads in the lower deck.

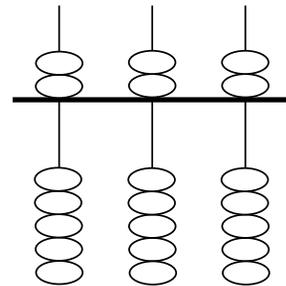


Instructions for making your own abacus can be found at
<http://www.ee.ryerson.ca/~elf/abacus/popsicle/>

Abacus Basics

1. Place the abacus so it lies flat on a desk. Make sure the lower deck is close to you and the upper deck is further from you.

2. Gently tilt the abacus up so that the upper beads rest on the separator beam and the lower beads rest at the base of the abacus, then lie the abacus flat again. This process is known as *resetting* the abacus.

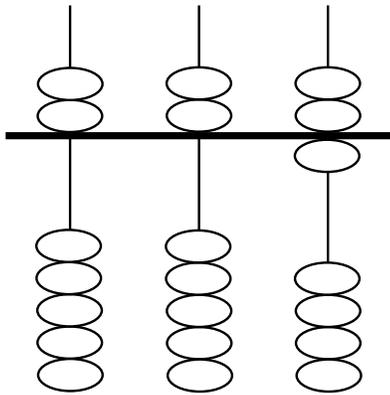


A reset abacus represents the value 0.

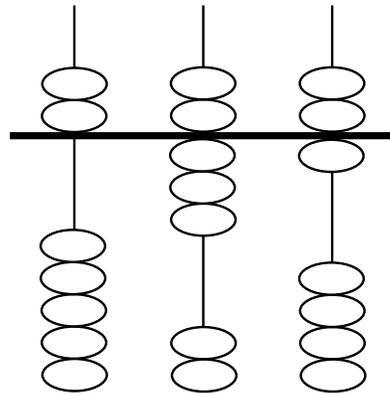
A reset abacus

3. Each rod represents a place holder: the rightmost rod represents the ones (or unit) digit; the rod to its left is the tens digit; the next left rod is the hundreds digit, and so on. Note that if you wish to work with two decimals, set the rightmost rod to the hundredths column, followed by tenths to the left, then ones, tens and so on.
4. Each bead in the lower deck represents a value of 1. If slid toward the separator beam that bead now changes the number displayed by the abacus. The rod determines the place of the value.

For example, sliding one lower bead from the rightmost rod gives a value of 1 since that rod displays the ones unit and one bead is at the beam. Now moving up three beads from the second rod we get the number 31 since there is a 3 in the tens place holder and a 1 in the ones place holder.



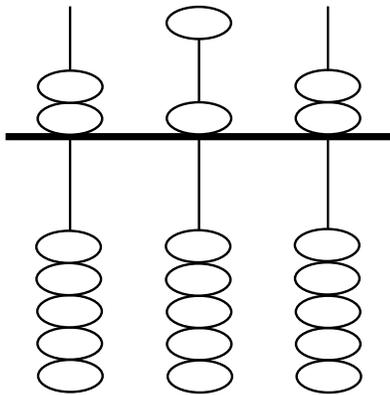
The number 1



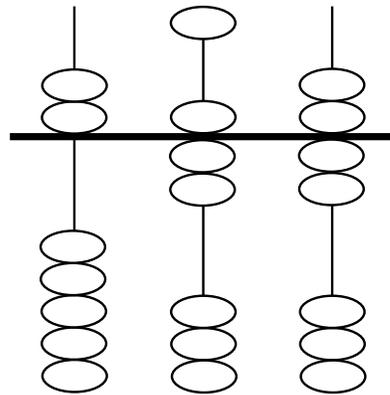
The number 31

5. Each bead in the upper deck represents a value of 5. If slid away from the separator beam, that bead now changes the number displayed on the abacus. The rod determines which place value is altered. If both upper and lower beads are used on the same rod, add the values together to get the place holder value in the final number.

For example (after resetting), sliding one upper bead on the second rod gives a value of 50 since the second rod is the tens place holder and one upper bead has moved. Now sliding two lower beads on the second rod and two lower beads on the first (rightmost) rod we get the number 72 since the value in the ones place holder is 2 and $5+2=7$ in the tens place holder.

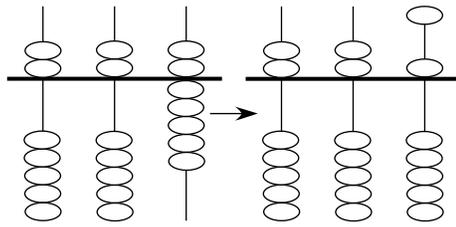


The number 50

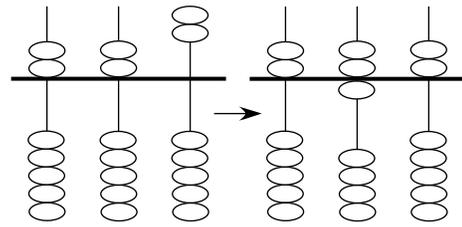


The number 72

6. Note there are multiple ways to show some numbers. As examples, 5 and 10.



The number 5 in two different ways.



The number 10 in two different ways.

Always have as few beads raised as possible when representing a number on an abacus.

If all lower beads or both upper beads on any rod are raised, then there is a way to represent the same number with fewer beads raised. This is called *simplifying*.

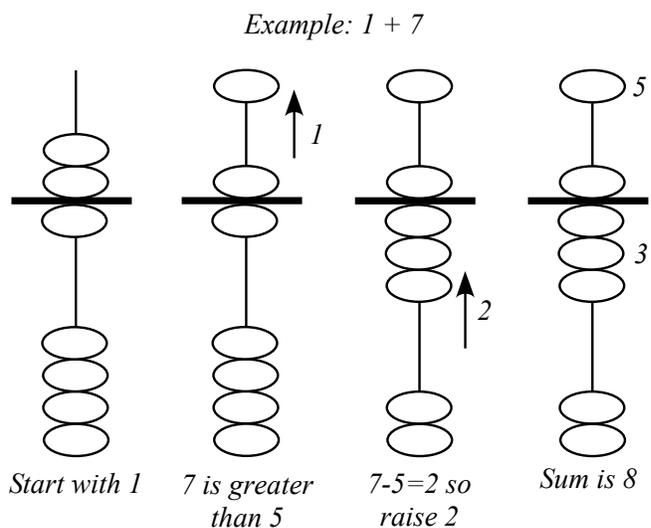
Using an Abacus

Addition

1. Represent (or *input*) the first number, called the *base*, into your abacus.
2. Observe the number you wish to add, called the *addend*. Look at the leftmost digit and the rod corresponding to its place value.

3. Start in the upper deck. If the addend digit is 5 or greater, raise one upper bead then go to the lower deck. If the digit is 4 or less, go right to the lower deck.

4. If the addend digit is 5 or greater, subtract 5 from the addend digit and raise that many lower beads. If the digit is 4 or less, raise that many lower beads.



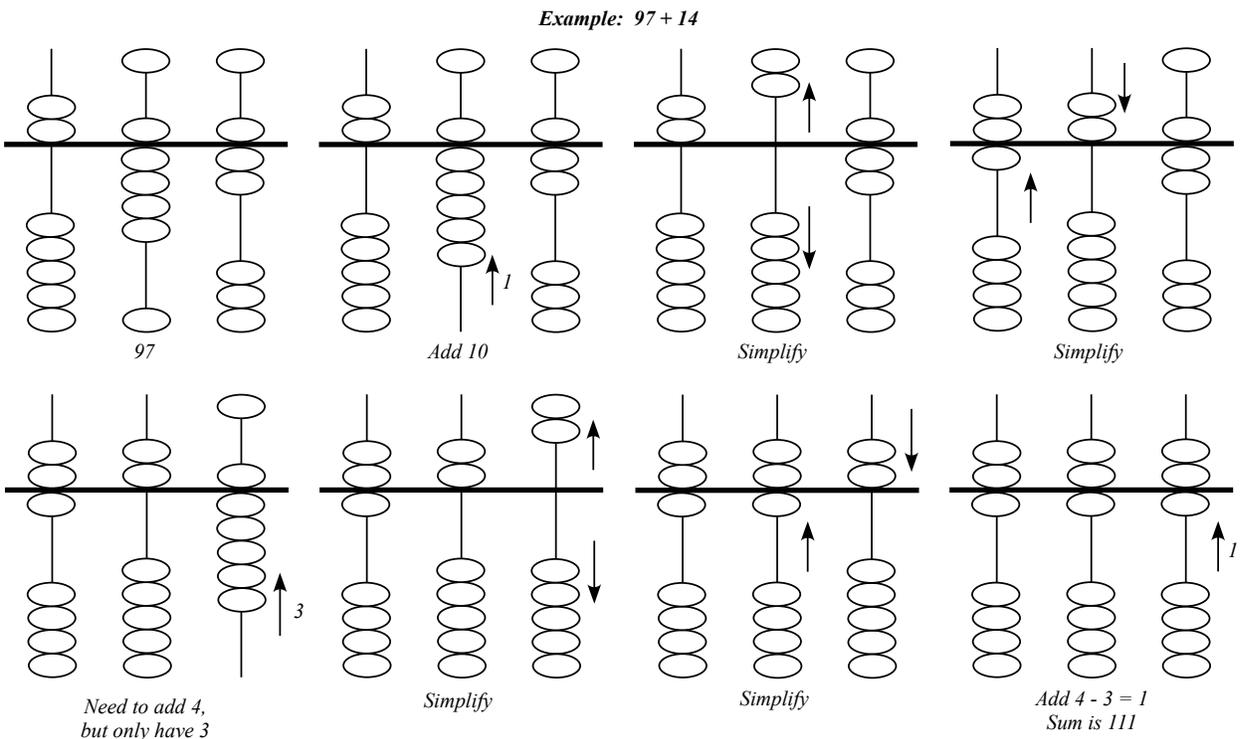
5. *ALWAYS* check this step after raising or dropping any beads!

- If both upper beads are raised, drop them and raise one lower bead *from the rod to the left*.
- If all lower beads are raised, drop them and raise one upper bead *from the same rod*.

6. If there are not enough lower beads on a rod to add the entire addend digit at once:

- Raise all five lower beads.
- You have added only part of the digit. The value you still must add is the addend digit minus the number of lower beads you raised.
- Follow *Step 5* which is to *simplify*.
- Raise the number you found in *Part b* lower beads.

7. Return to *Step 3* looking at the next digit to the right.



Note that adding in this way, you work from left to right, just as if you were reading. However, digits to the left of the rod on which you are working may change.

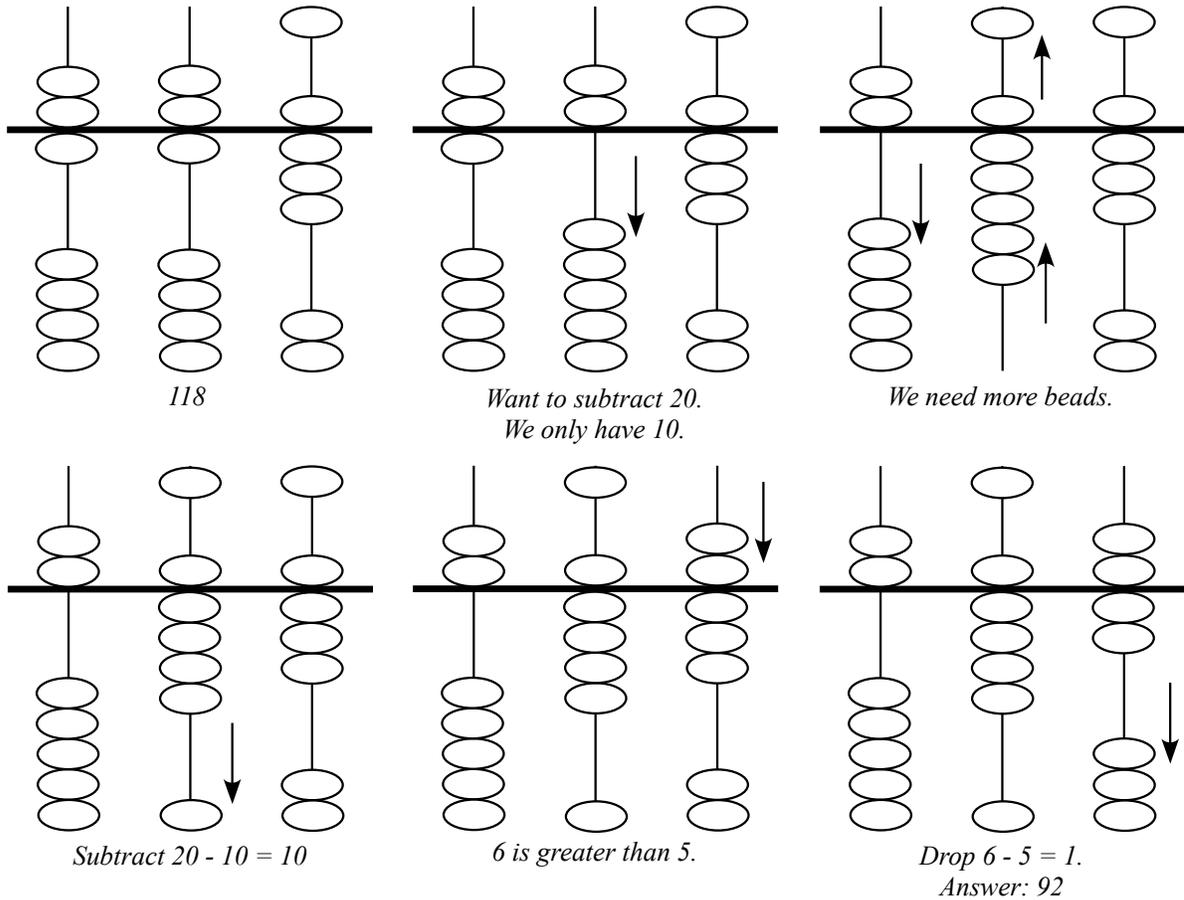
It is also possible to add starting at the rightmost digit and working to the left. Adding this way is similar to adding using a pencil and paper. However, get into the habit of working left to right as it is essential for multiplication and division.

Subtraction

The steps involved in subtraction are very similar to addition. However, you may *run out* of beads on the upper or lower deck. If this happens, you must represent the same number on your abacus, but using *more* beads, then continue with the subtraction.

1. Input the first number, or *base*, into your abacus.
2. Observe the *subtractor*, which is the number you wish to subtract from the base. Look at the leftmost digit of the subtracter and the rod corresponding to its place value.
3. If the subtracter digit is less than or equal to the value on the rod, drop the value of beads on the rod equal to the subtracter digit.
4. If the subtracter digit is greater than the value on the rod:
 - (a) Drop all beads on the rod.
 - (b) Drop one lower bead from the rod to the left. Raise one upper bead and all five lower beads on the rod you are working with. This is called *borrowing*.
 - (c) You've only subtracted part of the digit. Drop the number of beads needed to make the total value of beads you've dropped equal to the subtracter digit.

Example: $118 - 26$



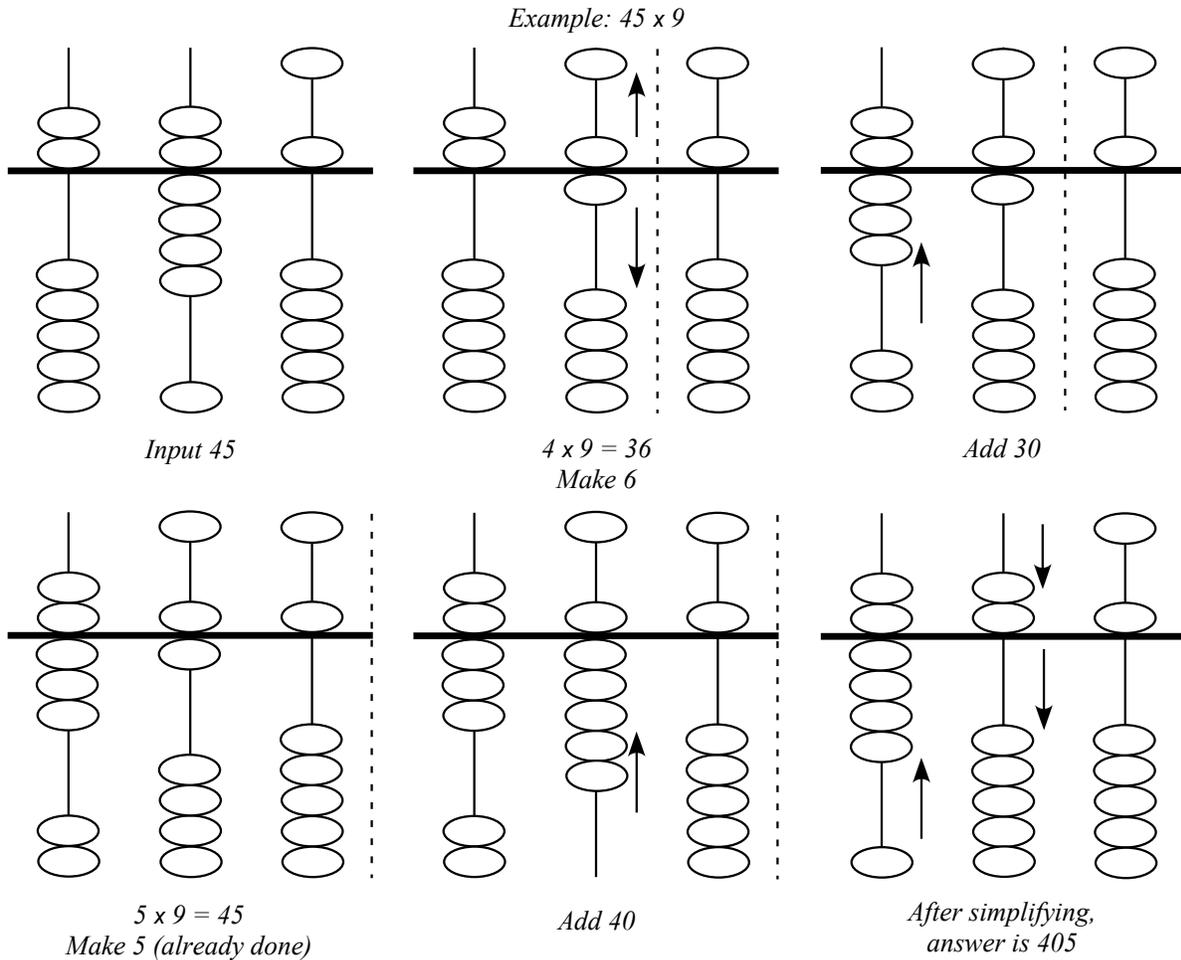
Multiplication

Due to time restrictions we will only multiply a number by a single digit.

1. Input the first number or *base* into the abacus. Make sure the *multiplier* (the other number) has **only one digit**.
2. Begin at the leftmost digit of the base. Multiply that digit and multiplier together.
3. Alter the beads on the rod you are using so it shows the value of the ones digit of the product.
4. If the product is greater than 9, temporarily let the rod you are working with be the ones digit, the rod to its left be tens, and so on. **Add** the respective non-ones digits

as normal, ensuring to carry over and simplify if necessary.

5. Continue to repeat the multiplication process with each digit to the right.



Division

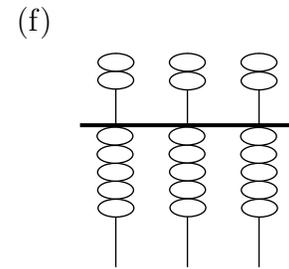
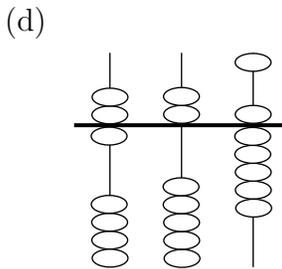
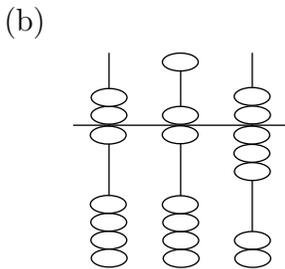
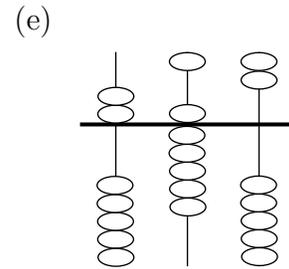
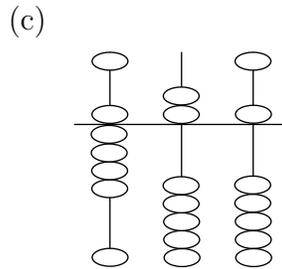
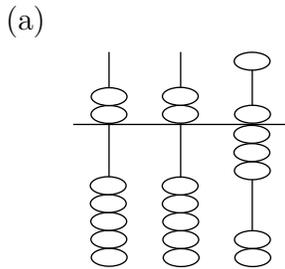
Division using an abacus is more difficult than the three operations covered so far. Division, as well as multi-digit multiplication, also requires an abacus with twice as many rods as there are digits. As such we will omit dividing on an abacus. If you are interested in division, visit

<http://scienceblogs.com/goodmath/2006/09/25/division-on-the-abacus/>

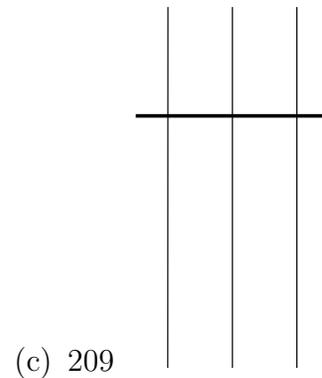
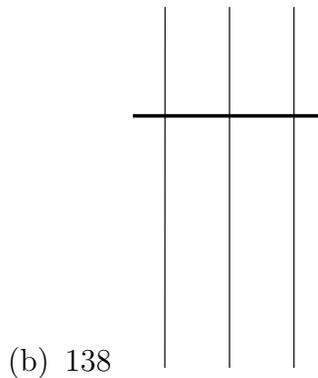
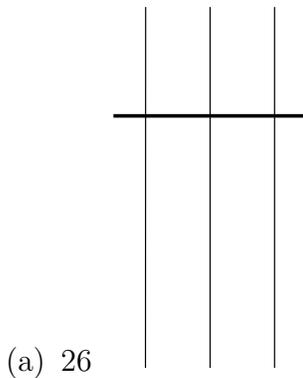
The more you practise with an abacus the faster you will get at using it. It can also help strengthen your mental math abilities which will be very helpful for the rest of your life!

Problems

1. What number is on each abacus?



2. Input each number into your abacus so that as few beads as possible are raised, then fill in your answer on the abacuses below.



3. On your abacus, represent each number in at least two different ways.

(a) 15

(b) 100

(c) 164

4. Solve the following addition problems using your abacus. Only write down the answer!

(a) $57 + 48$

(c) $164 + 338$

(e) $\$9.82 + \0.19

(b) $12 + 89$

(d) $16 + 491$

(f) $4963 + 3569$

5. Solve the following subtraction problems using your abacus. Only write the answer!

(a) $76 - 42$

(c) $100 - 9$

(e) $\$10.75 - \7.84

(b) $136 - 54$

(d) $514 - 16$

(f) $8462 - 5937$

6. Solve the following multiplication problems on your abacus. Only write the answer!

(a) 16×4

(c) 134×6

(e) 3166×3

(b) 52×7

(d) $\$2.81 \times 9$

(f) 2174×4

7. What is the product? As usual, don't write down any steps.

$$2 \times 8 \times 3 \times 4 \times 6 \times 7$$

8. CHALLENGE 1: Solve the following problem without writing down *any* steps!

$$4 + 7 - 2 + 35 - 16 + 487 - 276 + 6791 - 5796$$

9. CHALLENGE 2: Solve the following problem. Evaluate all the multiplication parts first and *write down your answers*, then do the addition and subtraction in between.

$$139 \times 7 \text{ less } 42 \times 5 \text{ less } 64 \times 8 \text{ add } 52 \times 8 \text{ less } 37 \times 3 \times 6$$