UNIVERSITY OF
WATERLOO
Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Centre for Education in
Mathematics and Computing

Grade 11/12 Math Circles
Symbolic Logic II Exercises
November 7, 2012

1. Let m and n be natural numbers (in other words, positive integers). Prove the following:

$$mn \text{ is odd if and only if } m \text{ and } n \text{ are both odd.}$$

Hint: As you know, you must prove both directions of implication. One of these is relatively easy, while the other one is more difficult to prove directly. For that case, try the contrapositive; it might be easier!

2. Prove the following:

$$(P \Rightarrow Q) \Leftrightarrow \sim (P \wedge \sim Q)$$

This is the logic behind what is known as "proof by contradiction". That is, if you want to prove that P implies Q, you could instead show that it is impossible for P and $\sim Q$ to be true at the same time. The common-sense reasoning usually goes like this: One assumes that P is true, and then assumes that Q is false (i.e., $\sim Q$ is true), and get to some contradiction which is obviously false. This shows that the assumption that "Q is false" must have been incorrect, so that Q must actually be true.