



Grade 6 Math Circles Kinematics

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Kinematics is all about solving math problems which involve the following three things:

- DISTANCE (d) (*how far apart or how far objects travel*)
- SPEED (s) (*how fast an object moves*)
- TIME (t) (*for how long the object moves*)

These quantities are related by the fundamental _____ formula:

$$\text{(Average) Speed} = \text{Distance} \div \text{Time}$$

$$s = d \div t$$

Example. *Superman flies at 220 km/h for 7 minutes. How far in km does he travel?*

Problem Solving Steps for Kinematics Problems

1. Identify which of the three quantities are given from the problem. s, t
2. Identify the quantity you do not have. d
3. * Convert all measurements into m, m/s and s (STANDARD units)
4. * Using the fundamental kinematics formula/kinematics triangle, solve for the unknown quantity. The units of the answer will be either m for distance, m/s for speed, or s for time. You can then convert these back into the appropriate units (such as km/h).

Step 3 - How to convert units

Converting units is all about understanding the conversion table.

1 km = 1000 m	1 min = 60 s	1 hr = 60 min	1 km/h = 0.28 m/s
1 m = 0.001 km	1 s = 0.0167 min	1 min = 0.0167 hr	1 m/s = 3.6 km/h

To convert your given units into the units that you want, find the appropriate entry. Multiply both sides by how much you have of your given unit. The value and units on the right side will be your answer.

Examples

1. Convert 27 m into km.
2. Convert 27 km into m.
3. Convert 5 m/s into km/h.
4. Convert 5 km/h into m/s

Exercises

1. Convert 17 m into km.
2. Convert 17 km into m.
3. Convert 5 m/s into km/h.
4. * Convert 5.5 hours into s

Complete step 3 by converting all given units to m , m/s and s .

Example. *Superman flies at 220 km/h for 7 minutes. How far in km does he travel?*

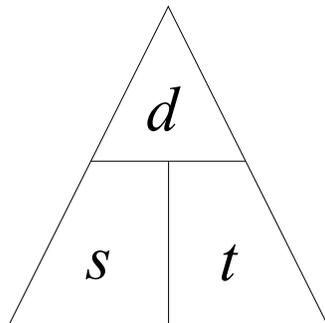
Known/Given Quantities: s (speed) and t (time)

$$s = 220 \text{ km/h} = 59.4 \text{ m/s}$$

$$t = 7 \text{ minutes} = 420 \text{ s}$$

Step 4 - How to use the fundamental kinematics formula

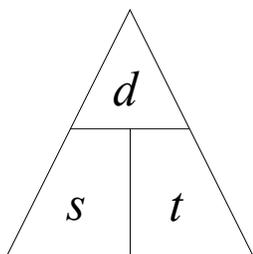
An easy way to remember the fundamental kinematics formula is using the **kinematics triangle**.



1. First, circle the quantity you are LOOKING FOR.
2. Next, find the letters which correspond to the KNOWN quantities.
 - (a) If they are **beside** each other, multiply them.
 - (b) If they are **above and below** each other, divide the top quantity by the bottom quantity.
3. The number you get in step 2 is the answer to step 1 in STANDARD units.

Complete step 4 of the “Problem Solving Steps” for our example by solving for the unknown quantity. Then convert the units (which should be standard) into the units asked for in the question.

Example. *Superman flies at 220 km/h for 7 minutes. How far in km does he travel?*



Known Quantities: s (speed) and t (time)

Unknown Quantity: d (distance)

$$s = 59.4 \text{ m/s}$$

$$t = 420 \text{ s}$$

Using the kinematics triangle: $d = s \times t = 59.4 \times 420 = 24\,948$ m.

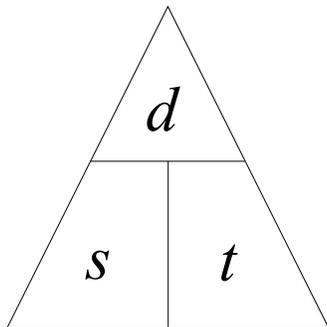
Your answer will be in *m*. But we asked for *km*! So you have to convert the answer into the proper units.

$24\,948$ m = $24\,948 \times 0.001$ km = 24.948 km.

Therefore Superman will travel 24.95 *km*.

Practice Exercise

Usain Bolt ran the 100 meters in 9.58 s at the 2009 World Championships. At what speed (in km/h) was he running?



1. Find the quantity you are LOOKING FOR.

2. What are the GIVEN quantities?

3. Solve - what operation do you use?

4. Convert to appropriate units.

A more physical problem...

The math behind kinematics is pretty simple - the tricky part is transferring a physical situation into a mathematical problem.

Example. *The Flash and Thor are having an argument. Thor claims his mighty hammer, Mjolnir, can fly to his hand faster than Flash can run. Flash calls his bluff and proposes the following:*

They will mark a starting line. Thor will stand 1 km away from this starting point, while Flash will stand at the line. Mjolnir will be placed on the ground, 250 m in front of Flash (a head start).

At the sound of Thor's thunder, Flash will race towards Thor, while Thor calls Mjolnir to him.

If Flash runs at 160 m/s and Mjolnir travels at 120 m/s, who will win the race?

Solution

Draw a picture of the situation:

Simplify the question:

“Who wins the race” is the same as asking “WHOSE FIRST”

“WHOSE FIRST” is the same as asking “WHO TAKES THE LEAST TIME”

Solve the new question:

Flash

Known Quantities:

Unknown Quantity:

Mjolnir

Known Quantities:

Unknown Quantity:

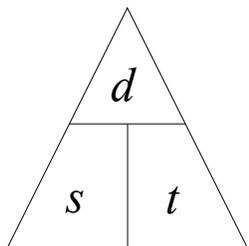
Convert units to STANDARD: Already standard.

Solve the problem: It's a tie - Flash takes 6.25 s and so does Mjolnir.

Exercise

Thor and Flash are unsatisfied with the result of the previous test. They do a retest, with the same conditions as before. However, Thor summons some extra power from Asgard, while Flash does some serious stretching to get his legs more limber.

If Flash now runs at 200 m/s and Mjolnir travels at 160 m/s , who will win the race?



Harder example...

Frodo is trying to climb Mount Doom. He runs at 6 m/s for the first 5 minutes, but soon hits a rocky patch, causing his speed to drop down to 3 m/s . He maintains this speed until he reaches the top. If the distance up Mount Doom is 3000 m in total, for how long was Frodo running on the rocky patch?

Problem Set

As always, questions marked with a “*” are more difficult and require more thinking. Don’t be afraid to WORK TOGETHER.

1. Harry is playing Quidditch. The Snitch is floating still, 500 m away, and his Firebolt is travelling at 15 m/s . How long will it take him to reach it, in seconds?
2. One of the world’s greatest soccer players, Diego Maradona, once dribbled his way up an entire soccer field, evading all the defenders and then scoring a goal. If the entire run took 1 minute, and the field was 120 m long, how fast was he going, in m/s ?
3. Batman is staring down a really deep cave, but doesn’t know how deep it is exactly. Being a kinematics expert, he drops a special rock which falls at a constant speed of 75 m/s . He times how long it takes for the rock to hit the ground: the time was 1.75 minutes. How deep was the cave, in km?
4. Pat Driver was caught speeding in a school zone and was called to court. She claims that she could not have been speeding; however, school cameras show her travelling 800 m over a 25 s period. The school speed limit is 20 km/h - is Pat lying?
5. Achilles and Hector are having a race. Hector can run at 25 m/s , while Achilles coasts at an easy 30 m/s . If they both start from the starting line (no head starts), how far ahead of Hector is Achilles after 30 seconds?
6. Bonnie and Clyde are having a race. Bonnie gives Clyde a 20 m head start. Bonnie runs at 7 m/s , while Clyde runs at 6 m/s . How long will it be until Bonnie catches up with Clyde? (Hint: first figure out how far Bonnie has to travel to catch Clyde.)
7. Ferris woke up late... again! He rushes to the bus stop, which is 300 m from his house. As he leaves the front door, he sees that the bus is 550 m away from the bus stop. If the bus is going at 10 m/s , and Ferris can sprint at 30 km/h , can he make it in time?
8. Wily E. Coyote is chasing the Roadrunner. He runs for 5 minutes at a speed of 7 m/s , at first. Suddenly, the Roadrunner quickens his pace, so Wily E. Coyote straps on some rocket skates and zooms off after him at 27 m/s for another 60 seconds, before running off a cliff. How far did Wily travel before falling off the cliff?
9. Two trains are 100 km apart. They leave their stations at the same time, heading towards each other at 100 km/h . How long will it be until they collide?

The problems on this page are more difficult than before. Work together.

10. * In a sprint (not a normal) triathlon, contestants swim, bike, then run. Andrea swims 600 m at 3 m/s , bikes 20 km at 20 m/s , and runs 5400 m at an unknown speed. If the entire sprint triathlon took her 35 minutes, how fast did she run, assuming she maintained constant speed?
11. * Rick and Shane are playing a dangerous game of “chicken.” They are driving cars initially 1000 m apart. Both drive forward at the same time; Rick forward at 12 m/s , Shane goes forward at 8 m/s . How far does each travel until they are about to collide? How long does this take? Give your answers in standard units.
12. ** A bank robber flees the bank in a car at a constant speed. 10 minutes later, a police patrol car arrives at the bank, and immediately pursues after him, moving at a speed 5 m/s greater than the bank robber’s speed. The police catch up the robber, 20 minutes later. How fast was each car going?

The following are optional challenge problems.

13. ** Two trains are 200 km apart initially, facing each other. There is a fly on one of the trains. At the same time, both trains start towards each other, each going at 45 km/h . While this is happening, the fly is flying back and forth between the trains, turning around every time it reaches one. If the fly is flying at 25 km/h , how far does it travel before the two trains collide?
14. *** Tyr, the Norse god of war, and Odin, Norse god of wisdom, want to do some Christmas shopping in the city for the other gods. Both of them leave the castle at the same time.

Tyr rides his pet, Fenrir, the great wolf, to the city, but Fenrir is not allowed in, so Tyr has to leave him at the gates and **walk 15 minutes to the store**. Odin runs the whole way (because he’s Odin) to the same store. **Both arrive at the store at the same time.**

If Odin runs at 20 m/s , Fenrir runs at 100 km/h , and Tyr walks at 7 m/s , how far is the castle from the store?