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Grade 6 Math Circles
Kinematics Solutions

1. Harry is playing Quidditch. The Snitch is floating still, 500 m away, and his Firebolt is travelling at 15 m/s. How long will it take him to reach it, in seconds?

   Solution

   Quantity we are looking for: **Time**

   Known quantities: Distance (500 m), Speed (15 m/s)

   The units are ALREADY standard. Using the kinematics triangle/kinematics formula, $t = d \div s = 500 \div 15 = 33.33$ s (remember, 2 decimal places). Therefore, Harry will take 33.33 seconds to reach the Snitch.

2. One of the world's greatest soccer players, Diego Maradona, once dribbled his way up an entire soccer field, evading all the defenders and then scoring a goal. If the entire run took 1 minute, and the field was 120 m long, how fast was he going, in m/s?

   Solution

   Quantity we are looking for: **Speed**

   Known quantities: Time (1 minute = 60 seconds), Distance (120 m)

   The units were converted to standard above. Using the kinematics triangle/kinematics formula, $s = d \div t = 120 \div 60 = 2$ m/s. Therefore, Maradona was going 2 m/s.

3. Batman is staring down a really deep cave, but doesn't know how deep it is exactly. Being a kinematics expert, he drops a special rock which falls at a constant speed of 75 m/s. He times how long it takes for the rock to hit the ground: the time was 1.75 minutes. How deep was the cave, in km?

   Solution

   Quantity we are looking for: **Distance**

   Known quantities: Speed (75 m/s), Time (1.75 minutes = 105 seconds)

   The units were converted to standard above. Using the kinematics triangle/kinematics formula, $d = s \times t = 75 \times 105 = 7875$ m.

   Convert this to km: 1 m = 0.001 km $\rightarrow$ 7875 m = 7.875 km. Round to 2 decimal places, 7.9 km.

   Therefore, the cave is 7.9 km deep.

4. Pat Driver was caught speeding in a school zone and was called to court. She claims that she could not have been speeding; however, school cameras show her travelling 800 m over a 25 s period. The school speed limit is 20 km/h - is Pat lying?

   Solution

   The question wants us to find Pat's actual speed in km/h, and then compare it to the speed limit of 20 km/h.

   Quantity we are looking for: **Speed**

   Known quantities: Distance (800 m), Time (25 s)

   The units are already standard. $s = d \div t = 800 \div 25 = 32$ m/s.

   Convert this to km/h: 1 m/s = 3.6 km/h $\rightarrow$ 32 m/s = 115.2 km/h. (Holy crap!)

   So Pat was DEFINITELY speeding; she was going 115.2 km/h.

5. Achilles and Hector are having a race. Hector can run at 25 m/s, while Achilles coasts at an easy 30 m/s. If they both start from the starting line (no head starts), how far ahead of Hector is Achilles after 30 seconds?

   Solution

   We want to find how far each has travelled after 30 seconds.

   Achilles travels $30 \times 30 = 900$ m.

   Hector travels $25 \times 30 = 750$ m.

   So after 30 seconds, Achilles is $900 - 750 = 150$ m ahead of Hector.

6. Bonnie and Clyde are having a race. Bonnie gives Clyde a 20 m head start. Bonnie runs at 7 m/s, while Clyde runs at 6 m/s. How long will it be until Bonnie catches up with Clyde? (Hint: first figure out how far Bonnie has to travel to catch Clyde.)

   Solution

   Bonnie catches Clyde when they travel the same distance from the starting line. So we need to set Bonnie's distance EQUAL to Clyde's distance. When Bonnie catches Clyde, we need to realize they travel the same amount of time.

   Let t be the amount of time Bonnie and Clyde travel. Let s_B be Bonnie's speed (7 m/s). Let s_C be Clyde's speed (6 m/s).

   The distance Bonnie travels is $d = s_B \times t = 7 \times t$. The distance Clyde travels is $d = (s_C \times t) + 20 = (6 \times t) + 20$ - remember, Clyde has a head start of 20 m. (This was a tricky question, I admit)

   Set these two equations equal to each other.

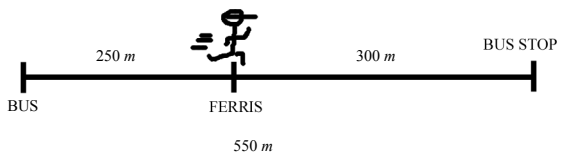
$$\begin{aligned} \text{Bonnie's distance} &= \text{Clyde's distance} \\ (7 \times t) &= (6 \times t) + 20 \\ (7 \times t) - (6 \times t) &= 20 \\ t &= 20 \end{aligned}$$

   Therefore, after 20 seconds, Bonnie catches Clyde.

7. Ferris woke up late... again! He rushes to the bus stop, which is 300 m from his house. As he leaves the front door, he sees that the bus is 550 m away from the bus stop. If the bus is going at 10 m/s, and Ferris can sprint at 30 km/h, can he make it in time?

   Solution

   Here is a picture of the situation.



   Can he make it in time is the same asking: "Who gets there first?".

   Ferris is going 30 km/h - convert this to STANDARD, which becomes 8.1 m/s. His distance is 300 m.

   The bus is going 10 m/s. Its distance is 550 m.

   So Ferris will take $t = d \div s = 300 \div 8.1 = 37.04$ seconds, while the bus will take $t = d \div s = 550 \div 10 = 55$ seconds. So Ferris WILL make it.

8. Wily E. Coyote is chasing the Roadrunner. He runs for 5 minutes at a speed of 7 m/s, at first. Suddenly, the Roadrunner quickens his pace, so Wily E. Coyote straps on some rocket skates and zooms off after him at 27 m/s for another 60 seconds, before running off a cliff. How far did Wily travel before falling off the cliff?

   Solution

   In the first section, before the rocket skates, Wily travelled for 5 minutes at 7 m/s. 5 minutes is 300 seconds, so Wily travelled a distance of $d = 300 \times 7 = 2100$ m.

   After Wily wears the rocket skates, he travels at 27 m/s for 60 seconds, so he goes $d = 27 \times 60 = 1620$ m. In total then, he travels $2100 + 1620 = 3720$ m (or 3.72 km) before falling off the cliff.

9. Two trains are 100 km apart. They leave their stations at the same time, heading towards each other at 100 km/h. How long will it be until they collide?

   Solution

   There were two approaches: common sense, and mathematical.

   Common sense is as follows - both are travelling at the same speed. So the distance they travel when they collide **must have been the same**. If they are 100 km apart, and they travel the same distance when they hit each other, then they must have each travelled 50 km. If they are going 100 km/h, it takes half an hour to travel 50 km. Therefore it will be half an hour.

   The mathematical approach is a little more involved (you need to know algebra), but it gives deeper understanding of similar problems. First, note that 100 km/h = 27 m/s.

   When the two trains collide, the TOTAL distance both travelled is 1 km. So, "Distance Train 1 travels" + "Distance Train 2 travels" = 1 km = 1000 m (think about it!). Also, and this is KEY - when the two trains hit, they travel the SAME amount of time.

Let t represent the amount of time until the trains collide.

Distance Train 1 travels: $d = s \times t = 27 \times t$.

Distance Train 2 travels: $d = s \times t = 27 \times t$.

$$\begin{aligned}
\text{“Distance Train 1”} + \text{“Distance Train 2”} &= 1000\,\text{m} \\
27 \times t + 27 \times t &= 1000\,\text{m} \\
54 \times t &= 1000\,\text{m} \\
t &= 1000 \div 54 \\
t &= 18.52\,\text{s} \qquad \text{(Rounded to 2 decimal places)}
\end{aligned}$$

So this is how long each train travels.

Then the DISTANCE each train travels is $d = s \times t = 27 \times 18.52 = 500$ m (rounded to nearest whole number, since we rounded 18.52).

The problems on this page are more difficult than before. Work together.

10. * In a sprint (not a normal) triathlon, contestants swim, bike, then run. Andrea swims 600 m at $3\,{}^{\mathrm{m}}/_{\mathrm{s}}$, bikes 20 km at $20\,{}^{\mathrm{m}}/_{\mathrm{s}}$, and runs 5400 m at an unknown speed. If the entire sprint triathlon took her 35 minutes, how fast did she run, assuming she maintained constant speed?

    Solution

    In the first section, Andrea swims 600 m at $3\,{}^{\mathrm{m}}/_{\mathrm{s}}$, so it takes her $t = d \div s = 600 \div 3 = 200$ s. In the second section Andrea bikes 20 km = 20 000 m at $20\,{}^{\mathrm{m}}/_{\mathrm{s}}$, so it takes her $t = d \div s = 20000 \div 20 = 1000$ s.

    At this point, she has travelled 1200 s. Her TOTAL time was 35 minutes = 2100 s. So in the running stage, she must have travelled for 900 s. If she ran 5400 m in the running stage, and it took her 900 s, then her speed is $s = d \div t = 5400 \div 900 = 6\,{}^{\mathrm{m}}/_{\mathrm{s}}$.

    Therefore her speed in the running stage was $6\,{}^{\mathrm{m}}/_{\mathrm{s}}$.

11. * Rick and Shane are playing a dangerous game of "chicken." They are driving cars initially 1000 m apart. Both drive forward at the same time; Rick goes forward at $12\,{}^{\mathrm{m}}/_{\mathrm{s}}$, Shane goes forward at $8\,{}^{\mathrm{m}}/_{\mathrm{s}}$. How far does each travel until they are about to collide? How long does this take? Give your answers in standard units.

    Solution

    Let d_R = distance Rick travelled.

    Let s_R = Rick's speed = $12\,{}^{\mathrm{m}}/_{\mathrm{s}}$.

    Let d_S = distance Shane travelled.

    Let s_S = Shane's speed = $8\,{}^{\mathrm{m}}/_{\mathrm{s}}$

    The total distance travelled when they hit is 1000 m, so $d_R + d_S = 1000$. The time travelled for each is the same, so let this be t.

    d_R = distance Rick travelled = Rick's speed $\times$ Rick's time = $s_R \times t$

    d_S = distance Shane travelled = Shane's speed $\times$ Shane's time = $s_S \times t$

    Substitute these into the equation $d_R + d_S = 1000$.

    $$s_R \times t + s_S \times t = 100012 \times t + 8 \times t \qquad = 100020 \times t = 1000t \qquad = 50$$

    So the time it takes for them to hit each other is 50 seconds. We can now substitute $t = 50$ into $d_R = s_R \times t$ and $d_S = s_S \times t$.

    So Rick travels $d_R = 12 \times 50 = 600$ m.

    So Shane travels $d_R = 8 \times 50 = 400$ m.

12. ** A bank robber flees the bank in a car at a constant speed. 10 minutes later, a police patrol car arrives at the bank, and immediately pursues after him, moving at a speed $5\,{}^{\mathrm{m}}/_{\mathrm{s}}$ greater than the bank robber's speed. The police catch up the robber, 20 minutes later. How fast was each car going?

Solution

30 minutes AFTER the robber flees, he is caught by the police. So the robber's time is 30 minutes = 1800 s.

20 minutes AFTER the police arrive, they catch the robber. So the police's time is 20 minutes = 1200 s.

When the police catches the robber, the two of them will have travelled the SAME distance from the bank. Therefore, "Distance Police" = "Distance Robber".

Let s be the speed of the robber. Since the police travel $5\,{}^{\mathrm{m}}/_{\mathrm{s}}$ faster, $s+5$ is the speed of the police car (think about this).

Then "Distance Police" = "Police Speed" × "Police Time" = $(s+15) \times 1200$.

Then "Distance Robber" = "Robber Speed" × "Robber Time" = $s \times 1800$.

Set these two equations equal to each other, since they are equal distance.

$$\begin{aligned}
\text{“Distance Police”} &= \text{“Distance Robber”} \\
(s+5) \times 1200 &= s \times 1800 \\
(s+5) \times 1200 \div 1200 &= s \times 1800 \div 1200 \\
(s+5) &= s \times 1.5 \\
s+5 &= 1.5s \\
s+5-s &= 1.5s - s \\
5 &= 0.5s \\
10 &= s
\end{aligned}$$

So s, the robber's speed, was $10\,{}^{\mathrm{m}}/_{\mathrm{s}}$. So the police car's speed was $s+5 = 10+5 = 15\ {}^{\mathrm{m}}/_{\mathrm{s}}$.

13. ** Two trains are 200 km apart initially, facing each other. There is a fly on one of the trains. At the same time, both trains start towards each other, each going at $45\,{}^{\mathrm{km}}/_{\mathrm{h}}$. While this is happening, the fly is flying back and forth between the trains, turning around every time it reaches one. If the fly is flying at $25\,{}^{\mathrm{km}}/_{\mathrm{h}}$, how far does it travel before the two trains collide?

Solution

The reason this was a challenge problem is because it required clever thinking. It doesn't matter if the fly turns around or not. What really matters is - how long does it take for the trains to hit each other?

The trains are travelling at $45\,{}^{\mathrm{km}}/_{\mathrm{h}}$, or $12.5\,{}^{\mathrm{m}}/_{\mathrm{s}}$. They are 200 km or 200 000 m apart. The time it takes them to hit each other is 8000 s (I won't explain how I got this part since it is very similar to #9's mathematical solution)

So, how long was the fly flying for? 8000 s!

The fly is flying $25\,{}^{\mathrm{km}}/_{\mathrm{h}}$, or $6.94\,{}^{\mathrm{m}}/_{\mathrm{s}}$. So if it flies for 8000 s at $6.94\,{}^{\mathrm{m}}/_{\mathrm{s}}$, the total distance it flew is $d = s \times t = 6.94\,{}^{\mathrm{m}}/_{\mathrm{s}} \times 8000 = 555200$ m, or 555.2 km.

14. *** Tyr, the Norse god of war, and Odin, Norse god of wisdom, want to do some Christmas shopping in the city for the other gods. Both of them leave the castle at the same time.

Tyr rides his pet, Fenrir, the great wolf, to the city, but Fenrir is not allowed in, so Tyr has to leave him at the gates and **walk 15 minutes to the store**. Odin runs the whole way (because he's Odin) to the same store. **Both arrive at the store at the same time.**

If Odin runs at $20\,{}^{\mathrm{m}}/_{\mathrm{s}}$, Fenrir runs at $100\,{}^{\mathrm{km}}/_{\mathrm{h}}$, and Tyr walks at $7\,{}^{\mathrm{m}}/_{\mathrm{s}}$, how far is the castle from the store?

Solution

Let t = Odin's time. Let $s = 20\,{}^{\mathrm{m}}/_{\mathrm{s}}$ = Odin's speed. Let $d = s \times t$ = Odin's distance.

Let t_F = amount of time that Tyr was on Fenrir. Let $s_F = 100\,{}^{\mathrm{km}}/_{\mathrm{h}} = 27\,{}^{\mathrm{m}}/_{\mathrm{s}}$ be Fenrir's speed. Let $d_F = t_F \times s_F = t_F \times 27$ be the distance Tyr travelled on Fenrir.

Let $t_W = 15\,\text{minutes} = 900\,\mathrm{s}$ = amount of time that Tyr was walking. Let $s_W = 7\,{}^{\mathrm{m}}/_{\mathrm{s}}$ be Tyr's walking speed. Let $d_W = s_W \times t_W = 7\,{}^{\mathrm{m}}/_{\mathrm{s}} \times 900 = 6300$ m be how far Tyr walked.

Notice that Tyr's total time is equal to $t_F + t_W$, and that this also equals t, since both of them took the same amount of time (they leave and arrive at the same time).

So $t = t_F + t_W = t_F + 900$. We can rearrange this to get $t_F = t - 900$ (Equation 1).

The distance from the castle to the store is the same for both of them. So set them equal to each other.

$$\begin{aligned}
\text{Odin's Distance} &= \text{Tyr's Distance on Fenrir} + \text{Tyr's Walking Distance} \\
s \times t &= t_F \times 27 + 6300 \\
20 \times t &= (t - 900) \times 27 + 6300 \qquad \text{(Equation 1)} \\
20 \times t &= 27 \times t - 900 \times 27 + 6300 \\
20 \times t &= 27 \times t - 24300 + 6300 \\
20 \times t + 18000 &= 27 \times t - 18000 + 18000 \\
20 \times t + 18000 &= 27 \times t \\
20 \times t + 18000 - 20 \times t &= 27 \times t - 20 \times t \\
18000 &= 7 \times t \\
2571.43 &= t \qquad \text{(2 decimal places)}
\end{aligned}$$

But the distance travelled is Odin's distance $= s \times t = 20 \times 2571.43 = 51428.6$ m. So the store is 51428.6 m away from their castle.