



## Grade 6 Math Circles Rates and Ratios NOVEMBER 15, 2012

### What are Rates and Ratios?



**Definition.** A *ratio* is a **COMPARISON** between two things (e.g. numbers, people, fruit, animals). They are written as

(how much of one thing) : (how much of the other thing)

A **RATE** is a special kind of ratio. It measures how much one quantity or thing changes with respect to another.

### How to use them?

Ratios are used to express proportions - how much we have of something **relative** to something else.

### Example



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## Solution

To approach this problem, we are going to find out the **UNIT COST**

### Unit Cost Method

The key to this method is to use **DIVISION**.

Write the ratio describing the situation. The left number represents the amount to be solved for: **5 : 2**

Divide the left number by the right number:  **$5 \div 2 = 2.5$** .

The answer you just obtained is your **unit cost**. The unit cost tells you how many parts **hydrofluoric** acid you need for every 1 part of **nitric** acid.

Multiply the unit cost by the **actual** amount of nitric acid:

$$2.5 \times 24 = 60.$$

This is your final answer!

Therefore, Walter needs **60** parts hydrofluoric acid.

## Another example...

In a survey of music taste, the ratio of Pink Floyd fans to Justin Bieber fans was 4 : 16 (this is not a realistic ratio, I hope). If there were 256 Justin Bieber fans in total, how many Pink Floyd fans were there?

**Set up ratio:** 4 : 16

**Divide to get unit cost:**  $4 \div 16 = 0.25$

**Multiply unit cost by how many J. B. fans in total:**  $0.25 \times 256 = 64$

**Conclude:** Therefore there are 64 Pink Floyd fans in total.

**Exercises** - Solve these problems using the unit ratio method on another sheet of paper.

1. Walter is back. For his new experiment, he needs 2 parts of water for every 8 parts of salt. If he has 7.5 parts of water, how many parts of salt did he use?

$8 : 2 \rightarrow 8 \div 2 = 4 \rightarrow 4 \times 7.5 = 30$  Therefore he used 30 parts of salt.

2. In another survey of music taste, the ratio of Carly Rae Jepsen fans to Miley Cyrus fans was found to be 5 : 8. If there were 176 Miley Cyrus fans, how many Carly Rae fans were there?

$5 : 8 \rightarrow 5 \div 8 = 0.625 \rightarrow 0.625 \times 176 = 110$  Therefore there were 110 Carly Rae fans.

Now that you've learned these two methods for ratios, rates will be a piece of cake! The only difference is that rates use **PHYSICAL UNITS** of **MEASUREMENT** (km, L and **minutes** are physical units of measurement; people, fruits are not. When money is involved, we count it is a **rate** as well).

### Example

Dexter is sailing on his boat in the middle of the ocean. He notices a small hole in the hull, leaking water. He immediately starts back to shore. For every 2 km, 15 L of water seep in. If Dexter is 15 km from shore, how much water will he take on when he reaches shore?



### Solution

We know that 15 L of water comes in for every 2 km. We are trying to solve for the amount of water after 15 km.

When you set up your ratio, don't forget units: 15 L : 2 km

Divide left number by right:  $15 \div 2 = 7.5$

This tells you how many L of water is taken on per 1 km.

To find how much water he takes on over 15 km, we multiply the unit cost by 15, since he travels 15 km.

Therefore, he will take on  $7.5 \times 15 = 112.5$  L by the time he reaches shore.

## One more example...

1. Johnny earns \$36 for every 3 hours he works in his dad's shop. If he works 45 hours a week, how much does he earn in a week?

$\$36 : 3 \text{ hours} \rightarrow 36 \div 3 = 12 \rightarrow 12 \times 45 = 540$ , since he works 45 hours in total. Therefore he makes \$540 a week.

**Now you try...** (on another sheet of paper)

1. From another of our examples, if the pipes in the basement leak 3 litres of water every 2 weeks, how much is lost over 9 weeks?

$3 : 2 \rightarrow 3 \div 2 = 1.5 \rightarrow 1.5 \times 9 = 13.5$ . Therefore 13.5 L are lost over 9 weeks.

## Extended Examples

1. Evariste and Gauss have challenged each other to a duel - Nerf guns at dawn! In this duel, they stand back to back facing away from each other, and then walk away from each other for 10 seconds. After 10 seconds, they will turn and fire.

Evariste walks at 6 ft/s; Gauss walks at 7 ft/s. How far apart will they be when they fire?

**Setup ratio:** Don't have to - unit cost is given!

Evariste walks 6 ft : 1 s, so the unit cost is  $6 \div 1 = 6$ . In 10 seconds, he walks  $6 \times 10 = 60$  ft. Gauss walks 7 ft/s, so in 10 seconds, he walks  $7 \times 10 = 70$  ft. These are in opposite directions, so in total, they are  $60 + 70 = 130$  ft apart

2. Ferris is late for school - again. As he leaves his house, he sees the bus pass right by him. If the bus is going 17 m/s, and Ferris (a track star) runs at 7.5 m/s, how far apart will they be after 15 s?

**Method 1 - Compare distances**

**How far does Ferris travel?:** Multiply unit cost (7.5 metres every 1 second) by the number of seconds  $7.5 \times 15 = 112.5$  m

**How far does bus travel?:** Multiply unit cost (17 metres every 1 second) by the number of seconds  $17 \times 15 = 255$  m

Therefore, they are  $255 - 112.5 = 142.5$  m apart after 15 s.

**Method 2 - Separation Rate**

**At what rate are they separating?** The bus goes 17 m/s, Ferris goes 7.5 m/s, so the distance between them grows at  $17 - 7.5 = 9.5$  m/s. This is the **separation rate** - the distance between them grows at 9.5 metres a second.

**Multiply separation rate by total time:**  $9.5 \times 15 = 142.5$  m/s.

3. Winter is coming... and driveways need to be shovelled. Ned knows he can finish shovelling a driveway in 4 hours by himself. Jaime knows he can finish shovelling it in 8 hours. Robert can finish shovelling it in 5 hours. If they all work together, how long will it take them to finish shovelling one driveway?

**Key Point - find how much driveway each shovels in one hour**

**Ned:** 1 driveway - 4 hours  $\rightarrow 1 \div 4 = 0.25$  driveways in 1 hour

**Jaime:** 1 driveway - 8 hours  $\rightarrow 1 \div 8 = 0.125$  driveways in 1 hour

**Robert:** 1 driveway - 5 hours  $\rightarrow 1 \div 5 = 0.2$  driveways in 1 hour

Then working together, they finish  $0.25 + 0.125 + 0.2 = 0.575$  driveways in 1 hour.

**How long does it take them to finish 1 driveway?**

*0.575 driveways in 1 hour... How long does it take them to finish 1 driveway?*

**Setup ratio:** 1 hour : 0.575 driveways

**Find unit cost:**  $1 \div 0.575 \approx 1.739 \approx 1.74$ . Therefore, the unit cost is 1.74 hours for 1 driveway.

**Conclude:** It takes them 1.74 hours to finish the driveway, if working together.

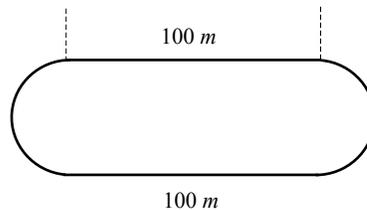
## Problems!

*As before, questions marked with one or more “\*” are considered more challenging and require you to think a little more.*

Solve these rate/ratio problems.

1. Tyson needs 5 cages to hold 12 carrier pigeons. If he has 48 carrier pigeons, how many cages does he need?
2. Muhammad Ali can run 5 miles in 40 minutes. Assuming he doesn't change his pace, how long does it take him to run 15 miles?
3. If the pipes in the basement leak 3 litres of water every 2 weeks, how much is lost over 6 months? Assume each month has 4 weeks.
4. Al and Bugsy are selling jeans. Al offers a deal - 5 jeans for \$130; Bugsy offers 2 jeans for \$50. From a strictly mathematical point of view, who offers the better deal? (Hint: Compare unit cost for one pair of jeans).
5. If 100 soldiers can eat 100 dozen eggs in a week, how long will it take 400 soldiers to eat 400 dozen eggs?
6. Ronny takes 5 weeks to paint 6 houses. Dylan takes 4 weeks to paint 5 houses. Who will finish painting 120 houses first, assuming they maintain the same rate and start at the same time?
7. The ratio of girls to boys in a class is 6 : 4. If the class has 30 students in total, how many girls and how many boys are there?
8. Dale drives east at 120 km/h; Jeff drives west at 150 km/h. If they leave at the same time, how far apart are they after half an hour?

9. Achilles and Hector are in a race. Achilles runs at 15 m/s, Hector at 13 m/s.
- How far apart will they be after 20 seconds?
  - How long does it take them to become 160 m apart?
10. Nancy can plant 50 flowers in 30 minutes. When she works together with Sid, they can plant 125 flowers in an hour. If Sid was working alone, how long would it take him to plant 50 flowers?
11. Six wolves can eat six sheep in six days. How many wolves does it take to eat sixty sheep in sixty days? How many days does it take three wolves to eat three sheep?
12. \* A high school track is 400 m long, with two straight away sections of 100 m and two semi-circle sections of 100 m, like below. Will runs at 7 m/s. Colin runs at 6 m/s.



- If they both start at the same time and maintain the same pace, which section of the track (there are four in total) will they be in after 4 minutes?
13. John, Cam, and Tara are painting the living room. Jon can paint a living room in 8 hours; Cam takes 6 hours; Tara takes 4 hours. How long does it take them to paint the living room if they work together?
14. Dale drives north at 120 km/h; Jeff drives west at 50 km/h. If they leave at the same time, how apart are they after half an hour? (It's not 85 km - hint: draw a picture and look up the Pythagorean Theorem)
15. \* The Three Stooges are having a pie eating contest. In 3 hours, Moe can eat 36 pies, Larry can eat 30, and Curly can eat 60. How many hours does it take them to eat 126 pies?

The following are **optional challenge problems**.

- \*\* One and a half hens can lay one and a half eggs in one and a half days. How many eggs does one hen lay in one day? (Your answer does not have to be a whole number)
- \*\*\* 13 men and 7 boys can finish a job in 7 days, while 6 boys and 13 women can finish the same job in 6 days. In how many days can 1 man, 1 boy and 1 woman working together finish the same job (assume constant rates each for men, women, and boys)?