Math Circles: Combinatorics III

Centre for Education in Mathematics and Computing

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1. Suppose I have 10 identical pennies, 20 identical nickels, 40 identical quarters, and 30 identical loonies. How many ways can I pay someone \( n \) cents? Express your answer in the form of a generating function.

2. Find the generating function for the number of ways to express a positive integer \( n \) as a sum of an odd number of positive integers each of the form \( 3k + 1 \).

3. Find a recurrence equation for the generating function of the previous problem.

4. Find a formula for \( c_n \) where \( c_n \) is given by the recurrence equation

   \[
   c_n - 4c_{n-1} + c_{n-2} + 6c_{n-3} = 0, \quad n > 2,
   \]

   with \( c_0 = 0, c_1 = 1, c_2 = 3 \).