UNIVERSITY OF WATERLOO
University of Waterloo
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

Grade 6 Math Circles
February 29, 2012
2D Geometry

What is Geometry?

Geometry is one of the oldest branches in mathematics. It studies the properties, measurement, and relationship between points, lines, angles, shapes, planes, etc. The word geometry comes from Greek words meaning "Earth measurement."

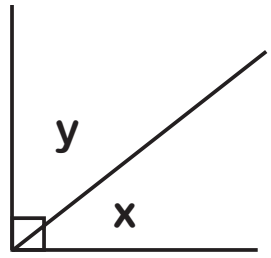
Types of Angles:

Angle Measurement	**Angle Classification**
Less than 90°	Acute Angle
90°	Right Angle
Between 90°and 180°	Obtuse Angle
180°	Straight Angle
Between 180°and 360°	Reflex Angle

Complementary Angles:

Complementary angles are angles that sum to 90°.

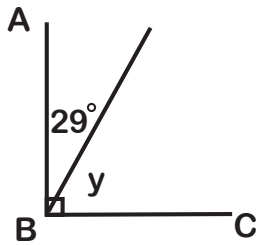
$$\angle x + \angle y = 90^\circ$$



Note: Both angles x and y are acute angles.

Example:

1) If $\angle ABC = 90^\circ$, determine the value of y.

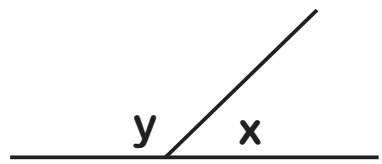


$29^\circ + y^\circ = 90^\circ$ (Complementary Angles)
$y^\circ = 90^\circ - 29^\circ$
$y^\circ = 61^\circ$
Thus, $y = 61$.

Supplementary Angles:

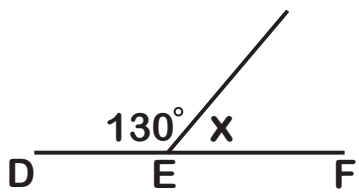
Supplementary angles are angles that sum to 180°.

$\angle x + \angle y = 180^\circ$



Note: Angle x is acute and y is obtuse.

Example:
1) If $\angle DEF = 180^\circ$, determine the value of x.

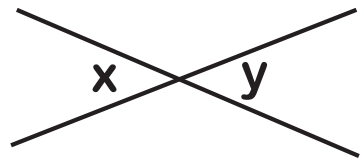


$130^\circ + x^\circ = 180^\circ$ (Supplementary Angles)
$x^\circ = 180^\circ - 130^\circ$
$x^\circ = 50^\circ$
Thus, $x = 50$.

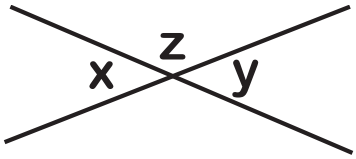
Opposite Angles:

Opposite angles that are formed by intersecting lines are equal.

$\angle x = \angle y$



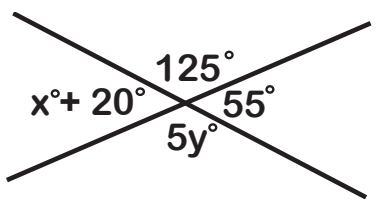
Why is this true?



Let z be the supplementary angle to x. This means that $\angle x + \angle z = 180$. But we can also see that z and y are supplementary angles. So $\angle y + \angle z = 180$.
Then since $\angle x + \angle z = 180$ and $\angle y + \angle z = 180$ we can see that $\angle y + \angle z = \angle x + \angle z$.
This can only be true if $\angle x = \angle y$.

Example:

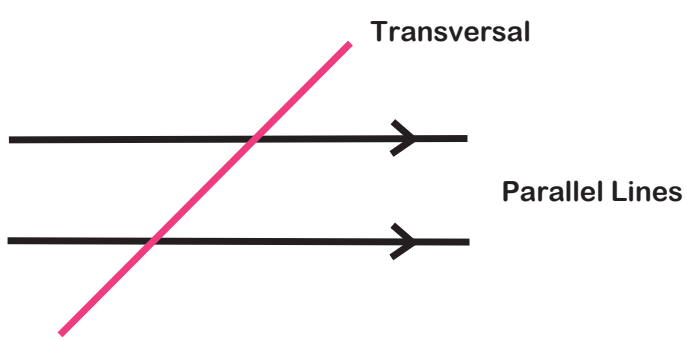
1) Determine the values of x and y.



$x^\circ + 20^\circ = 55^\circ$ (Opposite Angles)
$x^\circ = 55^\circ - 20^\circ$
$x^\circ = 35^\circ$
Thus, $x = 35$.
$5y^\circ = 125^\circ$ (Opposite Angles)
$y^\circ = 125^\circ \div 5^\circ$
$y^\circ = 25^\circ$
Thus, $y = 25$.

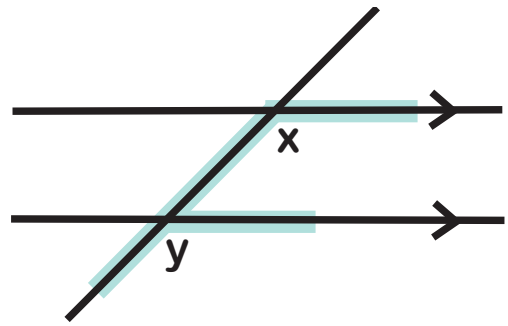
Angle Properties of Parallel Lines and a Transversal:

A **transversal** is a straight line that intersects two or more lines.



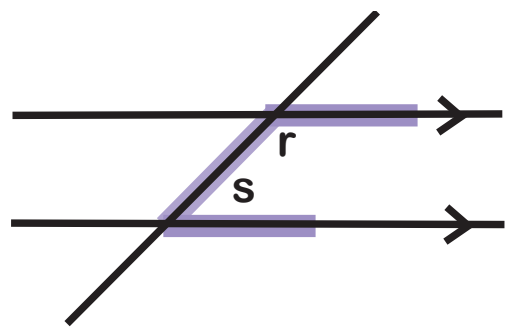
Corresponding Angles are Equal (F-Pattern):

$$\angle x = \angle y$$



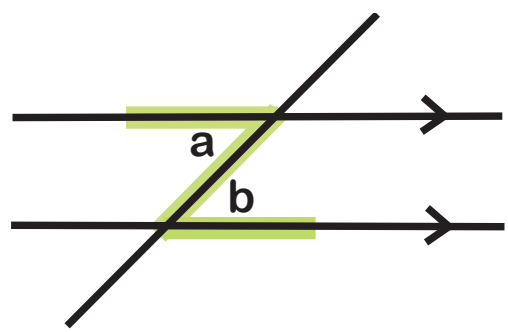
Interior Angles are Supplementary (C-Pattern):

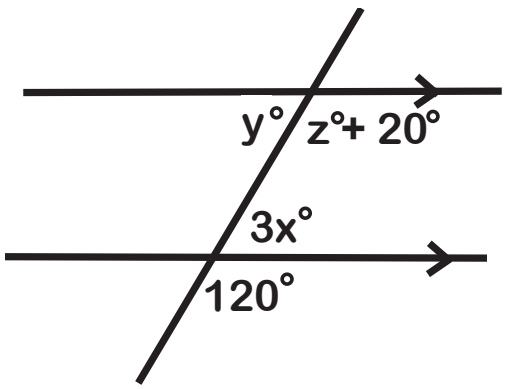
$$\angle r + \angle s = 180^\circ$$



Alternate Angles are Equal (Z-Pattern):

$$\angle a = \angle b$$





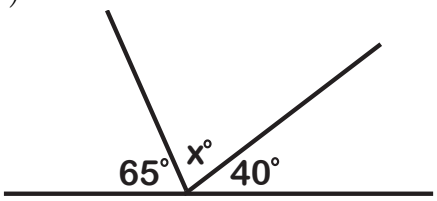
$z^\circ + 20^\circ = 120^\circ$ (F-Pattern)
$z^\circ = 120^\circ - 20^\circ$
$z^\circ = 100^\circ$
Thus, $z = 100$.

$z^\circ + 20^\circ + 3x^\circ = 180^\circ$ (C-Pattern)
$100^\circ + 20^\circ + 3x^\circ = 180^\circ$
$120^\circ + 3x^\circ = 180^\circ$
$3x^\circ = 180^\circ - 120^\circ$
$3x^\circ = 60^\circ$
$x^\circ = 60^\circ \div 3$
$x^\circ = 20^\circ$
Thus, $x = 20$.

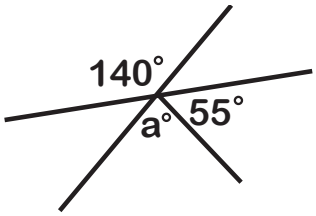
$y^\circ = 3x^\circ$ (Z-Pattern)
$y^\circ = 60^\circ$
Thus, $y = 60$.

Exercises:

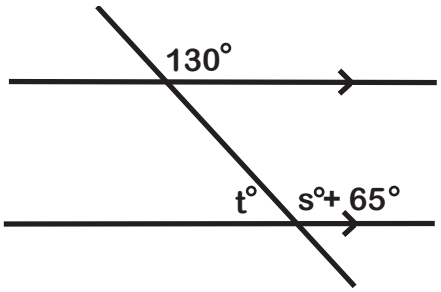
1) Determine the value of x.



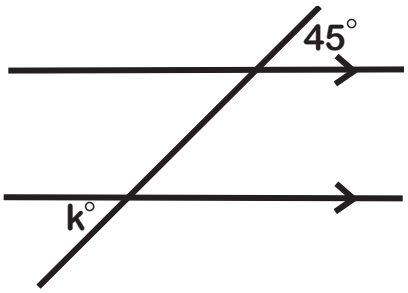
2) Determine the value of a.



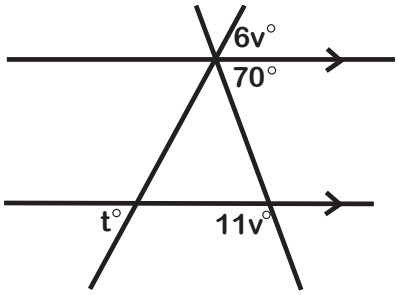
3) Determine the values of s and t.



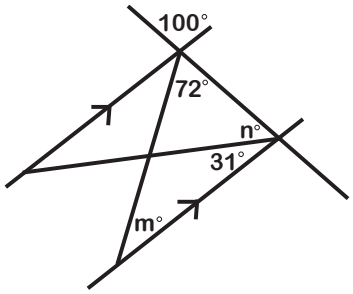
4) Determine the value of k.



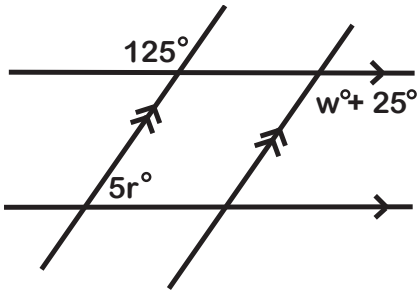
5) Determine the values of t and v.



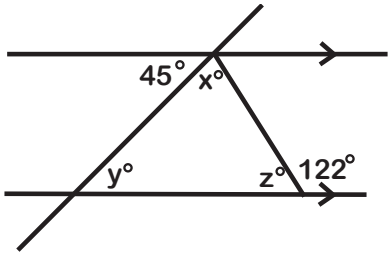
6) Determine the values of m and n.



7) Determine the values of r and w.



8) a) Determine the values of x, y, and z.



b) Determine the value of $\angle x + \angle y + \angle z$.

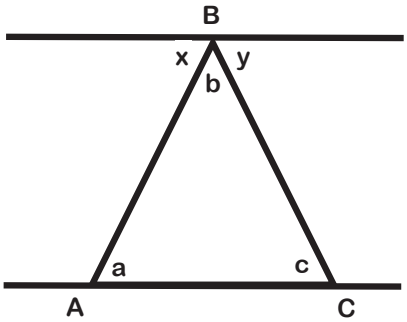
c) What is the sum of the interior angles of a triangle?

<u>Interior Angles of Polygons:</u>

<u>Triangles:</u>

The sum of the interior angles of a triangle is 180°.

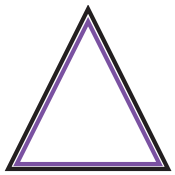
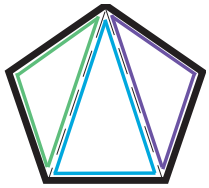
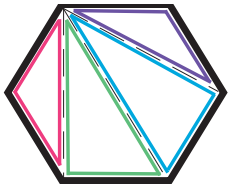
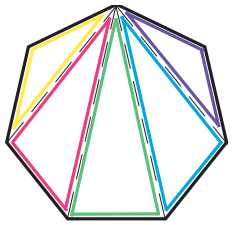
Why is this true?



From triangle ABC, if we draw a line parallel to the base of the triangle we can see that
$\angle x + \angle b + \angle y = 180°$
(since they are supplementary angles).
Then we can see that $\angle x = \angle a$ (from Z-Pattern).
Also, $\angle y = \angle c$ (from Z-Pattern).
The next step is to substitute $\angle a$ in for $\angle x$ and $\angle c$ for $\angle y$ to get:
$\angle a + \angle b + \angle c = 180°$
Thus, the sum of interior angles of a triangle is 180°.

What about other polygons?

To determine the sum of the interior angles of a polygon, draw a diagonal line from one vertex to all the other vertices that it is not already connected to. This divides the polygon into triangles. Since we already know the sum of interior angles of a triangle, we can use that to determine the sum of interior angles of the polygon. Multiply the number of triangles that you have divided your polygon into by 180°to get the sum of interior angles of the polygon.

Polygon	Picture	Number of Sides	Number of Angles	Number of Vertices	Sum of Interior Angles
Triangle		3	3	3	180°
Quadrilateral		4	4	4	360°
Pentagon		5	5	5	540°
Hexagon		6	6	6	720°
Heptagon		7	7	7	900°

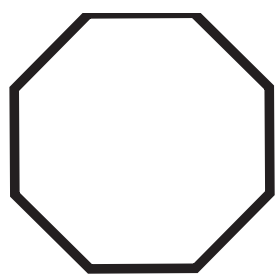
Looking at the chart above, do you notice a pattern?

The sum of interior angles increases by 180°as the number of sides increases by one. Notice that the number of triangles your polygon is divided into is two less than the number of sides.

The general formula for the sum of interior angles of an n-sided polygon is:
sum of interior angles $= (n-2)\times 180°$.

Examples:

1) a) Determine the sum of the interior angles of a regular octagon.
b) What is the measure of each interior angle?



a) Using the formula above, the sum of interior angles $= (n-2)\times 180°$.
Since an octagon has 8 sides, n=8.
Thus, the sum of interior angles $= (8-2)\times 180°$
$= 6\times 180°$
$= 1080°$.

b) Since the octagon is a regular octagon, all of the interior angles are the same size.
Let's call each angle x.
Then, $x° + x° + x° + x° + x° + x° + x° + x° = 1080°$
$8x° = 1080°$
$x° = 1080° \div 8$
$x° = 135°$
Therefore, the measure of each interior angle of a regular octagon is 135°.

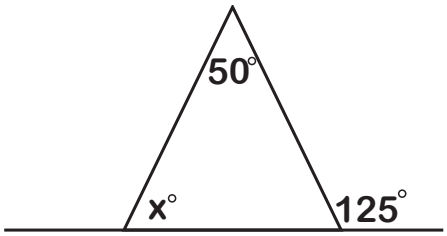
Exercises:

1) For each of the regular polygons in the chart above, determine the measure of each interior angle.

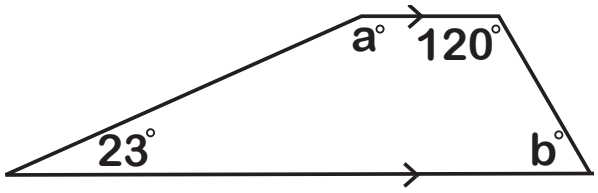
2) a) Determine the measure of each interior angle of a regular decagon.
   b) Using your protractor draw a regular decagon with side lengths of 3cm.

3) Determine the values of the variables in the diagrams below.

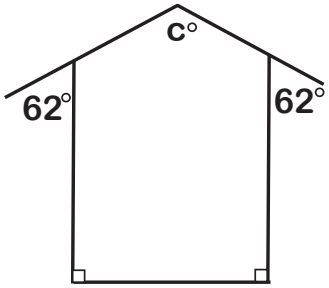
a)



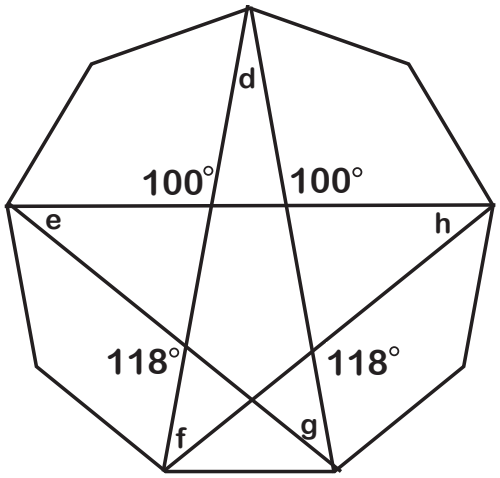
b)



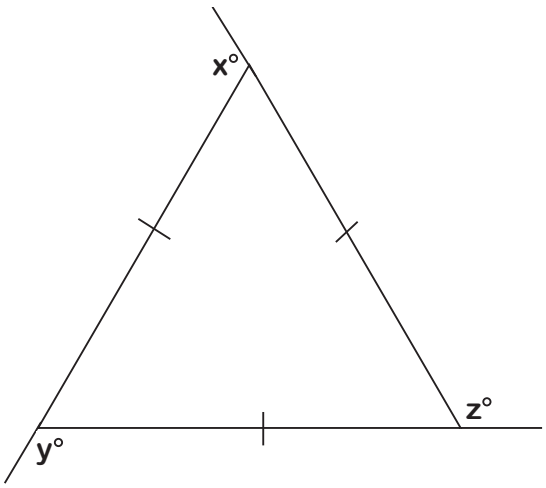
c)



d)

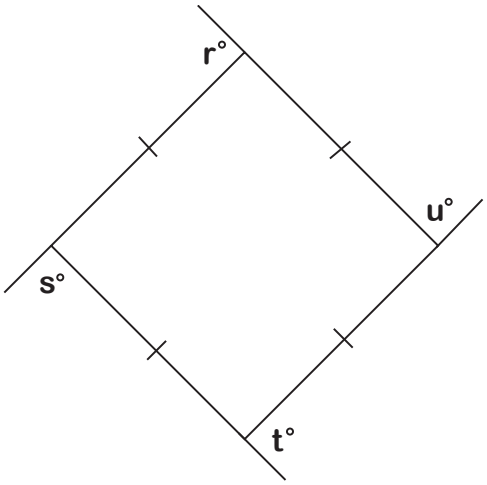


4)a)

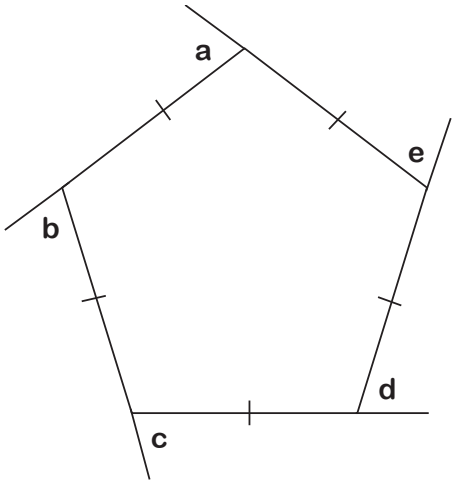


i) Determine the values of the variables in the diagram above.
ii) Determine the value of $\angle x + \angle y + \angle z$.

b)



i) Determine the values of the variables in the diagram above.
ii) Determine the value of $\angle r + \angle s + \angle t + \angle u$.



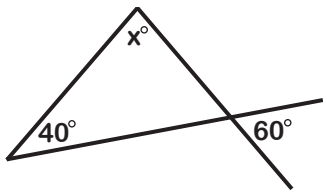
c) i) Determine the values of the variables in the diagram to the left.
ii) Determine the value of $\angle a + \angle b + \angle c + \angle d + \angle e$.

d) i) What do you notice about the sum of the exterior angles of the polygons above?
ii) Is this true for all polygons?

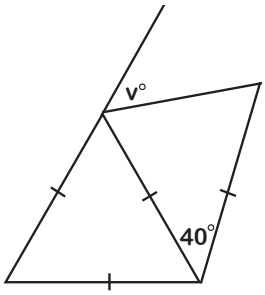
Extra Problems:

Determine the values of the variables in each of the diagrams below.

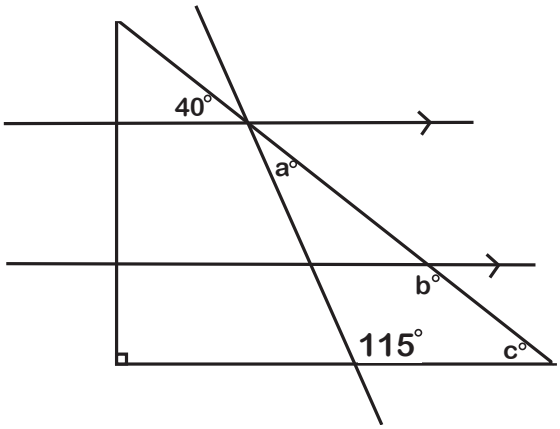
1)



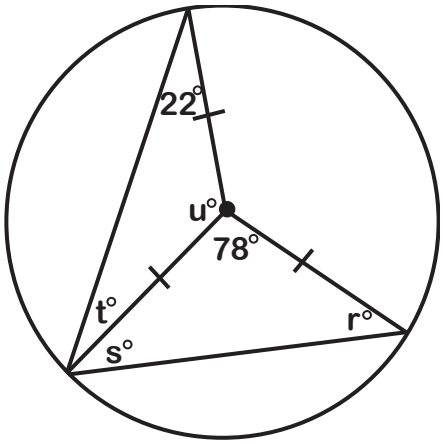
2)



3)



4)



5)