**Grade 6 Math Circles**  
**February 22, 2012**  
**Number Theory I**

**Greatest Common Factor Solutions:**

1) Determine the Greatest Common Factor for each of the following.
   a) $\text{GCF}(25,90)=5$  
   b) $\text{GCF}(64,82)=2$  
   c) $\text{GCF}(84,138)=6$  
   d) $\text{GCF}(342,243)=9$  
   e) $\text{GCF}(350,475)=25$  
   f) $\text{GCF}(1072,1112)=8$  
   g) $\text{GCF}(453,354)=3$  
   h) $\text{GCF}(121,212)=1$  

2) Determine the GCF for each of the following.
   a) $\text{GCF}(35,49,98)=7$  
   b) $\text{GCF}(27,57,78,96)=3$  
   c) $\text{GCF}(1212,2121)=303$  

3) $\text{GCF}(24,42)=6$. Therefore, Flo can create 6 bouquets with 4 tulips and 7 roses in each.

4) $\text{GCF}(84,108,128)=4$. So, Miss King will have 4 of each type of cupcake per box. She can create 21 boxes and will have 24 red velvet, and 44 vanilla cupcakes left over.

5) $\text{GCF}(18,24)=6$. The perimeter of the garden measures 84m. Thus, the minimum number of flowers that Poppy can plant around the perimeter of the garden is $84 \div 6 = 14$.

6) $\text{GCF}(48,72,84)=12$. So, Sally can create 12 loot bags with 6 candy bars, 7 lollipops, and 4 gum balls in each.
7) GCF(64,96)=32.

\[
\text{Area of floor} = 64 \times 96 = 6144\text{cm}^2 \\
\text{Area of each tile} = 32 \times 32 = 1024\text{cm}^2
\]

Thus, the minimum number of tiles needed to cover the floor is \(6144 \div 1024 = 6\) tiles.

8) GCF(14,42,21)=7. Thus, the dimensions of the cubes are \(7 \times 7 \times 7\text{cm}\) and there will be 36 pieces of cake.

**Least Common Multiple**

1) Determine the Least Common Multiple for each of the following.
   a) LCM(25,45)=225  
   b) LCM(20,50)=100  
   c) LCM(32,42)=672  
   d) LCM(16,56)=112  
   e) LCM(42,120)=840  
   f) LCM(80,115)=1840  
   g) LCM(165,300)=3300  
   h) LCM(180,1440)=14407

2) Determine the LCM for each of the following.
   a) LCM(35,49,98)=490  
   b) LCM(27,57,78,96)=213408  
   c) LCM(1212,2121)=8484

3) LCM(8,12)=24. Jim should buy 2 packs of burgers and 3 packs of buns in order to have no hamburgers or buns left over.

4) LCM(18,24,16)=144. Thus, the dimensions of the smallest cube shed that Ned could build is \(144\text{cm} \times 144\text{cm} \times 144\text{cm}\), and the volume is \(2985984\text{cm}^3\).

5) LCM(120,180,360)=360. Yes, they will have a break together at 3pm.

6) Complete the following table.

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>1st Number</td>
<td>2nd Number</td>
<td>GCF of A and B</td>
<td>LCM of A and B</td>
<td>A \times B</td>
<td>E \div C</td>
<td></td>
</tr>
<tr>
<td>136</td>
<td>420</td>
<td>4</td>
<td>14280</td>
<td>57120</td>
<td>14280</td>
<td></td>
</tr>
<tr>
<td>275</td>
<td>350</td>
<td>25</td>
<td>3850</td>
<td>96250</td>
<td>3850</td>
<td></td>
</tr>
<tr>
<td>228</td>
<td>462</td>
<td>6</td>
<td>17556</td>
<td>105336</td>
<td>17556</td>
<td></td>
</tr>
</tbody>
</table>
a) What do you notice about the relationship between columns D and column F? They’re the same!
b) Determine an equation to describe the relationship between column D and column F.
\[ \text{LCM}(A,B) = \frac{A \times B}{\text{GCF}(A,B)} \]

7) \[
\text{LCM}(76,42) = \frac{76 \times 72}{\text{GCF}(76,42)} = \frac{3192}{2} = 1596
\]

8) Let A represent the number that we know (140). Then, B represents the number we are trying to find.

\[ \text{LCM}(A,B) = \frac{A \times B}{\text{GCF}(A,B)} \quad \text{We need to re-arrange the equation for } B \]

\[
B = \frac{\text{LCM}(A,B) \times \text{GCF}(A,B)}{A} = \frac{9660 \times 4}{140} = 276
\]

**Extra Problems**

1) a) When \( a \) is a factor of \( b \), or \( b \) is a factor of \( a \).
   b) When \( a \) is a multiple of \( b \), or \( b \) is a multiple of \( a \).
   c) When the \( \text{GCF}(a,b)=1 \) (when \( a \) and \( b \) are co-prime).

2) The numbers are: 11, 13, 17, 31, 37, 71, 73, 79, 97.

3) The ages of the 8 students in the class are 13, 13, 15, 15, 16, 17, 17, and 19.